MATHEMATICS

For Class VII

L7W5Z1

ਸਮੱਗਰ ਸਿੱਖਿਆ ਅਭਿਆਨ

ਪੜ੍ਹੋ ਸਾਰੇ ਵਧੋ ਸਾਰੇ

ਸਿੱਖਿਆ ਅਤੇ ਭਲਾਈ ਵਿਭਾਗ, ਪੰਜਾਬ ਦਾ ਸਾਂਝਾ ਉਪਰਾਲਾ

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FOREWORD

The Punjab School Education Board has been continuously engaged in developing syllabi, producing and renewing text books according to the changing educational needs at the state and national level.

This book has been developed in accordance to the guidelines of National Curriculum Framework (NCF) 2005 and PCF 2013, after careful deliberations in workshops involving experienced teachers and experts from the board and field as well. All efforts have been made to make this book interesting with the help of activities and coloured figures. This book has been prepared with the joint efforts of subject experts of Board, SCERT and experienced teachers/experts of mathematics. Board is thankful to all of them.

The authors have tried their best to ensure that the treatment, presentation and style of the book in hand are in accordance with the mental level of the students of class VII. The topics, contents and examples in the book have been framed in accordance with the situations existing in the young learner's environment. A number of activities have been suggested in every lesson. These may be modified, keeping in view the availability of local resources and real life situations of the learners.

I hope the students will find this book very useful and interesting. The Board will be grateful for suggestions from the field for further improvement of the book.

Chairman

Punjab School Education Board

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CONTENTS

CHAPTER 1

F8T3H5

Integers

Learning Objectives :-

In this chapter you will learn :-

1. To define integers.
2. Addition, subtraction, multiplication and division of integers.
3. To understand and investigate the properties related to various operations on integers.
4. To realize the importance and use of integers in your day to day life.
5. To use number lines to represent integers.

OUR NATION'S PRIDE

Brahmagupta : Brahmagupta was the Indian Mathematician who defined zero and set the rules for its computation, which further led to make the mathematical problems real and easily solvable. Not only this, there are numerous areas in the field of mathematics where the contribution of Indian mathematician has been immense. It includes the discovery of zero, the rules of working with negative numbers and most importantly a system of expressing all the numbers using only ten symbols.

INTRODUCTION

As we know that the integers are the numbers used for counting forward as well as backward. The use of integers in our real life makes them important in mathematics too. They help in computing the efficiency in positions, to calculate how more or less measures to be taken for achieving better results and many more.

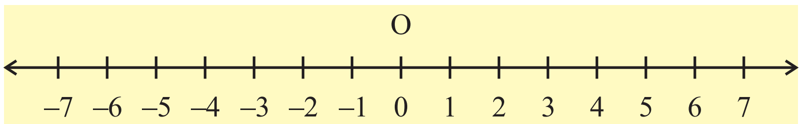
We have studied numbers used for counting called Natural numbers (N = 1, 2, 3, 4.........). All natural numbers along with 0 (Zero) are called whole numbers (W = 0, 1, 2, 3, 4..........) But these numbers cannot help us solve all our daily life problems. Therefore we shall study integers, which is a collection of whole numbers and the negatives of natural numbers.

.... –4, –3, –2, –1, 0, 1, 2, 3, 4..............

- 1, 2, 3, 4............. are positive integers.
- –1, –2, –3, –4............ are negative integers
- 0 (Zero) is an integer which is neither positive nor negative

REPRESENTATION OF INTEGERS ON A NUMBER LINE

Draw a line. Mark a point on the line. Label it as O. Now mark some more points at equal distances on the right as well as the left of 0. Label the point on the right side of 0 as 1, 2, 3, 4 .... and label the points on the left side as –1, –2, –3, –4.............. as shown below :



The arrowheads on both the sides of the number line indicate the continuation of integers infinitely on each side.

ABSOLUTE VALUE OF AN INTEGER

The absolute value of an integer '*a*' is the number value of '*a*' regardless of its sign. It is denoted by $|a|$, called the modulus of a

for example (*i*) $|5| = 5$ and $|-5| = 5$ (*ii*) $|-3| = 3$ and $|3| = 3$

Example-1 : Write all integers (*i*) between –5 and 5 (*ii*) between –20 and –13

Sol : (*i*) All integers between –5 and 5 are

–4, –3, –2, –1, 0, 1, 2, 3, 4

(*ii*) All integers between –20 and –13 are

–19, –18, –17, –16, –15, –14

Example-2 : Compare the integers

(*i*) –7 and 0 (*ii*) –5 and –13 (*iii*) –193 and –128 (*iv*) –26 and 23

Sol : (*i*) We know that every negative integer is less than 0

$\therefore \quad -7 < 0$

(*ii*) –5 lies to the right of –13 on number line.

$\therefore \quad -5 > -13$

(*iii*) –193 lies to the left of –128 on a number line

$\therefore \quad -193 < -128$

(*iv*) We know that every negative integer is less than every positive integer

$\therefore \quad -26 < 23$

Example-3 : Evaluate (*i*) $17 - |-12|$ (*ii*) $|-21|-|9|$ (*iii*) $|27-18| + |-9|$

Sol : We have

(*i*) $17 - |-12| = 17 - 12 = 5$ $\quad [\because |-12| = 12]$

(*ii*) $|-21|-|9| = 21 - 9 = 12$ $\quad [\because |-21| = 21$ and $|9| = 9]$

(*iii*) $|27 - 18| + |-9| = 9 + 9 = 18$ $\quad [\because |27-18| = |9| = 9$ and $|-9| = 9$

Example-4 : Arrange the following integers in ascending order.

135, –87, –9, 87, –23, 263, –172, 18

Sol : Given positive integers are 135, 87, 263, 18

In ascending order 18 < 87 < 135 < 263

Given negative integers are –87, –9, –23, –172

In ascending order –172 <–87 < –23 < –9

Hence all given integers in ascending order are

–172 < –87 < –23 < –9 < 18 < 87 < 135 < 263

i.e. –172, –87, –23, –9, 18, 87, 135, 263

EXERCISE - 1.1

1. Use the appropriate symbol >, < = to fill in the blanks

(*i*) –3 ☐ –5 (*ii*) –2 ☐ 5–4

(*iii*) 8–4 ☐ –3 (*iv*) –6 ☐ 5–0

(*v*) 5 ☐ 8–3 (*vi*) 0 ☐ –3

2. Arrange the following integers in ascending order.

(*i*) –2, 12, –43, 31, 7, –35, –10

(*ii*) –20, 13, 4, 0, –5, 5

3. Arrange the following integers in descending order.

(*i*) 0, –7, 19, –23, –3, 8, 46

(*ii*) 30, –2, 0, –6, –20, 8

4. Evalute :–

(*i*) 30 – |–21| (*ii*) |–25|–|–18| (*iii*) 6 – |–4| (*iv*) | –125| + |110|

5. Fill in the blanks :-

(*i*) 0 is greater than every .... integer.

(*ii*) Modulus of a negative integer is always ....

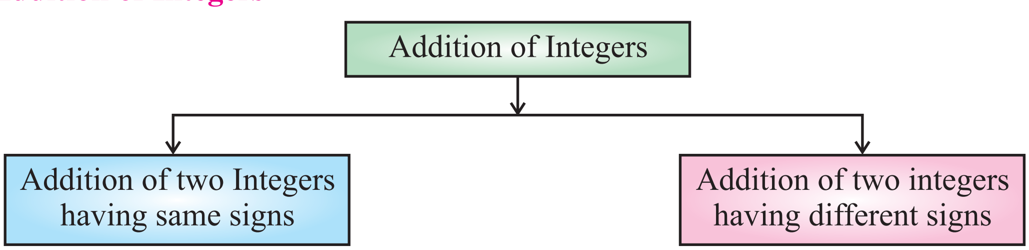
(*iii*) The smallest postive integer is ....

(*iv*) The largest negative integer is ....

(*v*) Every negative integer is less than every .... integer.

FOUR FUNDAMENTAL OPERATIONS

(*i*) Addition of Integers



1. Addition of two integers having same signs :

Step 1 : Add the values regardless of their sign.

Step 2 : Write the sum with sign of both the integers.

Example 1. Solve 10 + 23

Solution 10 + 23

= 33

Example 2. Solve 70 + 18

Solution 70 + 18

= 88

Both the numbers have same sign i.e. (+, +)

Example 3. Solve (–50) + (–32)

Solution (–50) + (–32)

= – 82

Example 4. Solve (–42) + (–60)

Solution (–42) + (–60)

= – 102

Both the numbers have same signs i.e. (–, –)

2. Addition of two integers having different signs :

Step 1 : Find the difference between the values regardless of their sign.

Step 2 : Write the difference with the sign of the integer having greater value.

Example 5. Solve (–17) + 35

Solution (–17) + 35

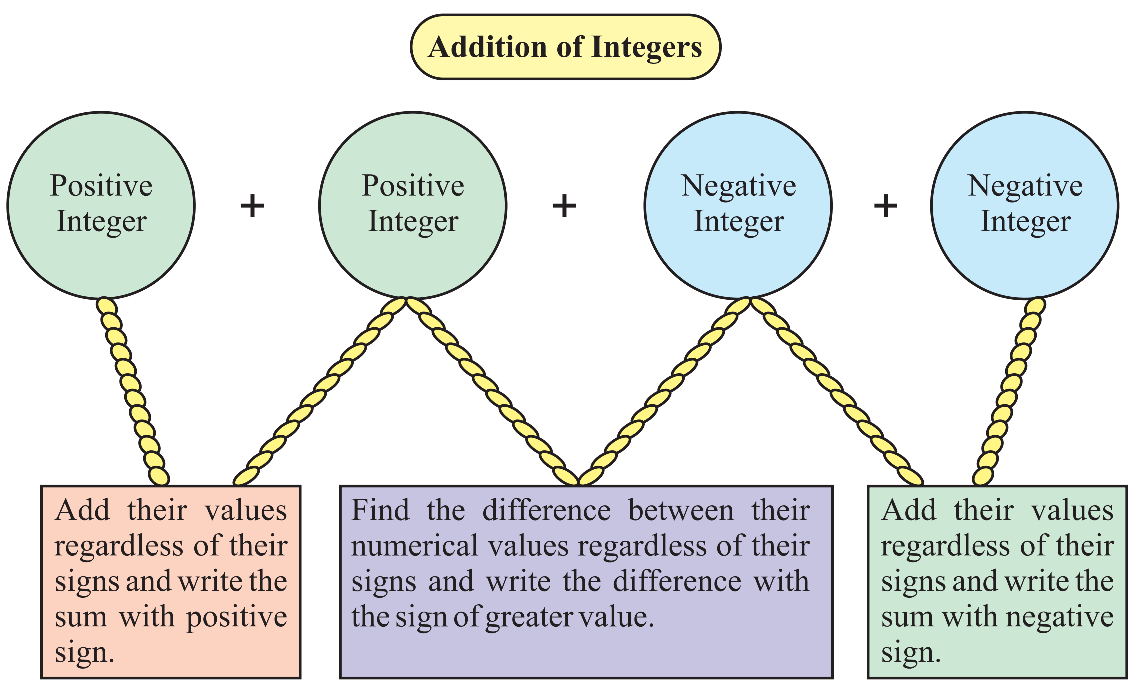
= 18

Example 6. Solve (–63) + 27

Solution (–63) + 27

= –36

The numbers are of opposite signs i.e. (+, –) or (–, +)



PROPORTIES OF ADDITION OF INTEGERS

1. **Closure property :** The sum of two integers is also an integer. *i.e* if a and b are integers then $a + b$ is also an integer. For example 2 + (–4) = –2, (–3) + (7) = 4, 8 + 5 = 13
2. **Commutative property :** For all integers a and b

$$a + b = b + a$$

For example 5 + 8 = 8 + 5 = 13
3. **Associative property :** For all integers a, b and c

$$a + (b + c) = (a + b) + c$$

For example (–2) + (5 + 9) | [(–2) + 5] + 9
= (–2) + (14) | = (3) + 9
= 12 | = 12
∴ (–2) + (5 + 9) = [(–2) + 5] + 9
4. **Additive identity :** The integer '0' is such that $a + 0 = 0 + a = a$ '0' is called the additive identity of integers.
5. **Additive Inverse :** For any integer a, we have $(-a) + a = 0 = a + (-a)$
The negative of an integer a is $(-a)$ and the sum of an integer and its negative is '0'
∴ Additive inverse of a is $(-a)$
Similarly additive inverse of $(-a)$ is $-(-a) = a$

Example-7 : In a quiz, Manjeet Singh scored 65, –30, 25, where as Ramandeep scored –30, 65, 25 in three rounds. Find who scored better ? What conclusion do you draw ?

Sol. Manjeet Singh's score = [(65 + (–30)] + 25
= 35 + 25
= 60
Ramandeep's score = (–30) + (65 + 25)
= –30 + 90
= 60

We see that both Manjeet and Ramandeep scored same. We conclude that addition of integers is associative.

Subtraction of Integers : Subtracting an integer from another integer is same as adding first integer with additive inverse of second integer. In other words, if a and b are two integers then $a - (+b) = a + (-b)$.

Example-8 : Solve 15 – (–8)

Sol. We have

15 – (–8) = 15 + 8
= 23
∴ 15– (–8) = 23

Example-9 : Solve (–3) – (+ 21)

Sol. We have

(–3) – (+ 21) = (–3) + (– 21)
= – 24

PROPERTIES OF SUBTRACTION OF INTEGERS :

1. Closure property : The difference of two integers is an integer i.e. if a and b are any two integers then $(a - b)$ is always an integer.

For example $-3 - 2 = -5$, $7 - (-4) = 11$

2. Subtraction of integers is not commutative

For Example	$(5 - 8)$		$(8 - 5)$
	$= -3$		3
$\therefore$	$5 - 8$	$\neq$	$8 - 5$

3. Subtraction of integers is not associative :

For example	$[7 - (-2)] - 1$		$7 - [(-2) - 1]$
	$= [7 + 2] - 1$		$= 7 - [-3]$
	$= 9 - 1$		$= 7 + 3$
	$= 8$		$= 10$
$\therefore$	$[7 - (-2)] - 1$	$\neq$	$7 - [(-2) - 1]$

4. For every integer a, $a - 0 = a \neq 0 - a$

Example-10 : Solve (–7) + (8) – (3)

Sol. $(-7) + (8) - (3)$

$= (-7) + (8) - (3)$

$= (-7) + 8 + (-3)$

$= -10 + 8$

$= -2$

[Add all positive numbers together and add all negative numbers seperately together]

$[(-7) + (-3) = -10]$

Example-11 : Solve 15 – (– 5) + 12 + (– 8) – (– 3)

Sol. $15 - (-5) + 12 + (-8) - (-3)$

$= 15 + (+5) + 12 + (-8) + (+3)$

$= 15 + 5 + 12 + 3 + (-8)$

$= 35 - 8$

$= 27$

[Add all positive numbers together and add all negative numbers seperately together]

$[15 + 5 + 12 + 3 = 35]$

Example-12 : The difference between two integers is – 7. If second integer is 23, then find the first integer

Sol.

Difference $= -7$

2^{nd} integer $= 23$

1^{st} integer $=$ Difference $+ 2^{nd}$ integer

$= -7 + 23 = 16$

EXERCISE - 1.2

1. Find the value of

(*a*) 32 + 15
(*b*) 17 + (–18)
(*c*) (–25) + (21)
(*d*) (–8) + (–11)
(*e*) (–13) + (21)
(*f*) (–19) + (0)
(*g*) (–85) – (–10)
(*h*) (15) – (6)
(*i*) (45) – (–27)
(*j*) (–62) – (52)

2. Solve the following

(*a*) (–3) + 7 + (–8)
(*b*) (–2) – (–1) – (4)
(*c*) 8 + (–7) – (–6)
(*d*) (–12) – (–17) + (–25)

3. Find the value of

(*a*) 15 – (–5) + 12 + (– 8) + (–3)
(*b*) (–32) – (–11) + (–25) + 27 – 13 + (–7)
(*c*) 160 + (–150) + (–130) – (–100)
(*d*) 25 – (–15) + (–12) + 21 – 65 – (–38)

4. Fill in the blanks using properties of adddition and subtraction of integers.

(*i*) 10 + [(–5) + (–7)] = [(10 + (–5)] + ☐

(*ii*) 25 – 10 = – 10 + ☐

(*iii*) 20 + ☐ = 15 + ☐

(*iv*) (–12) + 37 = 37 + ☐

(*v*) 13 + [☐ –2] = [13 + (–7)] + ☐

(*vi*) –17 + ☐ = – 17

5. The difference between two integers is –10. If first integer is 17, then find the other integer ?

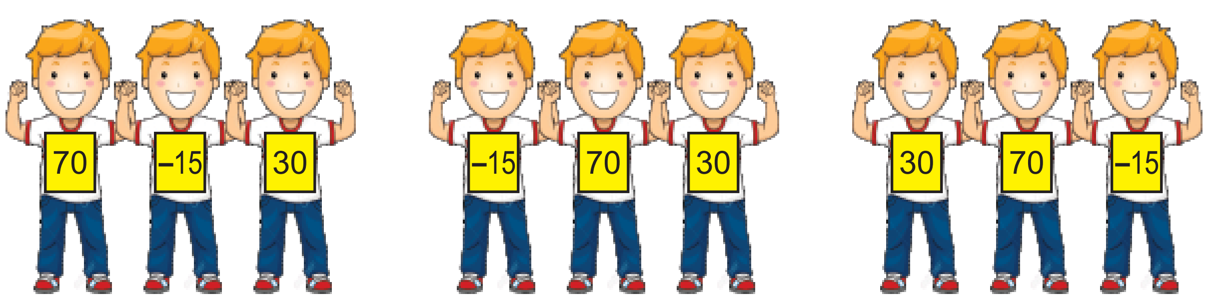
6. Write three consecutive odd integers succeeding (–93) ?

7. At sunrise, the outside temperature was 7° below zero. In the afternoon the temperature rose by 13° and then fell by 8° at night. What was the temperature at the end of the day ?

8. Manjeet Singh has a bank balance of ₹ – 430 at the start of the month ? What was the bank balance, after he deposited ₹ 250 ?

9. Mount Everest, the highest elevation in Asia, is 29028 feet above the sea level. The Dead Sea is 1312 feet below the sea level. What is the difference between these two elevations ?

10. In a quiz, Team A scored 70, – 15, 30. Team B scored –15, 70, 30 and team C score 30,70, –15. Which team scored better ? What conclusion do you draw ?



Team A Team B Team C

11. In a competition there are 5 Teams and three rounds. The scores of all the teams are given below in the table. Complete the table and find, the teams at 1st, IInd and IIIrd positions.

Round \ Teams	A	B	C	D	E
Round 1	7	–9	8	7	–6
Round 2	–3	5	–2	0	7
Total after two round					
Round 3	–2	–5	–3	–5	4
Round 4	6	7	4	3	–2
Final Score					

12. **Multiple choice questions :-**

(i) (–5) + (5) =

(*a*) –10 (*b*) 5

(*c*) 10 (*d*) 0

(*ii*) (–10) + (–12) =

(*a*) –2 (*b*) 22

(*c*) –22 (*d*) 2

(*iii*) (–1) – (–1) =

(*a*) –2 (*b*) –1

(*c*) 2 (*d*) None of these

(*iv*) Which of the following statements is incorrect ?

(*a*) Sum of two integers is also an integer.

(*b*) For all integers a and b, $a + b = b + a$.

(*c*) Difference of two integers is also an integer.

(*d*) Subtraction of integers is commutative.

(*v*) Which of the following is correct ?

(*a*) (–7) – (3) = 3 – (– 7)

(*b*) (–7) + 3 = 3 + (–7)

(*c*) (–1) + [(5) + (–3)] = [(–1) + (5)] – (–3)

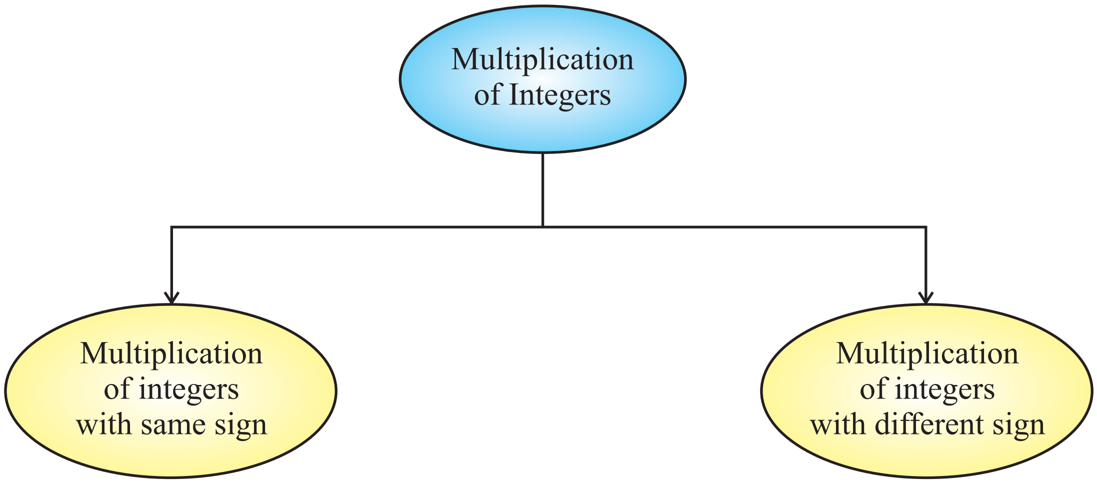
(*d*) None of these

MULTIPLICATION OF INTEGERS

Multiplication is Repeated addition : Let a and b are positive Integers then $a \times b$ is defined as addition of a to b times or addition of b to a times

For example $4 \times 3 = 4 + 4 + 4 = 12$ or $3 \times 4 = 3 + 3 + 3 + 3 = 12$

Multiplication of two integers :-



1. Multiplication of integers with same sign :-

Step 1 : Multiply the numbers regardless of their sign.

Step 2 : Write the product with a positive sign.

Example-1 : Find the product of 18 and 12.

Sol : Multiply the given Integer 18 and 12 we get

$$18 \times 12 = 216$$

Example-2 : Find the product of (–50) and (–8).

Sol : Multiply the given Integer –50 and –8 we get

$$-50 \times -8 = 400$$

2. Multiplication of integers with different sign :-

Step 1 : Multiply the numbers regardless of their sign.

Step 2 : Write the product with a negative sign.

Example-3 : Find product of 15 and –12.

Sol : Multiply given Integers 15 and –12 we get

$$\begin{aligned} 15 \times -12 &= -(15 \times 12) \\ &= -180 \end{aligned}$$

PRODUCT OF THREE OR MORE NEGATIVE INTEGERS

To find the product of three or more negative integers we can simply take two integers at a time and follow the rules as for multiplication of two integers.

$$\begin{aligned} (-a) \times (-b) \times (-c) &= [(-a) \times (-b)] \times (-c) \\ &= (a \times b) \times (-c) \\ &= -(a \times b \times c) \end{aligned}$$

Example-4 : Find the product of (–5) × (–4) × (–3)

Sol : $(-5) \times (-4) \times (-3)$

$$= (-5 \times -4) \times (-3)$$
$$= (20) \times (-3)$$
$$= -(20 \times 3)$$
$$= -60$$

PROPERTIES OF MULTIPLICATION OF INTEGERS

1. **Closure property :** If a and b are two integers then $a \times b$ is also an integer

   For example –5 and 8 are integers then $-5 \times 8 = -40$ is also an integer

2. **Commutative property :** If a and b are two integers. Then $a \times b$ is same as $b \times a$ i.e., $a \times b = b \times a$

   For example $2 \times 4 = 4 \times 2 = 8$

3. **Associative property for multiplication :** If a, b and c are three integers then

   $$a \times (b \times c) = (a \times b) \times c \quad \text{or } (a \times b) \times c = (a \times c) \times b$$

   For example $7 \times (6 \times 8) = (7 \times 6) \times 8 = 336$

4. **Distributive property :**

   (a) Distributive property of multiplication over addition. If a, b and c are three integers then

   $$a \times (b + c) = (a \times b) + (a \times c)$$

   For example
   $$10 \times (5 + 2) = (10 \times 5) + (10 \times 2)$$
   $$= 50 + 20$$
   $$= 70$$

   (b) Distributive property of multiplication over subtraction If a, b and c are three integers then

   $$a \times (b - c) = (a \times b) - (a \times c)$$

   For example
   $$6 \times (7 - 4) = (6 \times 7) - (6 \times 4)$$
   $$= 42 - 24$$
   $$= 18$$

5. **Multiplicative property of zero**

   For any Integer a we have

   $$a \times 0 = 0 \times a = 0$$

   For example $7 \times 0 = 0 \times 7 = 0$

6. **Multiplicative Identity**

   For any integer a we have

   $$a \times 1 = 1 \times a = a$$

   For example $8 \times 1 = 1 \times 8 = 8$

   For easier multiplication we can use commutative associative and distributive properties of integers

   For example
   $$50 \times 8 + 50 \times -2 = 50 \times (8 - 2)$$
   $$= 50 \times 6$$
   $$= 300$$

Example-1 : Find each of the following products

(*i*) (–15) × (–2) × (–5) × (6) **(*ii*) (–8) × (–5) × (–6) × (–1)**

Sol : (*i*) $(-15) \times (-2) \times (-5) \times (6) = [(-15) \times (-2)] \times [(-5) \times (6)]$

$= 30 \times (-30)$

$= -900$

(*ii*) $(-8) \times (-5) \times (-6) \times (-1) = [(-8) \times (-5)] [(-6) \times (-1)]$

$= 40 \times 6$

$= 240$

Example-2 : Verify (–20) × [15 + (–5)] = [(–20) × 15] + [(–20) × (–5)]

Sol : L.H.S $= (-20) \times [15 + (-5)]$

$= -20 \times (15 - 5)$

$= -20 \times 10$

$= -200$

R.H.S $= [(-20) \times 15] + [(-20) \times (-5)]$

$= (-300) + (100)$

$= -300 + 100$

$= -200$

L.H.S = R.H.S

Example-3 : In a class test containing 10 questions, 4 marks are awarded for every correct answer and –2 marks are awarded for every incorrect answer and 0 mark is given for the questions not attempted

(*i*) Smeep gets 8 correct and 2 incorrect answers. What is her score ?

(*ii*) Harmanjit gets 3 correct and 6 incorrect answers out of 9 questions he attempted. What is his score ?

Sol. (*i*) Marks given for one correct answer = 4

Marks given for 8 correct answers $= 4 \times 8$

$= 32$

Marks given for one incorrect answer $= -2$

Marks given for 2 incorrect answers $= -2 \times 2$

$= -4$

Total marks Smeep scored in the test $= 32 + (-4)$

$= 32 - 4$

$= 28$

(*ii*) Marks given for one correct answer = 4

Marks given for 3 correct answers $= 4 \times 3$

$= 12$

Marks given for one incorrect answer	=	–2
Marks given for 6 incorrect answers	=	–2 × 6
	=	–12
Marks given for one unattempted question	=	0
Total marks Harmanjit scored in the test	=	12 + (–12) + 0
	=	12 – 12 + 0
	=	0

EXERCISE - 1.3

1. Find the product of :-

(*i*) (–15) × 0 (*ii*) (–35) × 1

(*iii*) (–13) × (–12) (*iv*) (–20) × 16

(*v*) (–15) × (–4) × (–5) (*vi*) (–8) × (–5) × 9

(*vii*) (–2) × (–5) × (–4) × (–10) (*viii*) (–8) × 0 + [(–5) × (–4)]

2. (*i*) Verify : 15 × [9 + (–6)] = (15 × 9) + (15 × (–6)]

(*ii*) Verify : 18 × [(–5) + (–4)] = [(18 × (–5)] + [18 × (–4)]

3. Fill in the blanks :-

(*i*) 15 × ☐ = 0

(*ii*) –25 × ☐ = 25

(*iii*) (–15) × 18 = ☐ × (–15)

(*iv*) (–10) × [(–15) + (–5)] = (–10) × ☐ + (–10) × (–5)

(*v*) (–6) × [(–5) × (–18)] = (–6) × ☐ × – 18

4. Find product using properties

(*i*) 15 × (–20) + (–20) × (–5) (*ii*) (15 × 8) × 50

(*iii*) 8 × (40 –5) (*iv*) 510 × (–45) + (–510) × 55

5. In a class test containing 15 questions, 2 marks are awarded for every correct answer and (–1) mark awarded for every incorrect answer and 0 mark for questions not attempted.

(*i*) Kritika gets 5 correct and 10 incorrect answers. What is her score ?

(*ii*) Rohan gets 7 correct and 7 incorrect answers out of 14 questions he attempted. What is his score?

6. Multiple Choice Questions :-

(*i*) (–19) – (13) is equal to

(*a*) –32 (*b*) 6

(*c*) –6 (*d*) none of these

(*ii*) (–6) × (–5) × 0 is equal to

(a) 0 (*b*) –6

(*c*) –5 (*d*) 30

(*iii*) 0 ÷ (–10) is equal to

(*a*) 0 (*b*) –1

(*c*) –10 (*d*) none of these

(*iv*) (–33) × 102 + (–33) × (–2) is equal to

(*a*) 3300 (*b*) –3300

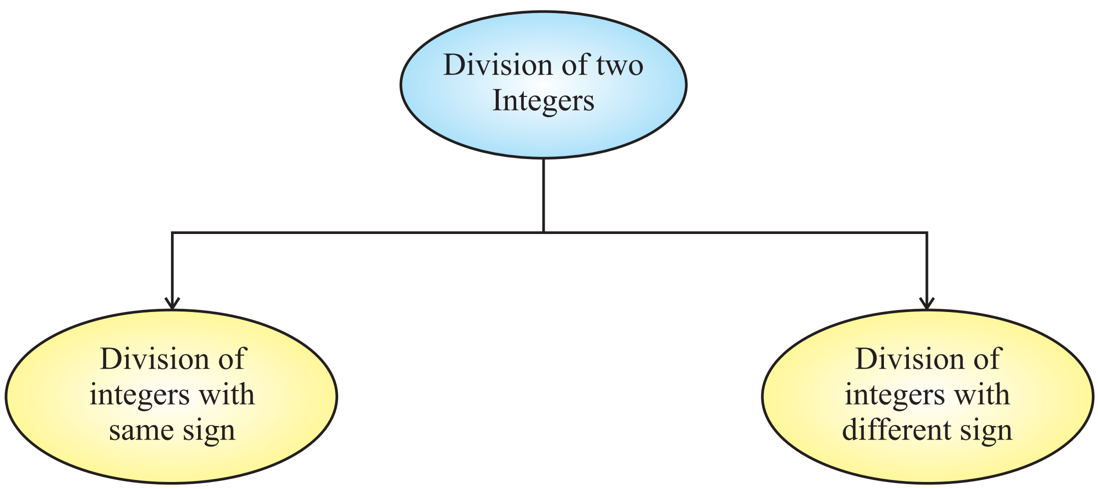
(*c*) 3432 (*d*) –3432

(*v*) 101 × (–1) + 0 × (–1) is equal to

(*a*) –101 (*b*) 101

(*c*) –102 (*d*) 102

DIVISION OF TWO INTEGERS



1. Division of integers with same signs :-

Step 1 : Divide the numbers regardless of their sign.

Step 2 : Write the quotient with a positive sign.

Example-1 : (*i*) (20) ÷ (5) = 4

5) 20 (4
20
×

(*ii*) (–15) ÷ (–3) = 5

3) 15 (5
15
×

2. Division of integers with different signs :-

Step 1 : Divide the numbers regardless of their sign.

Step 2 : Write the quotient with a negative sign.

Example-2 : (*i*) (–36) ÷ (12) = –3

12) 36 (3
36
×

(*ii*) (25) ÷ (–5) = –5

5) 25 (5
25
×

PROPERTIES OF DIVISION OF INTEGERS

(1) When an integer is divided by another, the quotient need not be an integer.

For example (*i*) 5 and 6 are two integers, but 5 ÷ 6 is not an integer *i.e* $\frac{5}{6}$ is not an integer.

(*ii*) –3 and 7 are two integers but (–3) ÷ 7 is not an integer i.e $\frac{-3}{7}$ is not an integer

(2) For every non zero integer a, we have $a \div a = 1$

For example (*i*) (+7) ÷ (+ 7) = 1

(*ii*) (–5) ÷ (–5) = 1

(3) For every non-zero integer a, we have $0 \div a = 0$

For example (*i*) 0 ÷ (+ 5) = 0

(*ii*) 0 ÷ (– 2) = 0

(4) For non-zero integers a and b, where $a \neq b$ we have

$a \div b \neq b \div a$ (i.e. commutative property does not hold)

For example 15 ÷ 5 = 3 but 5 ÷ 15 = $\frac{1}{3}$

(5) For non-zero integer a, b and c where $a \neq b \neq c$ we have

$(a \div b) \div c \neq a \div (b \div c)$ (i.e. Associative property does not hold)

EVEN AND ODD INTEGERS

Even Integers : All the integers, which are exactly divisible by 2 are called even integers.

.... –6, –4, –2, 0, 2, 4, 6, ...

are some even integers.

Odd Integers : All the integers which are not exactly divisible by 2 are called odd integers.

–5, –3, –1, 1, 3, 5, ...

are some odd Integers

Example-1 : Simplify (*i*) 63 ÷ (–7) (*ii*) (–80) ÷ 16 (*iii*) (72) ÷ (–9)

Sol. We have

(*i*) 63 ÷ (–7) = –9

$$7\,\overline{)\,63\,}\,(9$$
63
×

(*ii*) (–80) ÷ 16 = –5

$$16\,\overline{)\,80\,}\,(5$$
80
×

(*iii*) (–72) ÷ (–9) = 8

$$9\,\overline{)\,72\,}\,(8$$
72
×

Example-2 : Write all even integers between –20 and –10.

Sol. All even integers between –20 and –10 are –18, –16, –14, –12

Example-3 : Write all odd integers between –6 and 12

Sol. All odd integers between –6 and 12 are –5, –3, –1, 1, 3, 5, 7, 9, 11

EXERCISE - 1.4

1. **Evaluate each of the following :**

   (*i*) 76 ÷ 19 (*ii*) (–156) ÷ (–12)

   (*iii*) (–125) ÷ (–1) (*iv*) (125) ÷ (–25)

   (*v*) 0 ÷ (–5) (*vi*) (–15) ÷ (15)

2. Write all even integers between –18 and 0
3. Write all odd integers between –9 and 9
4. By what number should (–240) be divided to obtain 16 ?
5. **Find the value of :**

   (*i*) 125 ÷ [5 ÷ (–1)]

   (*ii*) [169 ÷ 13] ÷ [26 ÷ 2]

   (*iii*) [(–105) ÷ 3] ÷ 7

6. Simplify : 12 – [8 + 27 ÷ (2 × 8 – 7)]
7. Simplify : 10– [8 – {11 + 30 ÷ (4 + 2)}]

8. Multiple choice questions :-

(*i*) (–8) ÷ 2 =

(*a*) –16 (*b*) –4

(*c*) 4 (*d*) –8

(*ii*) (–7) ÷ (–7) =

(*a*) –1 (*b*) 49

(*c*) –49 (*d*) None of these

(*iii*) 0 ÷ 2 =

(*a*) 1 (*b*) 2

(*c*) –2 (*d*) 0

9. The quotient of two integers is always an integer. **(True/ False)**

10. If a and b are two unequal non-zero integers then $a \div b = b \div a$ **(True/ False)**

WHAT HAVE WE DISCUSSED ?

1. The numbers .... –3, –2, –1, 0, 1, 2, 3 .... are called integers.
2. The integers 1, 2, 3 .... etc. are called positive integers.
3. The integers –1, –2, –3, ... etc. are called negative integers.
4. 0 is neither positive nor negative.
5. To add two integers with same signs we add their numerical values and put the sign of addends with the sum.
6. To add two integers with different signs we take the difference of their numerical values and put the sign of integer with the greater numerical value.
7. For any integer a, we have $a + (-a) = 0$, We call $(-a)$ as the additive inverse of a.
8. Distributive law of multiplication over addition is $a \times (b + c) = (a \times b) + (a \times c)$

   Properties chart for four fundamental operations on Integers.

Operations / Properties	Addition	Subraction	Multiplication	Division
Clousre	Yes	Yes	Yes	No
Commutative	Yes	No	Yes	No
Associative	Yes	No	Yes	No
Identity	Yes	No	Yes	No

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Define integers and represent them on number line.
2. Perform basic operations of addition, subtraction, multiplication and division on integers.
3. Investigate the properties under addition, subtraction, multiplication and division of integers.

4. Use the properties of commutativity, associativity and the distributive property under addition and multiplication to make the calculations easier.
5. Make use of the knowledge of integers in their real life situations.

ANSWERS

EXERCISE 1.1

1. (*i*) > (*ii*) <
(*iii*) > (*iv*) <
(*v*) = (*vi*) >

2. (*i*) –43, –35, –10, –2, 7, 12, 31 (*ii*) –20, –5, 0, 4, 5, 13

3. (*i*) 46, 19, 8, 0, –3, –7, –23 (*ii*) –20, –6, –2, 0, 8, 30

4. (*i*) 9 (*ii*) 7
(*iii*) 2 (*iv*) 235

5. (*i*) Negative (*ii*) Positive
(*iii*) 1 (*iv*) –1
(*v*) Positive

EXERCISE 1.2

1. (*a*) 47 (*b*) –1
(*c*) –4 (*d*) –19
(*e*) 8 (*f*) –19
(*g*) –75 (*h*) 9
(*i*) 72 (*j*) –114

2. (*a*) –4 (*b*) –5
(*c*) 7 (*d*) –20

3. (*a*) 21 (*b*) –39
(*c*) –20 (*d*) 22

4. (*i*) –7 (*ii*) 25
(*iii*) 15, 20 (*iv*) –12
(*v*) –7, –2 (*vi*) 0

5. –27 **6.** –91, –89, –87

7. –2° **8.** –180°

9. 30 340 feet

10. Scores are equal, addition of integers is associative

11.

Team	A	B	C	D	E
After two rounds	4	–4	6	7	1
Final score	8	–2	7	5	3

Ist – A, IInd – C, IIIrd D

12. (*i*) *d* (*ii*) *c*
(*iii*) *d* (*iv*) *d*
(*v*) *b*

EXERCISE 1.3

1. (*i*) 0 (*ii*) –35
(*iii*) 180 (*iv*) –320
(*v*) –300 (*iv*) 360
(*viii*) 400 (*viii*) 20

3. (*i*) 0 (*ii*) –1
(*iii*) 18 (*iv*) –15
(*v*) –5

4. (*i*) 200 (*ii*) 6000
(*iii*) 280 (*iv*) 51000

5. (*i*) 0 (*ii*) 7

6. (*i*) *c* (*ii*) *a*
(*iii*) *a* (*iv*) *b*
(*v*) *a*

EXERCISE 1.4

1. (*i*) 4 (*ii*) 13
(*iii*) 125 (*iv*) –5
(*v*) 0 (*vi*) –1

2. –6, –4, –2, 0, 2, 4, 6

3. –7, –5, –3, –1, 1, 3, 5, 7

4. –15

5. (*i*) –25 (*ii*) 1
(*iii*) –5

6. 1 **7.** 18

8. (*i*) *b* (*ii*) *d*
(*iii*) *d*

9. False **10.** False

CHAPTER 2

T6C5A3

Fractions and Decimals

Learning Objectives :-

In this chapter you will learn :-

1. To find the reciprocal of a fraction.
2. To multiply and divide two (or more) fractions.
3. To multiply and divide decimal numbers.
4. To solve problems related to decimal numbers and fractions in daily life situations.

OUR NATION'S PRIDE

Bhaskara-I was a 7th century mathematician, who was the first to write numbers in the Hindu decimal system with a circle for the zero and who gave a unique and remarkable rational approximation of the sine function in his commentary on Aryabhata's work. He also wrote two astronomical works in the line of Aryabhata's school, the Mahabhaskariya and the Laghubhaskariya.

INTRODUCTION

We have already learnt about the basic concept of fractions and decimals in earlier classes. We studied Proper, Improper and mixed fractions as well as their addition and subtraction, comparison of fractions, equivalent fractions, representation of fractions on the number line and ordering of fractions. Also, we studied about decimals, comparison of decimals, addition and subtraction of decimals and representation of decimals on number line. Now, in this chapter, we will learn about multiplication and division of fractions as well as decimals.

First of all, we will review and revise what we have done about fractions in our previous class.

Fraction : Fraction is a number that represents part of a whole. $\frac{3}{4}$ represents the 3 parts when whole is divided into four equal parts. 4 is called the denominator and 3 is called the numerator.

Proper fraction : A fraction whose numerator is less than its denominator, is called a proper fraction. For example $\frac{2}{3}, \frac{5}{7}, \frac{9}{13}$ are all proper fractions.

Improper fraction : A fraction whose numerator is greater than its denominator is called an improper fraction. For example $\frac{3}{2}, \frac{8}{5}, \frac{7}{3}$ are all improper fractions.

Mixed fractions : A fraction which can be expressed as a sum of a natural number and a proper fraction is called a mixed fraction. For example :

$$1\frac{2}{5} = 1 + \frac{2}{5}$$

Like fractions : Fractions having the same denominators are called like fractions.

For Example : $\frac{1}{7}, \frac{2}{7}, \frac{8}{7}$ are like fractions.

Unlike fractions : Fractions having different denominators are called unlike fractions. For example $\frac{3}{8}, \frac{2}{5}, \frac{5}{7}$ are unlike fractions.

Decimal fraction : A fraction whose denominator is 10 or a power of 10 *i.e* 10, 100, 1000, ... is called a decimal fraction

For example : $\frac{3}{10}, \frac{22}{100}, \frac{732}{1000}$ are decimal fractions.

Simple fractions : A fraction whose denominator is a number other than 10, 100, 1000, ... is called a simple or vulgar fraction.

For example : $\frac{7}{15}, \frac{4}{25}, \frac{2}{17}$ are vulgar fractions.

Equivalent fractions : Equivalent fractions represent same part of a whole.

For example : $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ are all equivalent fractions :

Note : An improper fraction can be expressed as a mixed fraction and vice–versa. For example :

$\frac{15}{2}$ is an improper fraction also $\frac{15}{2} = 7\frac{1}{2}$ which is a mixed fraction.

$$2\overline{)15}(7$$
$$\underline{14}$$
$$\underline{1}$$

Now consider $3\frac{2}{5}$, which is a mixed fraction. It can also be written as $\frac{(3\times 5)+2}{5} = \frac{17}{5}$ an improper fraction.

Example-1 : Write five equivalent fractions of $\frac{2}{3}$.

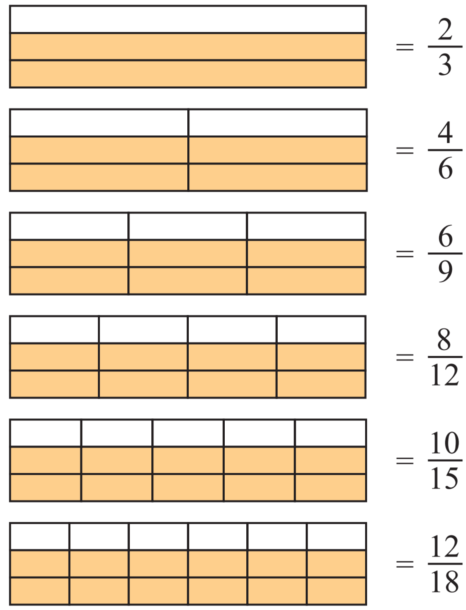
Sol. $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{4}{6}$

$\frac{2}{3} = \frac{2\times3}{3\times3} = \frac{6}{9}$

$\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$

$\frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}$

$\frac{2}{3} = \frac{2\times6}{3\times6} = \frac{12}{18}$



$\therefore$ $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}$ are five equivalent fractions of $\frac{2}{3}$

Example-2 : Add the fractions $2\frac{3}{4}$, $5\frac{5}{6}$, $\frac{3}{8}$

Sol. $2\frac{3}{4} + 5\frac{5}{6} + \frac{3}{8}$

$= \frac{11}{4} + \frac{35}{6} + \frac{3}{8}$

2	4, 6, 8
2	2, 3, 4
2	1, 3, 2
3	1, 3, 1
	1, 1, 1

L.C.M of (4, 6, 8) $= 2 \times 2 \times 2 \times 3$

$= 24$

Now $\frac{11}{4} + \frac{35}{6} + \frac{3}{8} = \frac{(11\times6)+(35\times4)+(3\times3)}{24}$

$= \frac{66+140+9}{24} = \frac{215}{24} = 8\frac{23}{24}$

$$24\overline{)215}(8$$
$$\underline{192}$$
$$\underline{23}$$

Example-3 : Raman purchased $3\frac{1}{2}$ kg oranges and $4\frac{3}{4}$ kg mangoes. What is the total weight of fruits purchased by him ?

Sol. The total weight of the fruits

$= \left(3\frac{1}{2} + 4\frac{3}{4}\right)\text{kg}$

$= \left(\frac{7}{2} + \frac{19}{4}\right)\text{kg}$

$$= \left(\frac{14+19}{4}\right)\text{kg}$$

$$= \frac{33}{4}\text{kg} = 8\frac{1}{4}\text{kg}$$

Example-4 : Dalbir exercised for $\frac{3}{4}$ of an hour while Ranjeet exercised for $\frac{7}{9}$ of an hour. Who exercised for a longer time ?

Sol. In order to find who exercised for a longer time, let us compare $\frac{3}{4}$ and $\frac{7}{9}$

L.C.M of 4 and 9 = 36

Converting $\frac{3}{4}$ and $\frac{7}{9}$ to like fractions we have

$$\frac{3}{4} = \frac{3\times 9}{4\times 9} = \frac{27}{36}$$

$$\frac{7}{9} = \frac{7\times 4}{9\times 4} = \frac{28}{36}$$

As 28 > 27,

$\therefore$ $$\frac{28}{36} > \frac{27}{36} \Rightarrow \frac{7}{9} > \frac{3}{4}$$

$\therefore$ Ranjeet exercised for longer time

EXERCISE - 2.1

1. **Solve the following fractions :-**

(*i*) $4 + \frac{7}{8}$

(*ii*) $\frac{9}{11} - \frac{4}{15}$

(*iii*) $\frac{11}{16} - \frac{2}{5} + \frac{8}{10}$

(*iv*) $2\frac{1}{5} + 6\frac{1}{2}$

(*v*) $8\frac{1}{2} - 3\frac{5}{8}$

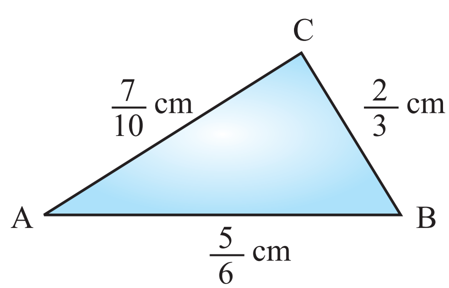
(*vi*) $\frac{9}{10} - \frac{9}{100} + \frac{9}{1000}$

2. **Arrange the following in ascending order :-**

(*i*) $\frac{2}{17}, \frac{10}{17}, \frac{3}{17}, \frac{16}{17}, \frac{5}{17}, \frac{8}{17}$

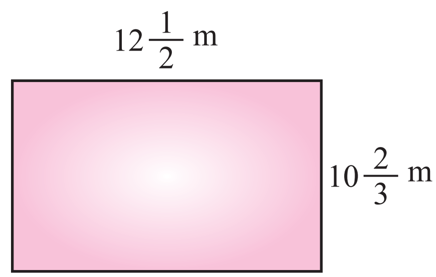
(*ii*) $\frac{1}{5}, \frac{3}{7}, \frac{7}{10}$

3. The three sides AB, BC and CA of a triangle ΔABC are $\frac{5}{6}\,cm$, $\frac{2}{3}\,cm$ and $\frac{7}{10}\,cm$ respectively. Find the perimeter of the triangle.



4. Ramesh studies for $5\frac{2}{3}$ hours daily. He devotes $2\frac{4}{5}$ hours of his time for science and mathematics. How much time does he devotes for other subjects ?

5. Sonia jogs once around the rectangular park of sides $10\frac{2}{3}\,m$ and $12\frac{1}{2}\,m$. Find the total distance covered by the Sonia.



6. Ritu coloured a picture in $\frac{7}{12}$ hours. Vaibhav coloured the same picture in $\frac{3}{4}$ hours. Who worked for a longer time and by what fraction ?

7. **Multiple Choice Questions :**

(*i*) Fractions $\frac{2}{5}, \frac{7}{5}$ are

(*a*) Like fractions (*b*) Unlike fractions

(*c*) Equivalent fractions (*d*) None of these

(*ii*) What fraction do 8 hours of a day represents

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{8}{60}$ (d) $\frac{2}{3}$

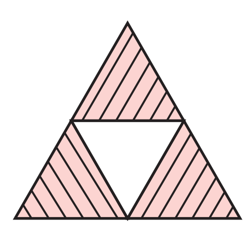
(*iii*) Equivalent fraction of $\frac{3}{5}$ is ....

(*a*) $\frac{13}{15}$ (*b*) $\frac{5}{3}$ (*c*) $\frac{9}{15}$ (*d*) $\frac{5}{13}$

(*iv*) Shaded area of given triangle represents the fraction

(*a*) $\frac{1}{3}$ (*b*) $\frac{3}{4}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{2}{3}$



(*v*) Sum of fractions $\frac{2}{7}$ and $\frac{3}{4}$ is equal to :

(*a*) $\frac{5}{28}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{5}{11}$ (*d*) $\frac{29}{28}$

MULTIPLICATION OF FRACTIONS

Multiplication of a fraction by a whole number : We know that multiplication means repeated addition. For example, 4 × 3 represents 4 times 3 *i.e.* 3 + 3 + 3 + 3 = 12

Now, to multiply $\frac{2}{9}$ with 4, we write $\frac{2}{9}$ four times and then add, that is

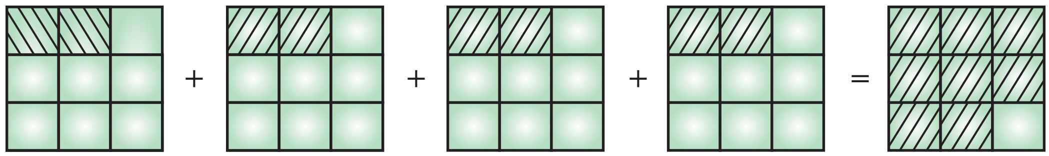


Figure : 2.1

$$4 \times \frac{2}{9} = \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{2+2+2+2}{9} = \frac{8}{9}$$

Let us consider one more example

$$4 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{2+2+2+2}{5} = \frac{8}{5} = 1\frac{3}{5}$$

$4 \times \frac{2}{5}$ can be represented in the following figure (2.2)

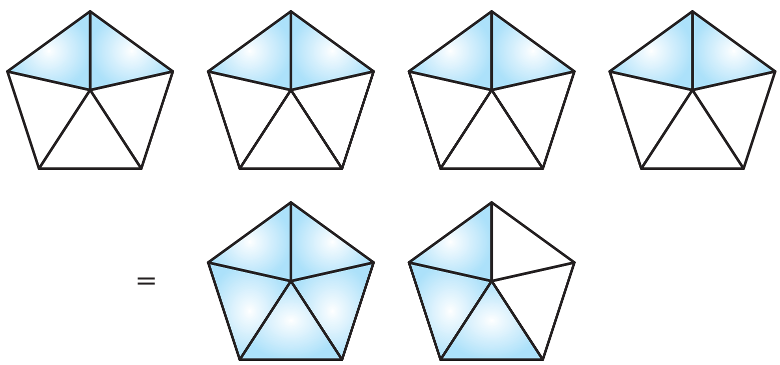


Figure : 2.2

Thus to multiply a whole number with a proper or an improper fraction, we multiply the numerator of fraction with the whole number. After multiplication we get new numerator, where as the denominator in the product remains the same as in the given fraction.

Example-1 : **Multiply whole number to a fraction and reduce it to lowest form. Also convert into a mixed fraction (If possible)**

(*i*) $7 \times \frac{3}{5}$

(*ii*) $\frac{2}{3} \times 4$

Sol. (*i*) $7 \times \frac{3}{5} = \frac{21}{5} = 4\frac{1}{5}$

(*ii*) $\frac{2}{3} \times 4 = \frac{8}{3} = 2\frac{2}{3}$

Example-2 : Multiply 4 by $6\frac{1}{3}$ and express the product as a mixed fraction

Sol . $4 \times 6\frac{1}{3} = 4 \times \left(\frac{18+1}{3}\right)$

$= 4 \times \frac{19}{3} = \frac{76}{3} = 25\ \frac{1}{3}$

FRACTION AS AN OPERATOR 'OF'

Observe the shaded portion of the circles in figure 2.3. Each shaded portion represents $\frac{1}{4}$ of 1

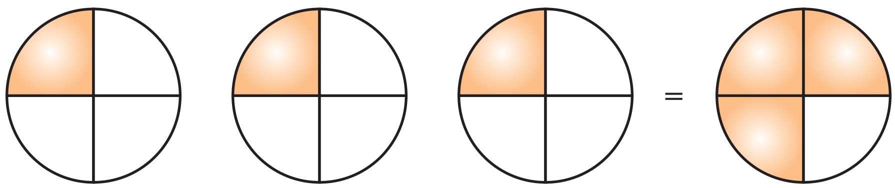


Figure : 2.3

So, three shaded parts represents $\frac{1}{4}$ of 3

After combining 3 shaded parts represent $\frac{3}{4}$ of 1

So $\frac{1}{4}$ of $3 = \frac{1}{4} \times 3 = \frac{3}{4}$

Thus, we observe that 'of' represent multiplication.

Example-3 : Solve the following

(*i*) $\frac{3}{4}$ of 16 **(*ii*) $\frac{1}{2}$ of $3\frac{5}{6}$**

Sol. (*i*) $\frac{3}{4}$ of 16

$= \frac{3}{4} \times 16 = \frac{48}{4} = 12$

(*ii*) $\frac{1}{2}$ of $3\frac{5}{6}$

$= \frac{1}{2}$ of $3\frac{5}{6}$

$= \frac{1}{2}$ of $\frac{23}{6}$ $\left[3\frac{5}{6} = \frac{18+5}{6} = \frac{23}{6}\right]$

$= \frac{1}{2} \times \frac{23}{6} = \frac{23}{12} = 1\ \frac{11}{12}$

Example-4 : Jasbir's monthly salary is ₹ 8400. He spends $\frac{1}{4}$th of his salary on food and $\frac{1}{7}$th of his salary on rent. Out of the remaining salary he spends $\frac{1}{3}$rd on the education of children. Find

(*i*) How much does he spend on each part ?

(*ii*) How much money is still left with him ?

Sol.

Money spent on food $= \frac{1}{4}$ of 8400

$= \left(\frac{1}{4} \times 8400\right) =$ ₹ 2100

Money spent on rent $= \frac{1}{7}$ of 8400

$= \left(\frac{1}{7} \times 8400\right) =$ ₹ 1200

Remaining money with him $= 8400 - 2100 - 1200$

= ₹ 5100

Money spent on education of children $= \frac{1}{3}$ of 5100

$= \left(\frac{1}{3} \times 5100\right) =$ ₹ 1700

Total money spent $= 2100 + 1200 + 1700$

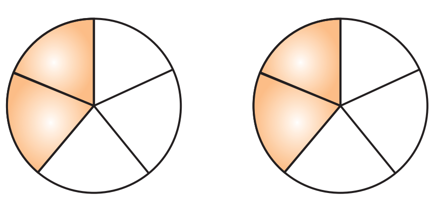
= ₹ 5000

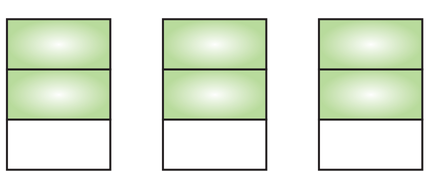
∴ Money left with him $= 8400 - 5000$

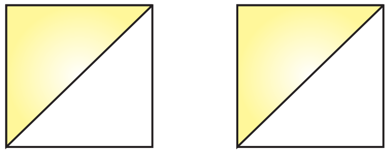
= ₹ 3400

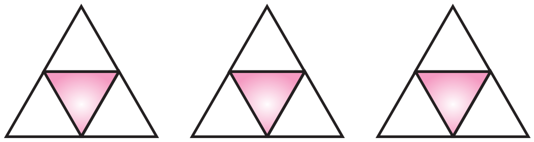
EXERCISE - 2.2

1. Match the following :-

(*a*)  (*i*) $3 \times \frac{1}{4}$

(*b*)  (*ii*) $2 \times \frac{2}{5}$

(*c*)  (*iii*) $3 \times \frac{2}{3}$

(*d*)  (*iv*) $2 \times \frac{1}{2}$

2. **Multiply whole number to a fraction and reduce it to the lowest form. Also convert into a mixed fraction :**

(*i*) $4 \times \frac{1}{3}$ (*ii*) $11 \times \frac{4}{7}$ (*iii*) $\frac{3}{4} \times 6$

(*iv*) $\frac{9}{7} \times 5$ (*v*) $2\frac{5}{6} \times 4$ (*vi*) $10\frac{5}{6} \times 5$

(*vii*) $5 \times 6\frac{3}{4}$ (*viii*) $3\frac{2}{5} \times 8$

3. **Solve :-**

(*i*) $\frac{1}{2}$ of 46 (*ii*) $\frac{2}{3}$ of 27 (*iii*) $\frac{1}{3}$ of 36

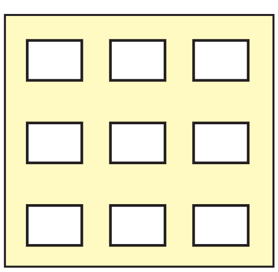
(*iv*) $\frac{3}{4}$ of 16 (*v*) $\frac{5}{7}$ of 35

4. **Shade :-**

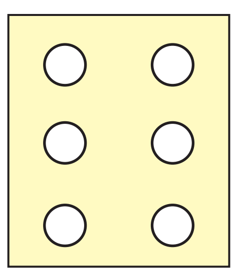
(*i*) $\frac{1}{3}$ of the rectangles in box (*a*)

(*ii*) $\frac{2}{3}$ of the circles in box (*b*)

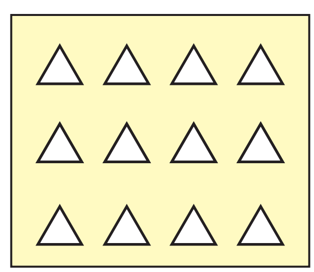
(*iii*) $\frac{1}{2}$ of the triangles in box (*c*)



(*a*)



(*b*)



(*c*)

5. Rahul earns ₹ 44,000 per month. He spends $\frac{3}{4}$th of his income every month and saves the rest of his earning. Find his monthly savings ?

6. The cost of a book is ₹ 117 $\frac{1}{2}$. Find the cost of 8 books ?

7. **Multiple choice questions :-**

(*i*) $\frac{1}{2} \times 8$ is equal to

(*a*) 8 (*b*) 2 (*c*) 4 (*d*) 1

(*ii*) $\frac{3}{2}$ of 16 is

(*a*) 48 (*b*) 8 (*c*) 3 (*d*) 24

(*iii*) What fraction of an hour is 40 minutes ?

(*a*) $\frac{2}{3}$ (*b*) 40 (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

(*iv*) What fraction does the shaded part of figure 2.4 represent ?

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{3}{4}$ (*d*) $\frac{1}{2}$

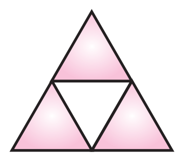


Figure : 2.4

MULTIPLICATION OF A FRACTION BY A FRACTION

We know that multiplication also represents the operator 'of'

So $\frac{3}{4} \times \frac{1}{3}$ represents $\frac{3}{4}$ of $\frac{1}{3}$

Let us now understand the meaning of $\frac{3}{4}$ of $\frac{1}{3}$ by following:

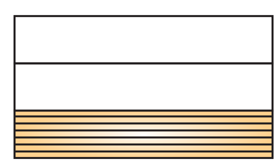


Figure : 2.5

(*i*) $\frac{1}{3}$ represents one part when the whole is divided into three equal parts (figure 2.5)

(*ii*) $\frac{3}{4}$ of $\frac{1}{3}$ represents the $\frac{3}{4}$ of the shaded region (fig 2.5)

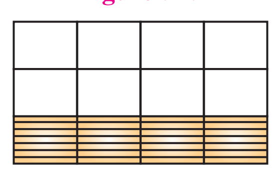


Figure : 2.6

(*iii*) To find $\frac{3}{4}$ of $\frac{1}{3}$ we divide each $\frac{1}{3}$ part of fig 2.5 in four equal parts (figure 2.6)

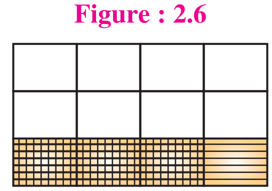


Figure : 2.7

(*iv*) $\frac{3}{4}$ of $\frac{1}{3}$ is represented by double shaded region in figure 2.7.

Thus double shaded region represents $\frac{3}{12}$ of the whole

so we observe that $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{\text{Product of numerators}}{\text{Product of denominators}}$

$\therefore$ Product of fractions $= \frac{\text{Product of numerators}}{\text{Product of denominators}}$

Example-1 : Solve $\frac{1}{2}$ of $\frac{3}{7}$

Sol. $\frac{1}{2}$ of $\frac{3}{7}$

$= \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

Example-2 : Multiply the fractions and reduce it to lowest form (If possible)

(*i*) $\frac{2}{3} \times 2\frac{2}{3}$ (*ii*) $6\frac{2}{5} \times \frac{7}{9}$

Sol : (*i*) $\frac{2}{3} \times 2\frac{2}{3}$

$= \frac{2}{3} \times \frac{8}{3} = \frac{16}{9} = 1\frac{7}{9}$ $\left[2\frac{2}{3} = \frac{3 \times 2 + 2}{3} = \frac{6+2}{3} = \frac{8}{3}\right]$

(*ii*) $6\frac{2}{5} \times \frac{7}{9}$

$= \frac{32}{5} \times \frac{7}{9} = \frac{224}{45} = 4\frac{44}{45}$

Example-3 : Raj reads $\frac{1}{5}$th part of a book in 1 hour. How much part of the book will he read in $3\frac{2}{3}$ hours ?

Sol. The part of the book read by Raj in 1 hour = $\frac{1}{5}$

so, the part of the book read by him in $3\frac{2}{3}$ hours = $3\frac{2}{3} \times \frac{1}{5}$

$= \frac{11}{3} \times \frac{1}{5} = \frac{11}{15}$ $\left[3\frac{2}{3} = \frac{9+2}{3} = \frac{11}{3}\right]$

VALUE OF THE PRODUCTS

(*i*) The value of the product of two proper fractions is smaller than each of the given fractions.

For example : $\frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

Here $\frac{3}{14} < \frac{1}{2}$ and $\frac{3}{14} < \frac{3}{7}$

(*ii*) The value of the product of two improper fraction is greater than each of the given fractions.

For example : $\frac{4}{3} \times \frac{5}{3} = \frac{20}{9}$

Here $\frac{20}{9} > \frac{4}{3}$ and $\frac{20}{9} > \frac{5}{3}$

(*iii*) The value of the product of a proper fraction and an improper fraction is less than the given improper fraction and greater than given the proper fraction.

For example : $\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

Here $\frac{3}{4} > \frac{1}{2}$ and $\frac{3}{4} < \frac{3}{2}$

EXERCISE - 2.3

1. Solve the following :

(*i*) $\frac{1}{3}$ of

(*a*) $\frac{1}{5}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{3}{2}$

(*ii*) $\frac{3}{4}$ of

(*a*) $\frac{2}{9}$ (*b*) $\frac{4}{7}$ (*c*) $\frac{8}{3}$

2. Multiply the fractions and reduce it to the lowest form (If possible) :

(*i*) $\frac{2}{7} \times \frac{7}{9}$ (*ii*) $\frac{1}{3} \times \frac{15}{8}$ (*iii*) $\frac{12}{27} \times \frac{3}{9}$

(*iv*) $\frac{2}{5} \times \frac{6}{4}$ (*v*) $\frac{81}{100} \times \frac{6}{7}$ (*vi*) $\frac{3}{5} \times \frac{5}{27}$

3. Multiply the following fractions :

(*i*) $\frac{3}{2} \times 5\frac{1}{3}$ (*ii*) $\frac{1}{7} \times 5\frac{2}{3}$ (*iii*) $2\frac{5}{6} \times 4$

(*iv*) $4\frac{1}{3} \times 9\frac{1}{4}$ (*v*) $2\frac{2}{3} \times 3\frac{5}{8}$ (vi) $3\frac{1}{5} \times 2\frac{1}{4}$

4. Which fraction is greater in the following fractions ?

(*i*) $\frac{3}{2}$ of $\frac{2}{7}$ or $\frac{5}{2}$ of $\frac{3}{8}$ (*ii*) $\frac{1}{2}$ of $\frac{6}{5}$ or $\frac{1}{3}$ of $\frac{4}{5}$

5. If the speed of a car is $105\frac{1}{3}$ km/hr, find the distance covered by it in $3\frac{2}{3}$ hours.

6. The length of a rectangular plot of land is $29\frac{3}{7}\,m$. If its breadth is $12\frac{8}{11}\,m$, find its area.

7. If a cloth costs ₹ $120\frac{1}{4}$ per metre, find the cost of $4\frac{1}{3}$ metre of this cloth.

8. Multiple choice questions :

(*i*) $\frac{1}{4}$ of $\frac{8}{3}$ is

(*a*) $\frac{9}{7}$ (*b*) $\frac{8}{4}$ (*c*) $\frac{2}{3}$ (*d*) 1

(*ii*) $\frac{3}{2} \times \frac{2}{3} = ?$

(*a*) 1 (*b*) $\frac{5}{6}$ (*c*) 3 (*d*) $\frac{6}{5}$

(*iii*) The value of the product of two proper fractions is :

(*a*) greater than both of the proper fractions.

(*b*) lesser than both of the proper fractions.

(*c*) lie between the given two fractions.

(*d*) none of these

9. Check if the following equations are 'True' or 'False' :-

(*i*) $1\frac{2}{3} \times 4\frac{5}{7} = 4\frac{10}{21}$? (True/ False)

(*ii*) $\frac{3}{4} \times \frac{2}{3} = \frac{2}{3} \times \frac{3}{4}$? (True/ False)

DIVISION OF FRACTIONS

Reciprocal of a fraction : Reciprocal of a fraction is a fraction obtained by interchanging the places of numerator and denominator. The product of a fraction and its reciprocal is always '1'.

For example; The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$

Also, we observe that, $\frac{3}{7} \times \frac{7}{3} = 1$

Hence, Fraction × reciprocal of fraction = 1

NOTE : Reciprocal of a fraction is also called multiplicative inverse of the fraction.

Example-1 : Find the reciprocal of (*i*) $\frac{2}{5}$ (*ii*) 3

Sol. (*i*) Reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$

(*ii*) Reciprocal of 3

$= \text{Reciprocal of } \frac{3}{1} = \frac{1}{3}$

Division of a whole number by a fraction

Let us find $1 \div \frac{1}{4}$

Clearly here we have to find number of $\frac{1}{4}$ that are included in 1. The number of such '$\frac{1}{4}$' parts would be $1 \div \frac{1}{4}$. Observe the figure 2.8. How many $\frac{1}{4}$ parts do you see ?

So $1 \div \frac{1}{4} = 4$

Also $1 \times \frac{4}{1} = 1 \times 4 = 4$

Thus $1 \div 4 = 1 \times \frac{1}{4}$

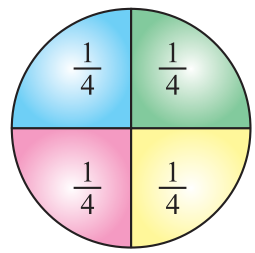


Figure : 2.8

We observe that while solving $1 \div \frac{1}{4}$ the (÷) sign is changed into (×) sign and $\frac{1}{4}$ is changed into $\frac{4}{1}$ (*i.e* reciprocal of $\frac{1}{4}$)

So, we conclude that to divide a whole number by any fraction, we have to multiply the whole number by the reciprocal of the given fraction.

Example-2 : Find $2 \div \frac{2}{3}$.

Sol. $2 \div \frac{2}{3}$

$= 2 \times (\text{Reciprocal of } \frac{2}{3})$

$= 2 \times \frac{3}{2} = 3$

Division of a whole number by a mixed fraction

To divide a whole number by a mixed fraction, first convert the mixed fraction into an improper fraction and then, multiply the whole number by the reciprocal of the improper fraction.

Example-3 : Solve $3 \div 2\frac{1}{4}$

Sol. $3 \div 2\frac{1}{4} = 3 \div \frac{9}{4}$ $\left[2\frac{1}{4} = \frac{8+1}{4} = \frac{9}{4}\right]$

$= 3 \times (\text{reciprocal of } \frac{9}{4})$

$= 3 \times \frac{4}{9} = \frac{4}{3}$

Division of a fraction by a non zero whole number

Example-4 : Solve (*i*) $\frac{5}{3} \div 2$ (*ii*) $2\frac{2}{3} \div 5$

Sol. (*i*) $\frac{5}{3} \div 2$

$= \frac{5}{3} \times \frac{1}{2}$ (Reciprocal of 2 = $\frac{1}{2}$)

$= \frac{5}{6}$

(*ii*) $2\frac{2}{3} \div 5$

$= \frac{8}{3} \div 5$ $\left(\frac{6+2}{3}\right) \div 5$

$= \frac{8}{3} \times \frac{1}{5}$ (Reciprocal of 5 = $\frac{1}{5}$)

$= \frac{8}{15}$

Division of a fraction by another fraction

To divide a fraction by another fraction, the first fraction is multiplied by the reciprocal of the second fraction.

Example-5 : Solve the following fractions

(*i*) $\frac{3}{5} \div \frac{1}{2}$ (*ii*) $2\frac{1}{2} \div \frac{3}{5}$ (*iii*) $\frac{2}{3} \div 2\frac{3}{4}$ (*iv*) $2\frac{3}{5} \div 2\frac{1}{5}$

Sol. (*i*) $\frac{3}{5} \div \frac{1}{2}$

$= \frac{3}{5} \times \frac{2}{1} = \frac{6}{5} = 1\frac{1}{5}$ (Reciprocal of $\frac{1}{2} = \frac{2}{1}$)

(*ii*) $2\frac{1}{2} \div \frac{3}{5}$ $\left[2\frac{1}{2} = \frac{4+1}{2} = \frac{5}{2}\right]$

$= \frac{5}{2} \div \frac{3}{5} = \frac{5}{2} \times \frac{5}{3} = \frac{25}{6} = 4\frac{1}{6}$

(iii) $\frac{2}{3} \div 2\frac{3}{4}$ $\left[2\frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}\right]$

$= \frac{2}{3} \div \frac{11}{4} = \frac{2}{3} \times \frac{4}{11} = \frac{8}{33}$

(iv) $2\frac{3}{5} \div 2\frac{1}{5}$ $\left[\begin{array}{l} 2\frac{3}{5} = \frac{10+3}{5} = \frac{13}{5} \\ 2\frac{1}{5} = \frac{10+1}{5} = \frac{11}{5} \end{array}\right]$

$= \frac{13}{5} \div \frac{11}{5} = \frac{13}{5} \times \frac{5}{11} = \frac{13}{11} = 1\frac{2}{11}$

First convert mixed fraction into improper fraction.

Multiply first fraction with reciprocal of second fraction.

EXERCISE - 2.4

1. **Find the reciprocal of each of the following fraction.**

(i) $\frac{2}{7}$ (ii) $\frac{3}{2}$ (iii) $\frac{5}{7}$

(iv) $\frac{1}{9}$ (v) $\frac{2}{3}$ (vi) $\frac{7}{8}$

2. **Solve the following (Division of a fraction by a non zero whole number)**

(i) $\frac{19}{6} \div 10$ (ii) $\frac{4}{9} \div 5$ (iii) $\frac{8}{9} \div 8$

(iv) $3\frac{1}{2} \div 4$ (v) $16\frac{1}{2} \div 5$ (vi) $4\frac{1}{3} \div 3$

3. **Solve the following (Division of a whole number by a fraction)**

(i) $8 \div \frac{7}{3}$ (ii) $5 \div \frac{7}{5}$ (iii) $4 \div \frac{8}{3}$

(iv) $3 \div 2\frac{3}{5}$ (v) $5 \div 3\frac{4}{7}$

4. **Solve the following (Division of a fraction by another fraction)**

(i) $\frac{2}{3} \div \frac{10}{9}$ (ii) $\frac{4}{9} \div \frac{2}{3}$ (iii) $2\frac{1}{2} \div \frac{3}{5}$

(iv) $\frac{3}{7} \div 1\frac{1}{5}$ (v) $5\frac{1}{2} \div 2\frac{1}{5}$ (vi) $3\frac{1}{5} \div 1\frac{2}{3}$

5. 11 small ropes are cut from $7\frac{1}{3}$ *m* long rope. Find the length of each of the small rope.

6. Multiple choice questions :

(*i*) Reciprocal of $\frac{3}{4}$ is

(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$ (*c*) 1 (*d*) none of these

(*ii*) $\frac{5}{7} \div \frac{7}{5} = ?$

(*a*) 1 (*b*) $\frac{49}{25}$ (*c*) $\frac{25}{49}$ (*d*) – 1

(*iii*) $\frac{5}{7} \div \frac{5}{7} = ?$

(*a*) 1 (*b*) $\frac{49}{25}$ (*c*) $\frac{25}{49}$ (*d*) –1

7. (*i*) The reciprocal of a proper fraction is an improper fraction (True/ False)

(*ii*) The reciprocal of a whole number is always a whole number (True/ False)

DECIMAL NUMBERS

You have learnt about decimal numbers in the previous class that a fraction with denominators 10,100, 1000, ... etc can be expressed in another form, called decimal numbers. For example : In 23.715, 23 is called whole number part and 715 is called decimal part.

Let us consider the number $38\frac{17}{100}$

Expanded form of this number is

$$38\frac{17}{100} = 30 + 8 + \frac{1}{10} + \frac{7}{100}$$

In decimal notation we can write

$$38\frac{17}{100} = 30 + 8 + 0.1 + 0.07$$

$$= 38.17$$

You can do the reverse too. For example $135.392 = 100 + 30 + 5 + \frac{3}{10} + \frac{9}{100} + \frac{2}{1000}$

Comparison of decimal numbers

To compare the two given decimal numbers, we first compare their whole number parts. The decimal number with the greater whole number part is greater of the two given decimals. For example 27. 75 > 22. 33 as whole number part of 27. 75 > whole number part of 22. 33

i.e. 27 > 22

If the whole number parts are equal, then we compare the digits on the right side of the decimal point starting from the tenth place. If the digit at the tenth place are equal, we compare the digits at the hundredth place and so on.

While converting lower units of length, weight, height and money to their higher units, we are required to use decimals.

Example-1 : Which is greater decimal number (*i*) 3.86 or 2.38 (*ii*) 5.32 or 5.3215

Sol. (*i*) 3.86 or 2.38

As whole number part of 3.86 is greater than whole number part of 2.38

∴ $3.86 > 2.38$

(*ii*) 5.32 or 5.3215

5.3200 or 5.3215

(To make decimal places equal in both numbers)

Whole number parts of both number are equal

So, we have to compare decimal part.

Also digits at tenths and hundredths place are equal

Thousandth part of 5.3215 is greater than

Thousandth part of 5.3200

∴ $5.3215 > 5.3200$

i.e. $5.3215 > 5.32$

Example-2 : Express 7 rupees 5 paise in rupees using decimals.

Sol. 7 rupees 5 paise

$$= ₹\,7 + ₹\,\frac{5}{100} = ₹\,7 + ₹\,0.05 = ₹\,7.05$$

Example-3 : Write the place value of 3 in the following decimal numbers :

(*i*) 3.472 (*ii*) 0.43 (*iii*) 54.2738

Sol. (*i*) Place value of 3 in 3.472 = 3

(*ii*) Place valueof 3 in 0.43 = $\frac{3}{100}$

(*iii*) Place value of 3 in 54.2738 = $\frac{3}{1000}$

EXERCISE - 2.5

1. Which is greater decimal number ?

(*i*) 0.9 or 0.4 (*ii*) 1.35 or 1.37

(*iii*) 10.10 or 10.01 (*iv*) 1735.101 or 1734.101

(*v*) 0.8 or 0.88

2. **Write the following decimal number in the expanded form :**

(*i*) 40.38 (*ii*) 4.038

(*iii*) 0.4038 (*iv*) 4.38

3. **Write the place value of 5 in the following decimal numbers :**

(*i*) 17.56 (*ii*) 1.253

(*iii*) 10.25 (*iv*) 5.62

4. **Express in rupees using decimals :**

(*i*) 55 paise (*ii*) 55 rupees 5 paise

(*iii*) 347 paise (*iv*) 2 paise

5. **Express in km :**

(*i*) 350 m (*ii*) 4035m

(*iii*) 2 km 5m

6. **Multiple choice questions**

(*i*) Place value of 2 in 3.02 is

(*a*) 2 (*b*) 20

(*c*) $\frac{2}{10}$ (*d*) $\frac{2}{100}$

(*ii*) The correct ascending order of 0.7, 0.07, 7 is

(*a*) 7 < 0.07 < 0.7 (*b*) 0.07 <0 . 7< 7

(*c*) 0.7 <0.0 7 < 7 (*d*) 0.0 7 < 7 < 0.7

(*iii*) Decimal expression of 5kg 20 gram is

(*a*) 5.2 kg (*b*) 5.20kg

(*c*) 5.02kg (*d*) None of these

(*iv*) Expanded form of 2.38 is

(*a*) $2 + \frac{38}{10}$ (*b*) $2 + 3 + \frac{8}{10}$

(*c*) $\frac{238}{100}$ (d) $2 + \frac{3}{10} + \frac{8}{100}$

MULTIPLICATION OF DECIMAL NUMBERS

Let us find 11.34 × 2.3

$$11.34 \times 2.3 = \frac{1134}{100} \times \frac{23}{10} = \frac{26082}{1000} = 26.082$$

In the above example, we converted the decimal numbers into fractions, multiplied the fractions and finally converted the product into decimal number.

- But there are quicker methods to multiply decimal numbers.

Multiplication of decimal number by 10,100 and 1000

(*i*) On multiplying a decimal number by 10, the decimal point is shifted to the right by one place.

(*ii*) On multiplying a decimal number by 100, the decimal point is shifted to the right by two places.

(*iii*) On multiplying decimal number by 1000, the decimal point is shifted to the right by three places and so on.

i.e. when a decimal number is multiplied by 10,100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as many of places as there are zeros over one.

Example-1 : Find the value of (*i*) 15.23 × 10 (*ii*) 2.457 × 1000 (*iii*) 3.7 × 100

Sol. (*i*) 15.23 × 10
= 152. 3

(we shift the decimal point by one place to the right as there in one zero in 10)

(*ii*) 2.457 × 1000
= 2457

(We shift the decimal point by three places to the right as there are three zeroes in 1000)

(*iii*) 3.7 × 100
We know 3.7 = 3.70
= 3.70 × 100
= 370

(We shift the decimal point by two places to the right as there are two zeros in 100)

MULTIPLICATION OF A DECIMAL BY A WHOLE NUMBER

To multiply a decimal number by a whole number :

(*i*) First, multiply decimal number by whole number ignoring the decimal point.

(*ii*) Then, put the decimal point in the product obtained such that there are as many decimal place on the right of the decimal point in the product as in the given decimal number that is multiplied.

Example-2 : Find the products.

(*i*) 1.3 × 7 (*ii*) 3.75 × 12 (*iii*) 0.02 × 15

Sol. (*i*) Ignoring the decimal point in 1.3
We get 13
Now 13 × 7 = 91
∴ 1.3 × 7 = 9.1 (∵ Number of decimal places in 1.3 is 1)

(*ii*) 3.75 × 12
Ignoring the decimal point in 3.75
We get 375
Now 375 × 12 = 4500
∴ 3.75 × 12 = 45.00 (∵ Number of decimal places in 3.75 is 2)
= 45

(*iii*) 0.02 × 15

Ignoring the decimal point in 0.02

We get 002 = 2

Now 2 × 15 = 30

∴ 0.02 × 15 = 0.30 (∵ Number of decimal places in 0.02 is 2)

MULTIPLICATION OF TWO DECIMAL NUMBERS

To multiply two decimal numbers :

(*i*) First, multiply the two decimal numbers without the decimal point (ignoring the decimal points) just like whole numbers.

(*ii*) In the product, place the decimal point leaving as many digits from the right as the total number of the decimal places in the given decimal numbers.

Example-3 : **Find the product.**

(*i*) 1.25 × 3.1 (*ii*) 1.01 × 10.01 (*iii*) 0.75 × 2.1

Sol. (*i*) 1.25 × 3.1

Ignoring the decimal points

First we shall multiply 125 by 31

Now 125 × 31 = 3875

Number of decimal places in 1.25 is 2

Number of decimal places in 3.1 is 1

Total number of decimal places in given decimal numbers = 2 + 1 = 3

∴ The decimal point in the product will be placed leaving three digits from the right.

∴ 1.25 × 3.1 = 3.875

(*ii*) 1.01 × 10.01

Ignoring the decimal points

First we shall multiply 101 by 1001

Now 101 × 1001 = 101101

Total number of decimal places in given decimal numbers = 2 + 2 = 4

∴ The decimal point in the product will be placed leaving four digits from the right.

1.01 × 10.01 = 10.1101

(*iii*) 0.75 × 2.1

Ignoring the decimal points

First we shall multiply 75 by 21

Now 75 × 21 = 1575

Total number of decimal places in given decimal numbers = 2 + 1 = 3

∴ The decimal point in the product will be placed leaving three digits from the right.

0.75 × 2.1 = 1.575

Example-4 : **The length of a rectangle is 8.5cm and its breadth is 5.7cm. What is the area of the rectangle ?**

Sol. Length of rectangle = 8.5cm

Breadth of rectangle = 5.7cm

∴ Area of rectangle = Length × Breadth

= 8.5 cm × 5.7cm

= 48·45cm^2.

EXERCISE - 2.6

1. Find the product of each of the following :-

(*i*) 1.31 × 10 (*ii*) 25.7 × 10
(*iii*) 1.01 × 100 (*iv*) 0.45 × 100
(*v*) 9.7 × 100 (*vi*) 3.87 × 10
(*vii*) 0.07 × 10 (*viii*) 0.3 × 100
(*ix*) 53. 7 × 1000 (*x*) 0.02 × 1000

2. Find the product of each of the following :-

(*i*) 1.5 × 3 (*ii*) 2.71 × 12
(*iii*) 7.05 × 4 (*iv*) 0.05 × 12
(*v*) 112.03 × 8 (*vi*) 3 × 7.53

3. Evaluate the following :-

(*i*) 3.7 × 0.4 (*ii*) 2.75 × 1.1
(*iii*) 0.07 × 1.9 (*iv*) 0.5 × 31.83
(*v*) 7.5 × 5.7 (*vi*) 10.02 × 1.02
(*vii*) 0.08 × 0.53 (*viii*) 21.12 × 1.21
(*ix*) 1.06 × 0.04

4. A piece of wire is divided into 15 equal parts. If length of one part is 2.03*m*, then find the total length of the wire.

5. The cost of 1 metre cloth is ₹ 75.80 Find the cost of 4.75 metre cloth.

6. Multiple choice questions :-

(*i*) 1.25 × 10 = ?
(*a*) 0.125 (*b*) 125
(*c*) 12.5 (*d*) 1.25

(ii) If *x* × 100 = 135.72 then value of *x* is equal to
(*a*) 13.572 (*b*) 1.3572
(*c*) 135.72 (*d*) 13572

(*iii*) The value of 1.5 × 8 is
(*a*) 1.2 (*b*) 120
(*c*) 12 (*d*) 0.12

7. (*i*) The product of a decimal number and zero is always zero. **(True/ False)**

(*ii*) On multiplying a decimal number by 10, the decimal point is shifted to the left by one place. **(True/ False)**

DIVISION OF DECIMAL NUMBERS

Division of a decimal number by 10,100 and 1000.

(*i*) On dividing a decimal number by 10, the decimal point is shifted to the left by one place.
(*ii*) On dividing a decimal number by 100, the decimal point is shifted to the left by two places.
(*iii*) On dividing a decimal number by 1000, the decimal point is shifted to the left by three places.

i.e. when a decimal number is divided by 10, 100 or 1000, the digits of the decimal number and the quotient are same but the decimal point in the quotient shifts to the left by as many places as there are zeroes over one.

Example-1 : Evaluate the following

(*i*) 25.73 ÷ 10 (*ii*) 15.3 ÷ 100 (*iii*) 3.25 ÷ 1000

Sol. (*i*) 25.73 ÷ 10

$= 25.73 \times \frac{1}{10}$

$= 2.573$

(We shift the decimal point by one place to the left as there is one zero in 10)

(*ii*) 15.3 ÷ 100

$= 15.3 \times \frac{1}{100}$

$= 0.153$

(we shift the decimal point by two places to the left as there are two zeroes in 100)

(*iii*) 3.25 ÷ 1000

$= 3.25 \times \frac{1}{1000}$

$= 0.00325$

(we shift the decimal point by three places to the left as there are three zeroes in 1000)

DIVISION OF A DECIMAL NUMBER BY A WHOLE NUMBER

To divide a decimal number by a whole number, write the given decimal number in fractional form with 10, 100, 1000 or so on as denominator. Multiply the fraction so found by the reciprocal of the given whole number (as we do in the division of a fraction by a whole number). Finally write the obtained fraction in the decimal form.

For example, to divide 3.45 by 5

i.e. 3.45 ÷ 5

Step I : Write 3.45 in fractional form as $\frac{345}{100}$

Step II : Multiple $\frac{345}{100}$ by the reciprocal of 5

i.e. $\frac{345}{100} \times \frac{1}{5}$ gives us $\frac{69}{100}$

Step I : Write $\frac{69}{100}$ in the decimal form as 0.69

∴ 3.45 ÷ 5 = 0.69

5) 345 (69
–30
45
45
0

Note : Here and in the next section (i.e. division of a decimal number by another decimal), Only those divisions will be considered in which, ignoring the decimal, the number in the numerators place would be completely divisible by the number in the denominators place. The situations in which the division of the numerator by the denominator leaves a remainder other than zero such situations will be studied in the next class. (For example 145 ÷ 7)

Example-2 : Find the value of (*i*) 13.6 ÷ 4 (*ii*) 73.282 ÷ 11

Sol. (*i*) 13.6 ÷ 4

$= \frac{136}{10} \div 4$

$= \frac{136}{10} \times \frac{1}{4}$

$= \frac{34}{10}$

= 3.4

```
4)136(34
 -12
  16
  16
   ×
```

(*ii*) 73.282 ÷ 11

$= \frac{73282}{1000} \div 11$

$= \frac{73282}{1000} \times \frac{1}{11}$

$= \frac{66\,62}{1000}$

= 6.662

```
11)73285(6662
  -66
    72
   -66
     68
    -66
      22
      22
       ×
```

DIVISION OF A DECIMAL NUMBER BY ANOTHER DECIMAL NUMBER

To divide a decimal number by another decimal number, write both the given numbers in fractional form. Then follow the method used to divide a fraction by another fraction. In case, the result is a fraction, change it into decimal form.

For example, To divide 2.55 by 0.5

i.e. 2.55 ÷ 0.5

$= \frac{255}{100} \div \frac{5}{10}$

$= \frac{255}{100} \times \frac{10}{5}$

$= \frac{51}{10}$

= 5.1

```
5)255(51
 -25
   05
   -5
    0
```

Example-3 : Find (*i*) 31.5 ÷ 1.5 (*ii*)12.42 ÷ 1.8

Sol. (*i*) 31.5 ÷ 1.5

$\frac{315}{10} \div \frac{15}{10}$

$= \frac{315}{10} \times \frac{10}{15}$

= 21

(*ii*) $12.42 \div 1.8$

$= \frac{1242}{100} \div \frac{18}{10}$

$= \frac{1242}{100} \times \frac{10}{18}$

$= \frac{69}{10}$

$= 6.9$

Example-4 : Find the average of 1.3, 3.2 1.7 and 0.6

Sol. The average of 1.3, 3.2, 1.7 and 0.6 is

$$= \frac{1.3 + 3.2 + 1.7 + 0.6}{4}$$

$$= \frac{6.8}{4} = 1.7$$

Example-5 : A car travelled 79.2 km in 4.5 litres of petrol. Find the distance travelled by the car in 1 litre petrol.

Sol. Distance travelled in 4.5 litres of petrol = 79.2km

Distance travelled in 1 litre of petrol = $\frac{79.2}{4.5}$ km = 17.6 km

EXERCISE - 2.7

1. Solve, dividing decimal number by 10, 100 or 1000 in the following.

(*i*) $2.7 \div 10$ (*ii*) $3.35 \div 10$ (*iii*) $0.15 \div 10$
(*iv*) $32.7 \div 10$ (*v*) $5.72 \div 100$ (*vi*) $23.75 \div 100$
(*vii*) $532.73 \div 100$ (*viii*) $1.321 \div 100$ (*ix*) $2.5 \div 1000$
(*x*) $53.83 \div 1000$ (*xi*) $217.35 \div 1000$ (*xii*) $0.2 \div 1000$

2. Solve, dividing decimal number by whole number.

(*i*) $7.5 \div 5$ (*ii*) $16.9 \div 13$ (*iii*) $65.4 \div 6$
(*iv*) $0.121 \div 11$ (*v*) $11.84 \div 4$ (*vi*) $47.6 \div 7$

3. Solve, dividing the decimal number by decimal number

(*i*) $3.25 \div 0.5$ (*ii*) $5.4 \div 1.2$ (*iii*) $26.32 \div 3.5$
(*iv*) $2.73 \div 13$ (*v*) $12.321 \div 11.1$ (*vi*) $0.0018 \div 0.15$

4. 25 steel chairs were purchased by a school for ₹ 11, 883.75. Find the cost of one steel chair.

5. A car covers a distance of 276.75km in 4.5 hours. What is the average speed of the car.

6. Multiple choice questions

(*i*) 27.5 ÷ 10 = ?

(*a*) 275 (*b*) 0.275

(*c*) 2.75 (*d*) none of these

(*ii*) The value of 1.5 ÷ 3 is

(*a*) 5 (*b*) 0.05

(*c*) 0.5 (*d*) 4.5

(*iii*) The average of decimal numbers 1.1, 2.1 and 3.1 is

(*a*) 2.5 (*b*) 1.1

(*c*) 2.1 (*d*) 6.3

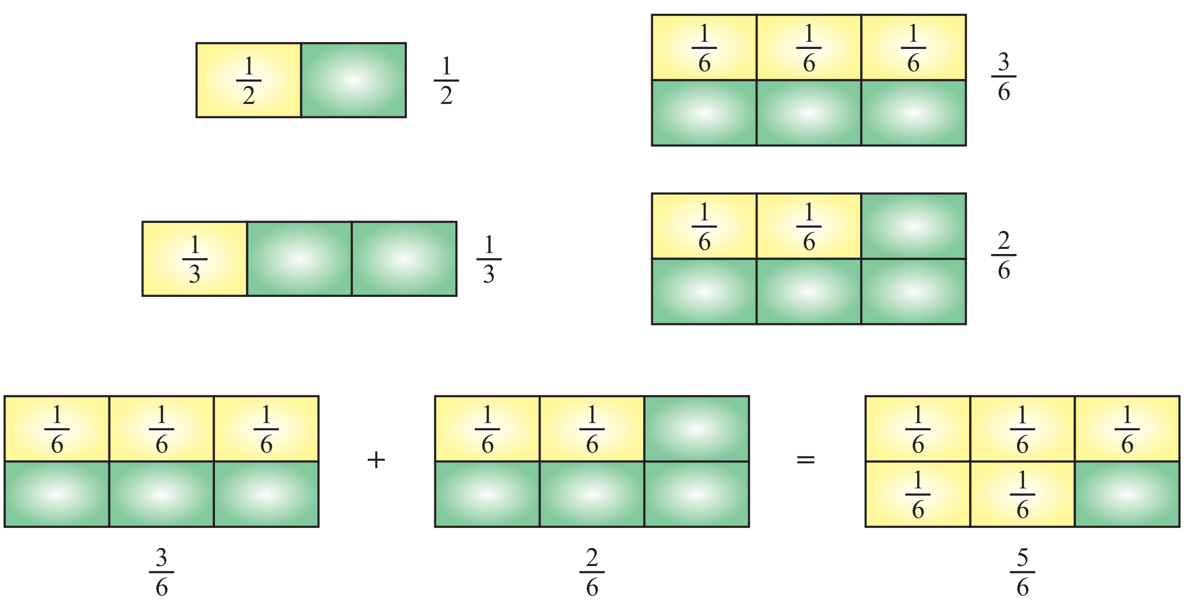
7. On dividing a decimal number by 100, the decimal point is shifted to the left by one place. **(True/ False)**

ACTIVITY

Objective : To find sum of two unlike fractions by an activity.

Material Required : Paper, Scale, Pencil, Coloured Pencil.

Procedure : Let two fractions to be added are $\frac{1}{2}$ and $\frac{1}{3}$. Take LCM of 2 and 3 which is 6. Now follow the steps as shown below :



Observation : we need to make their denominators same as the LCM of the denominators and then add numerators only.

Learning Outcomes : We learn that to find the sum of unlike fractions their denominators must be same.

VIVA VOCE

Q.1. What are unlike fractions ?

Ans. Two (or more) fractions having different denominators are called unlike fractions.

Q.2. What is L.C.M. of 2 and 5 ?

Ans. 10

Q.3. What is the denominator in $\frac{3}{5}$?

Ans. 5.

WHAT HAVE WE DISCUSSED ?

1. In the fraction $\frac{a}{b}$, we call a as numerator and b as denominator.

2. Classification of fractions

Type of fractions	Conditions
Proper fractions	Numerator is less than the denominator
Improper fractions	Numerator is greater than the denominator
Mixed fractions	Consists of a natural number and a proper fraction
Like fractions	Having the same denominators
Unlike fractions	Having different denominators.
Decimal fractions	Denominator is 10,100,1000.......... etc.
Simple fractions	Denominator is other than 10,100, 1000......... etc.
Equivalent fractions	Represent same part of a whole.

3. Product of fractions $= \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$

4. A fraction acts as an operator 'of' For example, $\frac{1}{3}$ of 3 is $\frac{1}{3} \times 3 = 1$

5. To find the reciprocal of a fraction, we interchange the numerator and the denominator.

6. To divide a whole number by a fraction, we multiply the whole number with the reciprocal of that fraction.

7. To divide a fraction by a whole number, we multiply the fraction by the reciprocal of the whole number.

8. To divide a fraction by another fraction, we multiply the first fraction by the reciprocal of the other fraction.

9. To multiply a decimal number by 10,100,1000 etc. we shift the decimal point to the right by as many places as the number of zeroes in 10,100, 1000 etc.

10. To multiply a decimal number by a whole number, multiply the given decimal number (without decimal point) by the given whole number. In the product, the number of decimal places will be same as that in the given decimal number.

11. To multiply a decimal number by another decimal number, multiply the two decimal numbers without the decimal point. In the product, place the decimal point, leaving as many digits from the right as the total number of decimal places in the given numbers.

12. To divide a decimal number by 10,100,1000 etc. we shift the decimal point to the left by as many places as the number of zeroes in 10, 100, 1000 etc.

13. To divide a decimal number by a whole number. Write the dividend in the fractional form and then multiply the fraction by the reciprocal of the whole number and convert the new fraction in decimal form.

14. To divide a decimal number by another decimal number write both the given numbers in fractional form. Then follow the method and to divide a fraction by another fraction. Finally change the new fraction in the decimal form.

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Find the reciprocal of a given fraction.
2. Multiply or divide two fractions and interpret them.
3. Use algorithms to multiply and divide fractions or decimals.
4. Solve the problems involving decimal numbers or fractions in daily life situations.

ANSWERS

EXERCISE 2.1

1. (*i*) $4\frac{7}{8}$ (*ii*) $\frac{91}{165}$ (*iii*) $1\frac{7}{80}$

(*iv*) $8\frac{7}{10}$ (*v*) $4\frac{7}{8}$ (*vi*) $\frac{819}{1000}$

2. (*i*) $\frac{2}{17}, \frac{3}{17}, \frac{5}{17}, \frac{8}{17}, \frac{10}{17}, \frac{6}{17}$ (*ii*) $\frac{1}{5}, \frac{3}{7}, \frac{7}{10}$

3. $2\frac{1}{5}$ cm **4.** $2\frac{13}{15}$ hours

5. $46\frac{1}{3}$ m **6.** Vaibhav, by $\frac{1}{6}$ of an hour

7. (*i*) *a* (*ii*) *b* (*iii*) *c*

(*iv*) *b* (*v*) *d*

EXERCISE 2.2

1. (*a*) (*ii*) (*b*) (*iii*)
(*c*) (*iv*) (*d*) (*i*)

2. (*i*) $1\frac{1}{3}$ (*ii*) $6\frac{2}{7}$ (*iii*) $4\frac{1}{2}$
(*iv*) $6\frac{3}{7}$ (*v*) $11\frac{1}{3}$ (*vi*) $54\frac{1}{6}$
(*vii*) $33\frac{3}{4}$ (*viii*) $27\frac{1}{5}$

3. (*i*) 23 (*ii*) 18 (*iii*) 12
(*iv*) 12 (*v*) 25

5. ₹ 11000

6. ₹ 940

7. (*i*) *c* (*ii*) *d*
(*iii*) *a* (*iv*) *c*

EXERCISE 2.3

1. (*i*) (*a*) $\frac{1}{15}$ (*b*) $\frac{2}{21}$ (*c*) $\frac{1}{2}$
(*ii*) (*a*) $\frac{1}{6}$ (*b*) $\frac{3}{7}$ (*c*) 2

2. (*i*) $\frac{2}{9}$ (*ii*) $\frac{5}{8}$ (*iii*) $\frac{4}{27}$
(*iv*) $\frac{3}{5}$ (*v*) $\frac{243}{350}$ (*vi*) $\frac{1}{9}$

3. (*i*) 8 (*ii*) $\frac{17}{21}$ (*iii*) $11\frac{1}{3}$
(*iv*) $\frac{481}{12}$ (*v*) $9\frac{2}{3}$ (*vi*) $7\frac{1}{5}$

4. (*i*) $\frac{5}{2}$ of $\frac{3}{8}$ (*ii*) $\frac{1}{2}$ of $\frac{6}{5}$

5. $386\frac{2}{9}$ km **6.** $374\frac{6}{11}$ *sq.m* **7.** ₹ $521\frac{1}{12}$

8. (*i*) *c* (*ii*) *a* (*iii*) *b*

9. (*i*) False (*ii*) True

EXERCISE 2.4

1. (*i*) $\frac{7}{2}$ (*ii*) $\frac{2}{3}$ (*iii*) $\frac{7}{5}$

(*iv*) 9 (*v*) $\frac{3}{2}$ (*vi*) $\frac{8}{7}$

2. (*i*) $\frac{19}{60}$ (*ii*) $\frac{4}{45}$ (*iii*) $\frac{1}{9}$

(*iv*) $\frac{7}{8}$ (*v*) $3\frac{3}{10}$ (*vi*) $1\frac{4}{9}$

3. (*i*) $3\frac{3}{7}$ (*ii*) $3\frac{4}{7}$ (*iii*) $1\frac{1}{2}$

(*iv*) $\frac{10}{13}$ (*v*) $\frac{7}{5}$

4. (*i*) $\frac{3}{5}$ (*ii*) $\frac{2}{3}$ (*iii*) $4\frac{1}{6}$

(*iv*) $\frac{5}{14}$ (*v*) $2\frac{1}{2}$ (*vi*) $1\frac{23}{25}$

5. $\frac{2}{3}m$

6. (*i*) *b* (*ii*) *c* (*iii*) *a*

7. (*i*) True (*ii*) False

EXERCISE 2.5

1. (*i*) 0.9 (*ii*) 1.37 (*iii*) 10.10

(*iv*) 1735.101 (*v*) 0.88

2. (*i*) $4 \times 10 + 0 + 3 \times \frac{1}{10} + 8\frac{1}{100}$

(*ii*) $4 + 0 \times \frac{1}{10} + 3 \times \frac{1}{100} + 8 \times \frac{1}{1000}$

(*iii*) $0 + 4 \times \frac{1}{10} + 0 \times \frac{1}{100} + 3 \times \frac{1}{1000} + 8 \times \frac{1}{10000}$

(*iv*) $4 + 3 \times \frac{1}{10} + 8 \times \frac{1}{100}$

3. (*i*) $\frac{5}{10}$ (*ii*) $\frac{5}{100}$ (*iii*) $\frac{5}{100}$ (*iv*) 5

4. (*i*) ₹ 0.55 (*ii*) ₹ 55.05
(*iii*) ₹ 3.47 (*iv*) ₹ 0.02

5. (*i*) 0.350*km* (*ii*) 4.035*km*
(*iii*) 2.005*km*

6. (*i*) *d* (*ii*) *b*
(*iii*) *c* (*iv*) *d*

EXERCISE 2.6

1. (*i*) 13.1 (*ii*) 257 (*iii*) 101
(*iv*) 45 (*v*) 970 (*vi*) 38.7
(*vii*) 0.70 (*viii*) 30 (*ix*) 53700
(*x*) 20

2. (*i*) 4.5 (*ii*) 32.52 (*iii*) 28.2
(*iv*) 0.6 (*v*) 896.24 (*vi*) 22.59

3. (*i*) 1.48 (*ii*) 3.025 (*iii*) 0.133
(*iv*) 15.915 (*v*) 42.75 (*vi*) 10.2204
(*vii*) 0.0424 (*viii*) 25.5552 (*ix*) 0.0424

4. 30.45m **5.** ₹ 360.05

6. (*i*) *c* (*ii*) *b* (*iii*) *c*

7. (*i*) True (*ii*) False

EXERCISE 2.7

1. (*i*) 0.27 (*ii*) 0.335 (*iii*) 0.015
(*iv*) 3.27 (*v*) 0.0572 (*vi*) 0.2375
(*vii*) 5.3273 (*viii*) 0.01312 (*ix*) 0.0025
(*x*) 0.05383 (*xi*) 0.21735 (*xii*) 0.0002

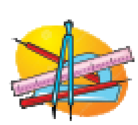
2. (*i*) 1.5 (*ii*) 1.3 (*iii*) 10.9
(*iv*) 0.011 (*v*) 2.96 (*vi*) 6.8

3. (*i*) 6.5 (*ii*) 4.5 (*iii*) 7.52
(*iv*) 0.21 (*v*) 1.11 (*vi*) 0.012

4. ₹ 475.35 **5.** 61.5 km/hr

6. (*i*) c (*ii*) c
(*iii*) c

7. False



CHAPTER 3

X9U8L3

Data Handling

Learning Objectives :-

In this chapter you will learn

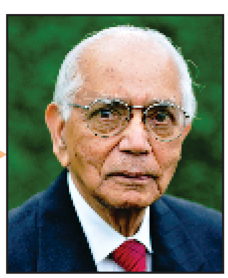
1. To collect the data.
2. To organise the collected data and interpret data for further references.
3. To find average or mean of the given data.
4. To arrange the data in ascending or descending order to find mode and median of the given data.
5. To draw the bar graph or double graph of the data.
6. About all the representative values of central tendency of a data *i.e.* mean, mode and median.
7. About the concepts of chance and probability in daily life situations.
8. About the possibilities of the events to be happen.

INTRODUCTION

In Class VI, you have learnt to collect data, organize various types of data and tabulate it and present it in the form of bar graphs. The collection, organization, tabulation and presentation of data help us in analyzing, interpreting and drawing inferences for further use.

Today data handling is one of the most important tasks in any organization - be it in hospitals where data needs to be maintained or in schools where cumulative records of students are kept for further reference. Data handling includes collection, interpretation or presentation of data by using various methods. It also forms an important part of a statistician's job. A statistician is a professional who collects, organises, analyses and interprets numerical data and uses it to explain what occured in the past or indicate what is likely to happen in future. Let us also attempt to understand the fundamentals of statistics. Who knows, tommorrow some of you might make a mark in the field of statistics just like Prof. C.R. Rao and Karen Dunnell.

Statisticians



Prof. C.R. Rao

Karen Dunnell

3.1 COLLECTION OF DATA

A data is a collection of facts in the form of numerical figures that is used to provide some information.

Each numerical figure (entry) in a data is called an observation (or variate) and the number of times a particular observation occurs in the data is called its frequency.

For example :

There are 25 employees in a government office. They were asked about the number of children they have. The results were :

1, 2, 3, 1, 0, 2, 0, 1, 2, 2, 1, 3, 5, 2, 4, 0, 3, 2, 4, 1, 1, 2, 2, 0, 3

We observe that each entry in this list is a fact in the form of a numerical figure, so it is data.

The data in this form is called raw data and it does not give much information. On the basis of these figures it will not be easy to answer the following questions :

(*i*) What is the maximum number of children that an employee has ?
(*ii*) How many employees have 2 children ?
(*iii*) How many employees have 2 or less than 2 children ?
(*iv*) How many employees have 2 or more than 2 children ?

3.2 ORGANISING AND TABULATING DATA

Let us arrange the above data in ascending or descending order.

The above data in ascending order is :

0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5

The data in this form is called an arrayed data.

The data in this form gives much better information. From this data it is easy to answer the first question - the maximum number of children, that an employes has, is 5. But answering the other three questions is still not easy. Moreover, arranging the data in this form is quite tedious and time consuming, particularly when the number of observations is large.

To answer the remaining three questions and to make the data more easily understandable, we shall arrange the data in tabular form.

A table showing the frequency of various observations is called a frequency distribution table or frequency table.

Number of Children (Variable)	Number of Employees (Frequency)
0	4
1	6
2	8
3	4
4	2
5	1
Total	**25**

Now, it is very easy to answer the above three questions :

The number of employees who have 2 children = 8

The number of employees who have 2 or less than 2 children = 8 + 6 + 4 = 18

The number of employees who hae 2 or more than 2 children = 8 + 4 + 2 + 1 = 15

REPRESENTATIVE VALUES

You must be aware of the term average and would have come across a number of statements involving the term 'average' in your day-to-day life :

- Geeta spends an average of about 6 hours daily for her studies.
- The average temperature of Punjab in the month of June is about 40 degree celsius. (40°C)
- The average age of students in my class is 13 years.
- The average attendance of students in a school during final examination was 96 percent.

Think about the statements given above.

Do you think that the child in the first statement studies exactly for 6 hours daily ? Or is the temperature of the given place during that particular time always 40 degrees ? Or, is the age of each student in that class 13 years obviously not.

Then what do these statements tell you ? By average, we understand Geeta, usually, studies for 6 hours. On some days, she may study for less number of hours and on the other days she may study longer.

Similarly, the average temperature of Punjab is 40 degree celsius, means that, very often, the temperature in the month of June is around 40 degree celsius. Sometimes, it may be less than 40 degrees celsius and at other times, it may be more than 40°C.

Thus we realise that average is a number that represents or shows the central tendency of a group of observations or data. Since average lies between the highest and the lowest value of the given data so, we say that average is a measure of central tendency of a group of data. Different forms of data need different forms of representative or central value to describe it. One of these representative values is the "Arithmetic mean".

ARITHMETIC MEAN

Arithmetic mean usually termed a mean, is one of the measures of central tendency as it gives the average value of the given data.

The average or Arithmetic Mean (A.M.) or simply mean is defined as follows :

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Example-1 : Aman studies for 3 hours, 5 hours, 2 hours and 6 hours on four consecutive days. How many hours does he study per day on an average ?

Sol. The average time for which Aman studies per day

$$= \frac{3+5+2+6}{4} \text{ hours} = \frac{16}{4} = 4 \text{ hours}$$

Example-2 : The heights (in cm) of 7 students of class VII in a school are 142, 153, 166, 161, 165, 149, 156. Find their mean height.

Sol.

$$\text{Mean height} = \frac{142+153+166+161+165+149+156}{7}$$

$$= \frac{1092}{7} \text{ cm} = 156 \text{ cm}$$

Example-3 : Find the mean of first five prime numbers.

Sol. The first five prime numbers are 2, 3, 5, 7 and 11

$$\text{Mean} = \frac{2+3+5+7+11}{5} = \frac{28}{5} = 5.6$$

Hence, the mean of first five prime numbers = 5.6

RANGE

The difference between the highest observation and the lowest observation is called the range of the data. [Range = Highest observation – Lowest observation]

Look at the following example.

Example-4 : The marks (out of 100) obtained by a group of students in a mathematics test are 85, 76, 90, 85, 39, 48, 56, 95, 81 and 75. Find the

(*i*) **The highest and the lowest marks obtained by students.**

(*ii*) **Range of the marks obtained**

(*iii*) **Mean marks obtained by the group.**

Sol. (*i*) Arranging the marks in ascending order, we get :

39, 48, 56, 75, 76, 81, 85, 85, 90, 95

Highest Marks = 95

Lowest Marks = 39

(*ii*) Range of the marks obtained = 95 – 39 = 56

(*iii*) Mean marks obtained by the group = $\frac{85+76+90+85+39+48+56+95+81+75}{10}$

$$= \frac{730}{10} = 73$$

EXERCISE - 3.1

1. **Find the mean of the following data :**

   (*i*) 3, 5, 7, 9, 11, 13, 15

   (*ii*) 40, 30, 30, 0, 26, 60

2. Find the mean of the first five whole numbers.

3. **A batsman scored the following number of runs in six innings :**

   36, 35, 50, 46, 60, 55

   Calculate the mean runs scored by him in an inning.

4. **The ages in years of 10 teachers of a school are :**

   32, 41, 28, 54, 35, 26, 23, 33, 38, 40

   (*i*) What is the age of the oldest teacher and that of the youngest teacher ?

   (*ii*) What is the range of the ages of the teachers ?

   (*iii*) What is the mean age of these teachers ?

5. The rain fall (in mm) in a city on 7 days of a certain week was recorded as follows :

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Rain fall (in mm)	0.01	12.2	2.1	0.0	20.5	5.5	1.0

(*i*) Find the range of the rain fall in the above data.

(*ii*) Find the mean rainfall for the week.

(*iii*) How many days had the rainfall less than the mean rainfall ?

MODE

The observation which occurs maximum number of times in the given data is called mode (or modal value) of the data.

Look at the example.

The owner of a readymade dress shop says, "The most popular size of dress I sell, is the size 90 cm.

Observe that here also, the owner is concerned about the number of dresses of different sizes sold. He is however looking at the shirt dress that is sold the most. This is another representative value for the data. The highest occuring event is the sale of size 90 cm. This representative value is called the mode of the data.

MODE OF LARGER DATA

Putting the same observations together and counting them is not easy if the number of observations is large. In such cases we tabulate the data. Tabulation can begin by putting tally marks and finding the frequency.

Example-1 : Find the mode of the following numbers

1, 1, 2, 4, 3, 2, 1, 2, 2, 4

Sol. Arranging the given data in ascending order, we get

1, 1, 1, 2, 2, 2, 2, 3, 4, 4.

In the given data, 2 occurs more number of times than any other number.

$\therefore$ Mode of given data = 2.

Example-2 : Following are the margins of victory (in goals) in the hockey matches of a league :

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2 6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2

Find the mode of this data.

Sol. As the given data has a large number of observations, we shall put it in tabular form

Margin of Victory (Observations)	Tally marks	Number of matches (frequency)
1	~~||||~~ ||||	9
2	~~||||~~ ~~||||~~ ||||	14
3	~~||||~~ ||	7
4	~~||||~~	5
5	|||	3
6	||	2
Total		**40**

Looking at the table, we find that the observation 2 has maximum frequency.

∴ Mode of the given data = 2

Example-3 : Find the mode of the numbers

1, 2, 2, 2, 3, 3, 5, 5, 5, 6, 8, 8, 10

Sol. Here 2 and 5 both occur three times.

Therefore both the numbers 2 and 5 are modes of the given data.

> Can a set of numbers have more than one mode ?

MEDIAN

We have seen that in some situations, mean is an appropriate measure of central tendency where as in some other situations, mode is an appropriate measure of central tendency. There may be some other situations where none of these measures can be a representative value. In that case we need to think of an alternative measure of central tendency. Look at the following data, which shows the heights (in cm) of 17 students of a class.

108, 112, 106, 125, 123, 119, 116, 114, 118, 115, 104, 102, 116, 101, 116, 120, 125

Let's arrange these observations in ascending order.

101, 102, 104, 106, 108, 112, 114, 115, 116, 116, 116, 118, 119, 120, 123, 125, 125

The middle value in this data is 116 because this value divides the students in two equal groups of 8 students each. This value is called is Median. **Median refers to the value which lies the middle of the data (when arranged in ascending or descending order)** with half of the observation above it and the other half below it.

Here, we consider only those cases, where the number of observations is odd.

Example-4 : Find the median of the data :

24, 36, 46, 17, 18, 25, 35.

Sol. Arranging the data in ascending order, we get

17, 18, 24, 25, 35, 36, 46

Median is the middle observation

∴ 25 is the median.

Example-5 : Find the median of the following data.

2, 0, 4, 12, 10, 6, 8, 5, 7

Sol. Arranging the given data in ascending order, we get

0, 2, 4, 5, 6, 7, 8, 10, 12

Median is the middle observation

∴ 6 is the median.

EXERCISE - 3.2

1. **Find the median of the following data :**

3, 1, 5, 6, 3, 4, 5

2. **Find the mode of the following numbers :**

2, 2, 2, 3, 4, 5, 5, 5, 6, 6, 8

3. **The scores in mathematics test (out of 25) of 15 students are as follows :**

19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20

Find the mean mode and median of this data. Are they same ?

4. **The weight (in kg) of 15 students of a class are :**

38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47.

(*i*) Find the mode and median of this data.

(*ii*) Is there more than one mode ?

5. **Find the mode and median of the following data :**

13, 16, 12, 14, 19, 12, 14, 13, 14

6. **Find the mode of the following data :**

12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 15, 16, 16, 15

17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14

Multiple Choice Questions :

7. **The mode of the data :**

3, 5, 1, 2, 0, 2, 3, 5, 0, 2, 1, 6 is

(*i*) 6 (*ii*) 3 (*iii*) 2 (*iv*) 1

8. **A cricketer scored 38, 79, 25, 52, 0, 8, 100 runs in seven innings, the range of the runs scored is :**

(*i*) 100 (*ii*) 92 (*iii*) 52 (*iv*) 38

9. **Which of the following is not a central tendency of a data ?**

(*i*) Mean (*ii*) Median (*iii*) Mode (*iv*) Range

10. **The mean of 3, 1, 5, 7 and 9 is**

(*i*) 6 (*ii*) 4 (*iii*) 5 (*iv*) 0

BAR GRAPHS

Drawing bar graph vertically or horizontally is a simple and effective way of representing data visually. In bar graphs, the height (or length) of a bar represents the frequency of the corresponding observation. All bars must be of equal width and there should be equal gap between the adjoining bars. In a bar graph, the breadth of bars has no significance. It is only for eye attraction. In a bar graph, it must be clearly mentioned on both axes what is being represented.

Choosing a scale : We know that a bar graph is representation of numbers using bars of uniform width and length of the bars depend upon the frequency and scale you have chosen. For example, in a bar graph where numbers in units are to be shown, the graph represents one unit length for one observation and if it has to show numbers in tens or hundreds, one unit length can represent 10 or 100 observations. Consider the following examples.

Example-1 : The number of students in six different classes of a school is given below. Represent the data by a bar graph.

Classes	V	VI	VII	VIII	IX	X
Number of students	135	120	95	100	90	80

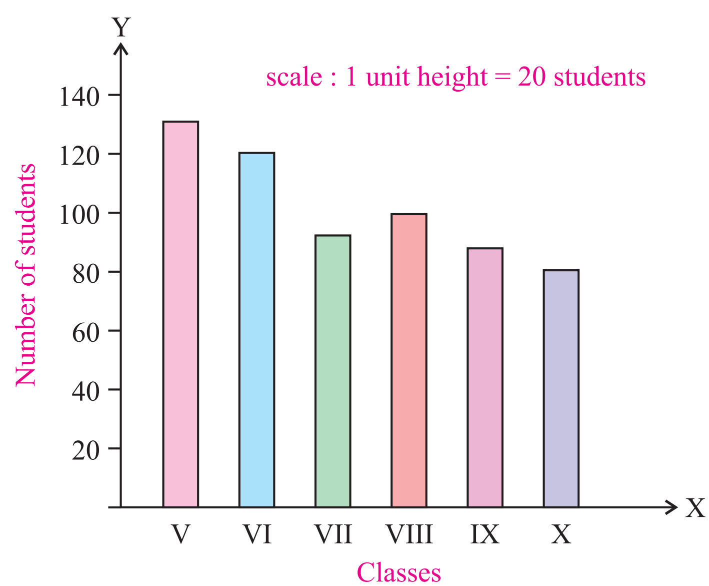
(*i*) How would you choose a scale ?

(*ii*) Which class has the maximum number of students ? And which class has the minimum ?

(*iii*) Find the ratio of the students of class VI to students of class VIII.

Sol. (*i*) **Choosing a proper scale :** Start the scale at 0. As the gratest value in the given data is 135, so end of the scale at a value little higher than 135, say at 140. Use equal divisions along vertical axis to choose a scale such that the length between 0 and 140 is neither too long nor too short. Take 1 unit height of bars = 20 students.

Mark different classes along horizontal axis. The bar graph representing the given data is shown below :



(*ii*) Class V has maximum number of students and class X has minimum number of students.

(*iii*) Ratio of number of students of class VI to number of students of class VIII
= 120 : 100 *i.e.* 6 : 5.

DOUBLE BAR GRAPHS

A graph that displays two sets of data using two bars drawn besides each other is called a double bar graph. This graph helps us to compare two collections of data at a glance.

Example-2 : The performance of a student in Ist term and IInd term is given below :

Subject :	English	Hindi	Maths	Science	S. Science
Ist Term (Out of 100) :	67	72	88	81	73
IInd Term (Out of 100) :	70	65	95	85	75

Draw a double bar graph choosing appropriate scale and answer the following questions :

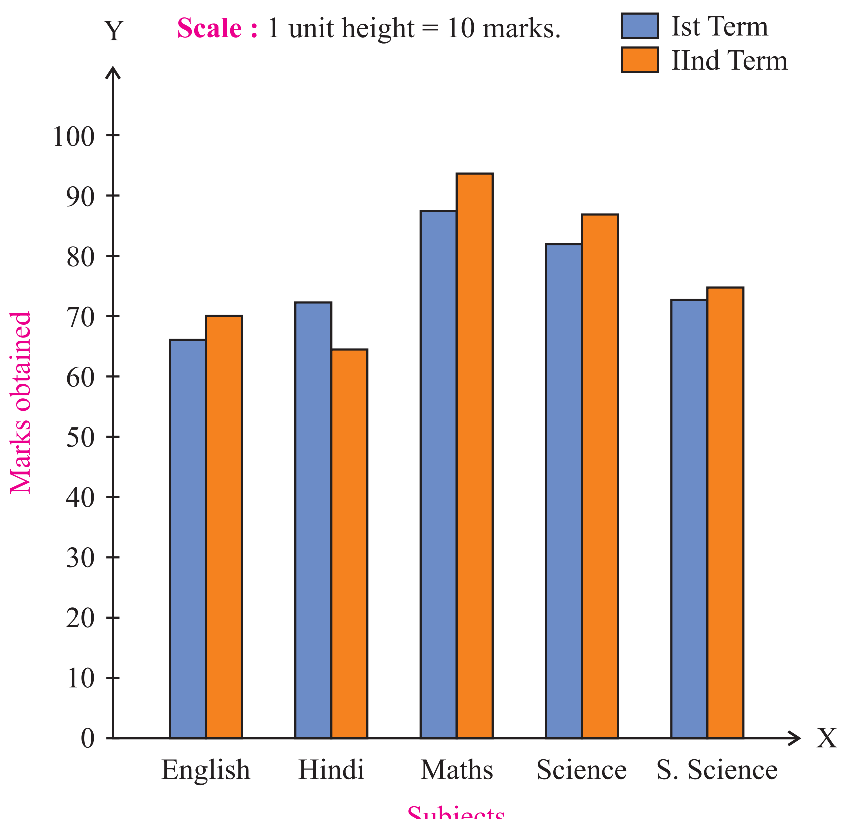
(*i*) In which subject has the student improved his performance the most ?

(*ii*) In which subject the improvement is least ?

(*iii*) Has the performance gone down in any subject ? Name the subject ?

Sol. Take the different subjects along X-axis and the marks obtained in different subjects along Y-axis.

Scale : Take 1 unit height along y-axis = 10 marks. The double bar graphs representing the given data is shown below :



(*i*) Maths

(*ii*) S. Science

(*iii*) Yes, Hindi

EXERCISE - 3.3

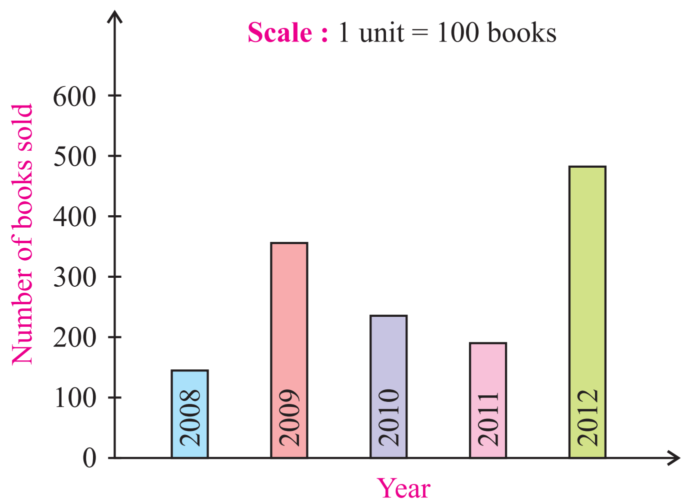
1. **Following data gives total marks (out of 600) obtained by six students of a particular class. Represent the data on a bar graph.**

Students :	Ajay	Bali	Dipti	Faiyaz	Geetika	Hari
Marks obtained :	450	500	300	360	400	540

2. **The following bar graph shows the number of books sold by a bookstore during five consecutive years. Read the bar graph and answer the following questions :**

(*i*) About how many books were sold in 2008, 2009 and 2011 years ?

(*ii*) In which year about 475 books were sold ? And in which year about 225 books were sold ?



3. **Two hundred students of 6th and 7th class were asked to name their favourite colour so as to decide upon what should be the colour of their school building. The resutls are shown in the table :**

Favourite Colour :	Red	Green	Blue	Yellow	Orange
Number of Students :	43	19	55	49	34

Represent the data on a graph.

Answer the following questions with the help of bar graph:

(*i*) Which is the most preferred colour ?

(*ii*) Which is the least preferred colour ?

(*iii*) How many colours are there in all ? What are they ?

4. **Consider the following data collected from a survey of a colony :**

Favourite Sport	**Cricket**	**Basket Ball**	**Swimming**	**Hockey**	**Athletics**
Watching	1240	470	510	430	250
Participating	620	320	320	250	110

Draw a double bar graph choosing an appropriate scale.

What do you infer from the bar graph :

(*i*) Which sports is the most popular ?

(*ii*) Which is more preferred, watching or participating in sports ?

5. The following table shows the time (in hours) spent by a student of class VII in a day.

Activity	School	Sleeping	Playing	Watching television	Studying	Others
Time (in hours)	8	8	1	3	2	2

Draw a bar graph to represent the above data. What do you infer from the above table ?

CHANCE

Chance is the occurrence of events. It is simply the possibility of something happening. Consider the statements given below and try to understand these terms a bit more ;

(*i*) The sun rises from the west.

(*ii*) An ant growing to 3m height.

(*iii*) India winning the next cricket match.

If we look at the statements given above you would say that the sun rises from the West is impossible, an ant growing to 3m is also not possible. On the other hand, India can win the match or lose it. Both are possible.

If you toss a coin, can you always correctly predict what you will get ? We toss a coin 10 times and write our observations in the following table.

Toss Number	1	2	3	4	5	6	7	8	9	10
Outcome	H	H	T	T	T	H	T	T	H	H

Where H represents head and T represents tail.

What does this data tell you ? Can you find a predictable pattern for head and tail ?

Clearly there is no fixed pattern of occurrence of head and tail. When you throw the coin each time the outcome of every throw can be either head or tail. It is matter of chance that in one particular throw you get either head or tail.

You must have played with a die. A die is a cube having six faces marked with numbers 1, 2, 3, 4, 5, 6.

When you throw a die, can you predict the number that will be obtained ? It is matter of chance that in one particular throw you get any of the numbers 1, 2, 3, 4, 5, 6 as outcome.

PROBABILITY

We know that when a coin is thrown, it has two possible outcomes, head or tail and for a die, we have 6 possible outcomes. We also know from experience that for a coin, head or tail is equally likely to be obtained. We say that probability of getting head or tail is equal and is $\frac{1}{2}$ for each.

For a die, probability of getting either of 1, 2, 3, 4, 5 or 6 is equal. For a die, there are 6 equally likely possible outcomes. We say that each of 1, 2, 3, 4, 5, 6 has one - sixth $\left(\frac{1}{6}\right)$ probability.

The probability of an event E, written as P (E), is defined as

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

We shall study more about this in next classes. This definition is just for information.

Events that have many possibilities can have probability between 0 and 1. Those which have no chance of happening have probability 0 and those that are bound to happen have probability 1.

- An experiment is a situation that involves a chance of the occurence of a particular event.
- An **outcome** is the result of an experiment.
- **Sample space** is the set of all possible outcomes in an experiment.
- An **event** is a specific outcome of an experiment.

Let us consider the following experiment.

Experiment : Rolling a die

Out comes : 1, 2, 3, 4, 5 or 6

Sample space : [1, 2, 3, 4, 5, 6]

Event : Getting an even number.

Example-1 : A bag contains 5 white and 9 red balls. One ball is drawn at random from the bag. Find the probability of getting (*i*) a white ball (*ii*) a red ball.

Sol. (*i*) Total number of balls in the bag = 5 + 9 = 14

Probability that the event is getting a white ball = $\frac{5}{14}$.

(*ii*) The event is getting a red ball.

Probability (getting a red ball) = $\frac{9}{14}$.

We have seen above that probability of getting a white ball and red ball lies between 0 and 1.

EXERCISE - 3.4

1. State whether the following is certain to happen, impossible to happen, may happen.

(*i*) Two hundred people sit in a Maruti car.

(*ii*) You are older than yesterday.

(*iii*) A tossed coin will land heads up.

(*iv*) A die when rolled shall land up 8 on top.

(*v*) Tommorrow will be a cloudy day.

(*vi*) India will win the next test series.

(*vii*) The next traffic light seen will be green.

2. **There are 6 marbles in a box with numbers 1 to 6 marked on them.**

(*i*) What is the probability of drawing a marble with number 5 ?

(*ii*) What is the probability of drawing a marble with number 2 ?

3. There are two teams A and B. A coin is flipped to decide which team starts the game. What is the probability that team A will start ?

4. A bag contains 3 red and 7 green balls. One ball is drawn at random from the bag. Find the probability of getting (*i*) a red ball (*ii*) a green ball.

Multiple Choice Questions :

5. The probability of an impossible event is

(*i*) – 1 (*ii*) 0 (*iii*) $\frac{1}{2}$ (*iv*) 1

6. The probability of selecting letter G from the word 'GIRL' is

(*i*) 1 (*ii*) $\frac{1}{2}$ (*iii*) $\frac{1}{4}$ (*iv*) $\frac{1}{3}$

7. When a die is thrown, the probability of getting a number 4 is

(*i*) $\frac{1}{2}$ (*ii*) $\frac{1}{3}$ (*iii*) $\frac{4}{6}$ (*iv*) $\frac{1}{6}$

8. A bag contains 5 white balls and 10 black balls. The probability of drawing a white ball from the bag is

(*i*) $\frac{5}{10}$ (*ii*) $\frac{5}{15}$ (*iii*) $\frac{10}{15}$ (*iv*) 1

WHAT HAVE WE DISCUSSED ?

1. A collection of facts regarding some information in the form of numerical figures is called data.

2. The numbers gathered in the data are called observations.

3. A data arranged in ascending or descending order is called arrayed data.

4. The number of times a particular observation occurs in a data is called its frequency.

5. Tally marks (|) are useful in counting observations. We write tally marks in bunches of five like ~~||||~~ .

6. A table showing the frequency of various observations is called frequency distribution table.

7. Data can be presented visually by bar graphs, drawn vertically or horizontally.

8. Double bar graphs help in comparing two collections of data at a glance.

9. Mean, median and mode are the representative values or measures of central tendency of a data.

10. For simple data, Mean = $\frac{\text{Sum of observations}}{\text{Number of observations}}$

11. Mode of a data is the observation with highest frequency.

12. Median is the value that lies in the middle of the data (data which is written either in ascending or in descending order)
13. Mode is always one of the given observations where as mean and median can be a value which is not an observation in the given data.
14. The value of mean, median and mode always lies between the lowest and the highest observations.
15. Probability is a measure of uncertainty.

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Collect the data.
2. Organise and interpret the data.
3. Find the mean of the given and simple data.
4. Arrange the data for finding mode and median of the data.
5. Interpret the data using bar graph and double graph according the given data.
6. Find the outcomes of an event.
7. Find the probability of an event.

ANSWERS

EXERCISE 3.1

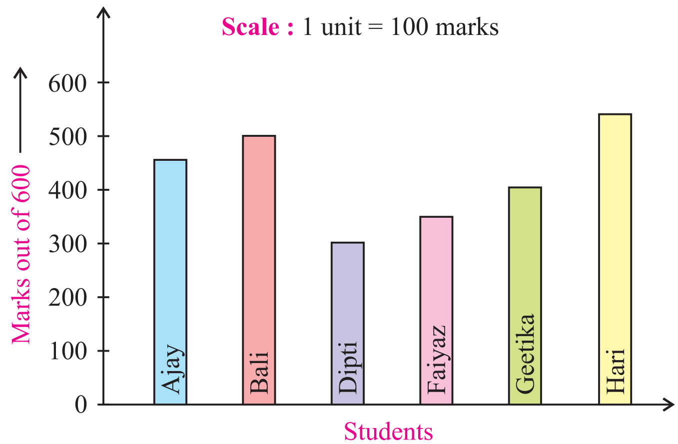
1. (*i*) 9 (*ii*) 31
2. 2
3. 47
4. (*i*) 54 years (*ii*) 31 years (*iii*) 35 years
5. (*i*) 20.5 (*ii*) 5.9 (*iii*) 5 days

EXERCISE 3.2

1. (*i*) 4 (*ii*) 4.5
2. 2 and 5
3. Mode = 20, Median = 20 ; Yes
4. (*i*) Mode = 38 kg, 43 kg, Median = 40 kg (*ii*) Yes
5. Median = 14 Mode = 14
6. 15.
7. (*iii*)
8. (*i*)
9. (*iv*)
10. (*iii*)

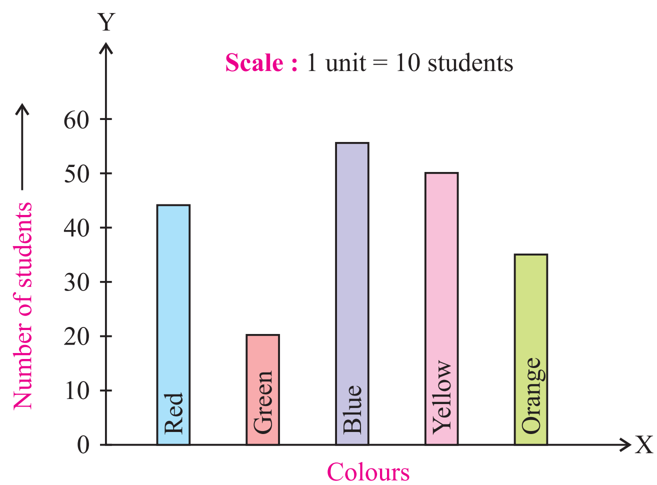
EXERCISE 3.3

1.



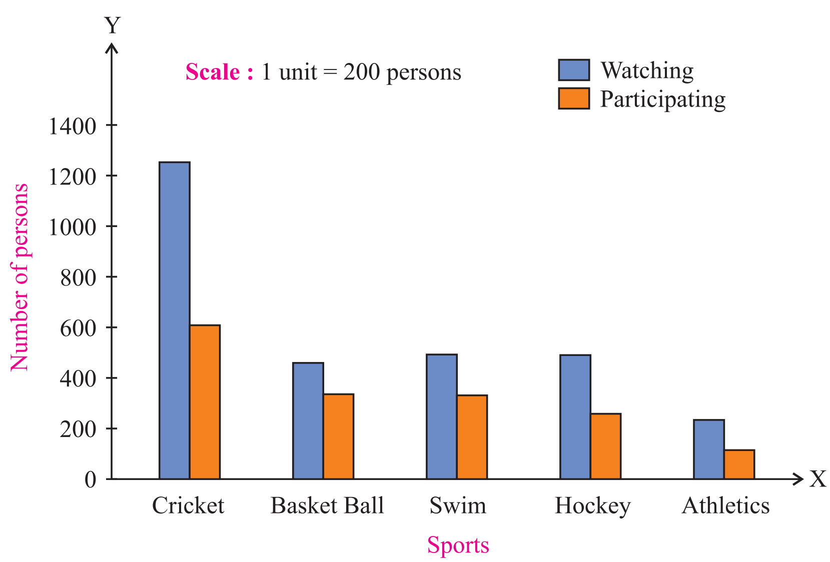
2. (*i*) 140 ; 360 ; 180 (*ii*) 2012 ; 2010

3.



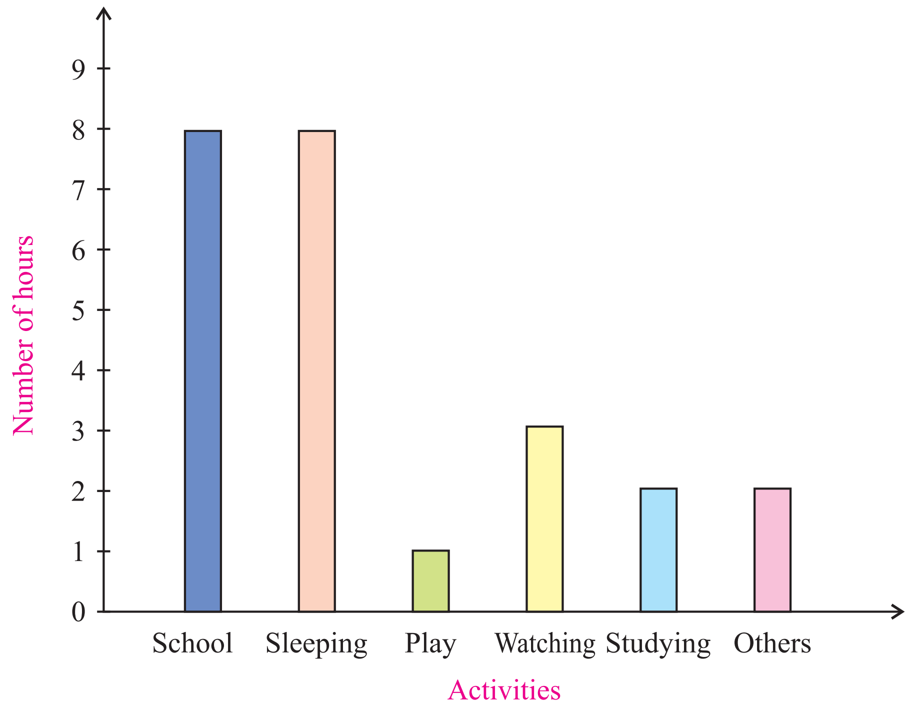
(*i*) Blue (*ii*) Green (*iii*) 5 ; Red, Green, Blue, Yellow and Orange.

4.



(*i*) Cricket (*ii*) Watching

5.



EXERCISE 3.4

1. (*i*) Impossible to happen

(*ii*) Certain to happen

(*iii*) Can happen but not certain

(*iv*) Impossible to happen

(*v*) Can happen but not certain

(*vi*) Can happen but not certain

(*vii*) Can happen but not certain

2. (*i*) $\frac{1}{6}$ (*ii*) $\frac{1}{6}$

3. $\frac{1}{2}$

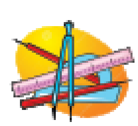
4. (*i*) $\frac{3}{10}$ (*ii*) $\frac{7}{10}$

5. (*ii*)

6. (*iii*)

7. (*iv*)

8. (*ii*)



CHAPTER 4

A8J5P5

Simple Equations

Learning Objectives :-

In this chapter you will learn :-

1. To verify if a number is a solution of an equation or not.
2. To form a linear equation from a given statement.
3. To convert an equation in to a statement.
4. To solve the simple equation by three methods i.e. trial and error method, balancing method and transposing the terms.
5. To use simple equations to solve their daily life problems.

INTRODUCTION

In class VI we have learnt that an equation is a statement of equality of two algebraic expressions. We have also learnt about linear equations, their formulation and solution by trial and error method. In this chapter we shall review the topics studied in class VI. We shall also learn to solve a linear equation by the method of transposition and about the practical utility of simple linear equations.

RECALL (EQUATIONS)

In primary classes we have solved problems on numbers like what must be added to 7 to get the sum 13. This problem can be written in another way as follows :

$\square + 7 = 13$

Here $\square$ is the unknown number which we have to find. We can easily find the number to fill the box so that the left hand side and the right hand side of the equality sign "=" are equal.

So the number 6 must be placed to fill $\square$ as $6 + 7 = 13$.

The same problem can also be written as $x + 7 = 13$.

Here we have used the letter x for the unknown number. Here 'x' is called a literal number.

We can use any other letters like y, z, a, b, c, etc. to denote the unknown.

The above expression of equality i.e., $x + 7 = 13$ is called an equation. Thus, in general a statement of equality combining one or more variable is called an equation.

The literal number 'x' used in above equation is called a variable.

An equation containing only one variable is called an equation in one variable. For example

$$2y + 6 = 7,\ p = \frac{7}{2},\ 2q + 10 = 0$$

$$3x^2 + 2x + 6 = 0,\ 2x^2 = 8 \text{ are all equations in one variable.}$$

If in an equation of one variable, the highest power of the variable is 1, the equation is called a linear equation in one variable.

The equations $2y + 6 = 7$, $p = \frac{7}{2}$, $2q + 10 = 0$ are all linear equations in one variable.

The equations $3x^2 + 2x + 6 = 0$ and $2x^2 = 8$ are not linear equations.

FORMATION OF AN EQUATION

Step 1 : Read the problem carefully and identify the unknown quantity or quantities.

Step 2 : Denote the unknown quantities by *x*, *y*, *z*.......... or *a*, *b*, *c*, ... etc.

Step 3 : Write the given statements in the form of expressions using mathematical symbols like, +, –, × and ÷.

Step 4 : Write the equation by equating the expressions according to the given problem.

Example-1 : Form the equations for the following statements.

(*a*) Seven times a number is 42.

(*b*) Two added to the half of a number gives the sum 17.

(*c*) If you subtract 5 from 6 times a number, you get 7.

(*d*) the sum of numbers *x* and 6 is 9.

Sol. (*a*) Let the unknown number be *x*.

∴ $7x = 42$ is the required equation.

(*b*) Let the unknown number be *y* then half of the number = $\frac{1}{2}y$.

2 added to it = $2 + \frac{1}{2}y$

According to given, $2 + \frac{1}{2}y = 17$

Which is the required equation.

(*c*) Let the number be *z*, *6 times z* is 6*z*.

Subtracting 5 from 6*z*, one gets $6z–5$.

According to the given statement, $6z – 5 = 7$ is the required equation.

(*d*) $x + 6 = 9$

Example-2 : Convert the following equations in statement form

(*i*) $x + 4 = 15$ **(*ii*)** $x – 7 = 3$

(*iii*) $2m = 8$ **(*iv*)** $\frac{p}{5} – 2 = 6$

Sol. (*i*) Add x and 4 to get 15

(*ii*) Taking away 7 from x gives 3

(*iii*) 2 times a number m is 8

(*iv*) subtracting 2 from one-fifth of a number *p* gives 6.

Example-3 : Write an equation for the following statement.

Laxmi's father's age is 5 years more than 3 times Laxmi's age. Laxmi's father is 44 years old.

Sol. We do not know Laxmi's age. Let us take it to be x years. Three times Laxmi's age is $3x$ years. Laxmi's father's age is 5 years more than $3x$; that is, Laxmi's father age is $(3x + 5)$ years. It is also given that Laxmi's father is 44 years old.

Therefore, $3x + 5 = 44$

This is an equation in x. It will give Laxmi's age when solved.

EXERCISE - 4.1

1. Complete the following :

Sr. No.	Equation	Value	Say, whether the equation statisfied (Yes/No)
(*i*)	$x + 5 = 0$	$x = 5$	
(*ii*)	$x + 5 = 0$	$x = -5$	
(*iii*)	$x - 3 = 1$	$x = 3$	
(*iv*)	$x - 3 = 1$	$x = -3$	
(*v*)	$2x = 10$	$x = 5$	
(*vi*)	$\frac{x}{3} = 2$	$x = -6$	
(*vii*)	$\frac{x}{3} = 2$	$x = 0$	

2. Check whether the value given in the brackets is a solution to the given equation or not.

(*i*) $x + 4 = 11$ $(x = 7)$ (*ii*) $8x + 4 = 28$ $(x = 4)$

(*iii*) $3m - 3 = 0$ $(m = 1)$ (*iv*) $\frac{x}{5} - 4 = -1$ $(x = 15)$

(*v*) $4x - 3 = 13$ $(x = 0)$

3. Solve the following equations by trial and error method

(*i*) $5x + 2 = 17$ (*ii*) $3p - 14 = 4$

4. Write equations for the following statements.

(*i*) the sum of numbers x and 4 is 9. (*ii*) 3 subtracted from y gives 9

(*iii*) Ten times x is 50 (*iv*) Nine times x plus 6 is 87

(*v*) One fifth of a number y minus 6 gives 3.

5. Write the following equations in statement form :

(*i*) $x - 2 = 6$ (*ii*) $3y - 2 = 10$

(*iii*) $\frac{x}{6} = 6$ (*iv*) $7x - 15 = 34$

(*v*) $\frac{x}{2} + 2 = 8$

6. Write an equation for the following statements :

(*i*) Raju's father's age is 4 years more than five times Raju's age. Raju's father is 54 years old.

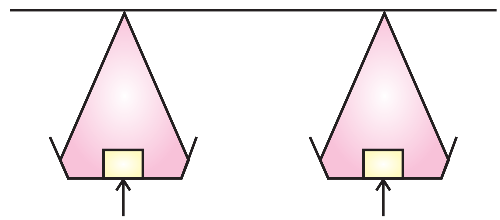
(*ii*) A teacher tells that the highest marks obtained by a student in his class is twice the lowest marks plus 6. The highest score is 86. (Take the lowest score to *be* x).

(*iii*) In an isosceles triangle, the vertex angle is twice either base angle (Let the base angle be x in degrees. Remember that the sum of angles of a triangle is 180 degrees).

(*iv*) A shopkeeper sells mangoes in two types of boxes. One small and one large. The large box contains as many as 8 small boxes plus 4 loose mangoes. The number of mangoes in a large box is given to be 100.

SOLVING AN EQUATION (BY BALANCING)

An equation is like a weighing balance having equal weights on both of its pans, where the arm of the balance is exactly horizontal. But the horizontal arm is not disturbed, if LHS and RHS are interchanged or if same amount of weight is added to both sides. Similarly if the same amount of weight is removed from both the pans the arm will remain balanced.



A balanced equation is like a weighing balance with equal weights in the two pans.

We can use this principle for solving an equation maintaining the sign of equality between the left side and the right side.

Consider the equation $x + 4 = 6$, we shall subtract 4 from both sides of this equation. The new LHS (left hand side) will be $x + 4 - 4 = x$.

In the same way the new RHS will be $6 - 4 = 2$

Hence LHS and RHS are still balanced (not changed), *i.e* $x = 2$

Rules for solving an equation :

(*i*) The same quantity can be added to both the sides of an equation without disturbing the balance.

(*ii*) The same quantity can be subtracted from both the sides of an equation without disturbing the balance.

(*iii*) Both the sides of an equation may be multiplied by the same non-zero number without disturbing the balance.

(*iv*) Both the sides of an equation may be divided by the same non-zero number without disturbing the balance.

(*v*) If we fail to do the same mathematical operation on both sides of an equation, the equality does not hold.

REMEMBER

Sometimes, we may have to carryout more than one mathematical operation for solving an equation. We should make an attempt that the variable in the equation gets separated.

Example-1 : Solve the equation $6x - 4 = 22$

Sol. Given equation is $6x - 4 = 22$

We will try to reduce the LHS of this equation by adding 4 to both sides

$$6x - 4 + 4 = 22 + 4$$
$$6x = 26$$

Now, on dividing both sides by 6, we get

$$\frac{6x}{6} = \frac{26}{6}$$

or $$x = \frac{26}{6} = \frac{13}{3}.$$

Thus, $x = \frac{13}{3}$ is the solution of the given equation

Example-2 : Solve (*i*) $2x + 6 = 12$ (*ii*) $\frac{p}{4} = 5$

Sol. (*i*) **Step I :** Subtract 6 from both the sides.

$$2x + 6 - 6 = 12 - 6$$
$$2x = 6$$

Step II : Divide both sides by 2

$$\frac{2x}{2} = \frac{6}{2}$$

or $x = 3$, which is the solution of the given equation.

One good practice you should develop is to check the solution you have obtained.

Let us put the solution $x = 3$ back into the equation

$$\text{LHS} = 2x + 6 = 2 \times 3 + 6 = 6 + 6$$
$$= 12 = \text{RHS}$$

The solution is thus checked for its correctness.

(*ii*) $\frac{p}{4} = 5$

Multiplying both sides by 4

$$\frac{p}{4} \times 4 = 5 \times 4$$
$$p = 20$$

∴ $p = 20$ is a solution of the given equation.

EXERCISE - 4.2

1. Write the first step that you will use to separate the variable and then solve the equation.

(*i*) $x + 1 = 0$ (*ii*) $x - 1 = 5$

(*iii*) $x + 6 = 2$ (*iv*) $y + 4 = 4$

(*v*) $y - 3 = 3$

2. Write the first step that you will use to separate the variable and then solve the equation:

(*i*) $3x = 15$ (*ii*) $\frac{p}{7} = 4$

(*iii*) $8y = 36$ (*iv*) $20x = -10$

3. Give the steps you will use to separate the variable and then solve the equation.

(*i*) $5x + 7 = 17$ (*ii*) $\frac{20x}{3} = 40$

(*iii*) $3p - 2 = 46$

4. Solve the following equations :

(*i*) $10\,x + 10 = 100$ (*ii*) $\frac{-p}{3} = 5$

(*iii*) $3x + 12 = 0$ (*iv*) $2q - 6 = 0$

(*v*) $3\,p = 0$ (*vi*) $3s = -9$

SOLVING EQUATIONS (BY TRANSPOSING)

Let's practice solving some more equations, by transposing numbers, i.e. moving it from one side to the other.

While transposing numbers, follow these :-

1. When number or a term added on one side is transposed (taken) to the other side it is subtracted.

i.e. $x + 4 = 10$

$\Rightarrow$ $x = 10 - 4 = 6$ (4 is transposed)

2. When number or a term subtracted on one side is transposed to the other side, it is added.

i.e. $y - 6 = 8$

$\Rightarrow$ $y = 8 + 6$ (6 is transposed)

3. When number or a term multiplied on one side is transposed to the other side, it divides the terms on other side.

i.e. $7z = 14$

$\Rightarrow$ $z = 14 \div 7$ or $\frac{14}{7} = 2$ (7 is transposed)

4. When number or a term dividing other numbers on one side is transposed to the other side, it is multiplied to the number or terms on the otherside.

i.e. $\frac{y}{8} = 5$

$\Rightarrow$ $y = 5 \times 8 = 40$ (8 is transposed)

Example-1 : Solve $12x - 3 = 21$

Sol. Transposing (–3) from LHS to RHS (on transposing –3 becomes + 3)

$$12x = 21 + 3 \text{ or } 12x = 24 \text{ (12 is transposed)}$$

$$x = \frac{24}{12} = 2$$

To check, put $x = 2$ in the LHS of equation

$$\text{L.H.S.} = 12x - 3 = 12(2) - 3 = 24 - 3 = 21 = \text{RHS}$$

Example-2 : Solve 3 (*y* + 7) = 15

Sol. Divide both sides by 3.

$$y + 7 = \frac{15}{3}$$

or $y + 7 = 5$

Transposing 7 to RHS

$$y = 5 - 7$$

or $y = -2$ which is the required solution.

To Check, Put $y = -2$ in the LHS

$$\text{L.H.S.} = 3(y + 7) = 3(-2 + 7) = 3(5) = 15 = \text{RHS}$$

Example-3 : Solve (*i*) $\frac{x}{5} + 3 = 1$ (*ii*) 3 (*x* – 2) = 2 (*x* + 1) –3. Also verify your answer.

Sol. Given $\frac{x}{5} + 3 = 1$

Transposing 3 from LHS to RHS

$$\frac{x}{5} = 1 - 3$$

$$\frac{x}{5} = -2$$

$$x = -2 \times 5 \text{ (5 is transposed)}$$

$$x = -10$$

To Check, Put $x = -10$ in the LHS of equation

$$\text{LHS} = \frac{x}{5} + 3 = \frac{-10}{5} + 3 = -2 + 3 = 1 = \text{RHS.}$$

(*ii*) $3(x - 2) = 2(x + 1) - 3$

Here we should remove the bracket first

$$3x - 6 = 2x + 2 - 3$$

Transposing 6 from LHS to RHS

$$3x = 2x + 2 - 3 + 6$$

$$3x = 2x + 5$$

Transposing $2x$ from RHS to LHS

$$3x - 2x = 5$$

$$x = 5$$

To Check, Put $x = 5$ in the L.H.S.

$$\begin{aligned} \text{L.H.S. } 3(x-2) &= 2(x+1)-3 \\ 3(5-2) &= 3 \times 3 = 9 \\ \text{R.H.S. } 2(x+1)-3 &= 2(5+1)-3 \\ &= 12-3 = 9 \\ \text{L.H.S.} &= \text{R.H.S.} \end{aligned}$$

From Solution to equation

Equation $\rightarrow$ solution (normal path)

Solution $\rightarrow$ Equation (reverse path)

Example-4 : Construct 3 equations starting with $x = 5$

Solution start with	$\downarrow$	$x = 5$	$\uparrow$	Divide both sides by 4
Multiplying both sides by 4		$4x = 20$		
Subtract 3 from both sides	$\downarrow$	$4x - 3 = 17$	$\uparrow$	Add 3 to both sides
Adding 4 to both side	$\downarrow$	$4x + 1 = 21$	$\uparrow$	Subtract 4 from both side

EXERCISE - 4.3

1. Solve each of the following equation.

   (i) $6x + 10 = -2$ (ii) $2y - 3 = 2$

   (iii) $\frac{a}{5} + 3 = 2$ (iv) $\frac{3x}{2} = \frac{2}{3}$

   (v) $\frac{5}{2}x = -5$ (vi) $2x + \frac{5}{2} = \frac{37}{2}$

2. Solve the following equation

   (i) $5(x + 1) = 25$ (ii) $2(3x - 1) = 10$

   (iii) $4(2 - x) = 8$ (iv) $-4(2 + x) = 8$

3. Solve the following equations :

   (i) $4 = 5(x - 2)$ (ii) $-4 = 5(x - 2)$

   (iii) $4 + 5(p - 1) = 34$ (iv) $6y - 1 = 2y + 1$

4. (i) Construct 3 equations starting with $x = 2$

   (ii) Construct 3 equation starting with $x = -2$

Multiple Choice Questions :-

5. If $7x + 4 = 39$, then x is equal to

   (i) 6 (ii) –4 (iii) 5 (iv) 8

6. If $8m - 8 = 56$ then m is equal to

   (i) –4 (ii) –2 (iii) –14 (iv) 8

7. Which of the following number satisfies the equation $-6 + x = -18$?

   (i) 10 (ii) –13 (iii) –12 (iv) –16

8. If $\frac{x}{2} = 14$, then the value of 2x + 6 is equal to.

(*i*) 62 (*ii*) –64 (*iii*) 16 (*iv*) –62

9. If 3 subtracted from twice a number is 5, then the number is

(*i*) –4 (*ii*) –2 (*iii*) 2 (*iv*) 4

10. If 5 added to thrice an integer is –7, then the integer is

(*i*) –6 (*ii*) –5 (*iii*) –4 (*iv*) 4

APPLICATION OF SIMPLE EQUATIONS TO PRACTICAL SITUATIONS

Due to the wide variety of word (or applied) problems, there is no single technique that works in all problems. However, the following general suggestion may prove helpful.

- Read the statement of the problem carefully and determine what quantity must be found.
- Represent the unknown quantity by a letter.
- Determine which expressions are equal and write an equation.
- Solve the resulting equation.

Example-1 : If 5 is added to twice a number, the result is 29. Find the number.

Sol. Let the required number be x

Twice the number $= 2x$

5 added to twice the number $= 2x + 5$.

According to the problem

$$2x + 5 = 29$$
$$2x = 29 - 5$$
$$2x = 24$$
$$x = \frac{24}{2}$$
$$\Rightarrow \quad x = 12$$

Hence the required number is 12.

Example-2 : Find a number, such that one fourth of the number is 10.

Sol. Let us take the unknown number to be x ; one fourth of x is $\frac{x}{4}$.

Hence we get the equation $\frac{x}{4} = 10$

Transposing 4,

$$x = 10 \times 4$$

Or $x = 40$ which is the required number. Let us check the solution. Putting value of x in the equation.

$$\text{LHS} = \frac{x}{4}$$
$$\frac{40}{4} = 10 = \text{RHS}$$

Example-3 : Radha's father's age is 49 years old. He is 4 years older than three times Radha's age. What is Radha's age ?

Sol. Let Radha's age be x years, then

Radha's father's age = $(3x + 4)$ years but Radha's father's age is 49 years.

According to question

$$3x + 4 = 49$$

$$\Rightarrow \quad 3x = 49 - 4$$

$$3x = 45$$

Dividing both sides by 3

$$\frac{3x}{3} = \frac{45}{3}$$

$$x = 15$$

$\therefore$ Radha's age = 15 years.

EXERCISE - 4.4

1. If 7 is added to five times a number, the result is 57. Find the number.
2. 9 decreased from four times a number yields 43. Find the number
3. If one-fifth of a number minus 4 gives 3, find the number.
4. In a class of 35 students, the number of girls is two-fifth the number of boys. Find the number of girls in the class.
5. Sham's father's age is 5 years more than three times Sham's age. Find Sham's age, if his father is 44 years old.
6. In an isoscels triangle the base angles are equal, the vertex angle is 40°. What are the base angles of the triangle ? (Remember, the sum of three angles of a triangle is 180°)
7. Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. How many marbles does Parmit have ?
8. The length of a rectangle is 3 units more than its breadth and the perimeter is 22 units. Find the breadth and length of the rectangle.

WHAT HAVE WE DISCUSSED ?

1. An (algebraic) equation is a mathematical statement that sets two expressions equal. It may involve one or more unknowns called variables or literal numbers.
2. An equation containing only one variable (literal) with highest power 1 is called a linear equation in one variable.
3. A number which satisfies the given equation is called a solution of the equation.
4. The process of finding the particular value of the variable (literal) which makes both sides of the equation equal is called solving the equation.
5. An equation remains the same if the LHS and RHS are inter changed.
6. In case of a balanced equation, if we
   (*i*) add the same number to both the sides, or
   (*ii*) subtract the same number from both the sides, or

(*iii*) multiply both sides by the same number, or

(*iv*) divide both sides by the same number, the balance remains undisturbed, i.e, the value of the LHS remains equal to the value of the RHS.

7. While solving simple (or applied) word problems involving one unknown, we first have to write an equation corresponding to the given statement and then solve this equation to find the value of the unknown.

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Identify the unknown quantity.
2. Understand the meaning of linear equation.
3. Find the solution of an equation.
4. Determine whether the given value of an unknown is a solution or not.
5. Represent daily life situations in the form of simple equations and solve them.
6. Convert a statement into an equation.
7. Apply the three methods of solving an equation.

ANSWERS

EXERCISE 4.1

1. (*i*) No (*ii*) Yes (*iii*) No
(*iv*) No (*v*) Yes (*vi*) No
(*vii*) No

2. (*i*) Yes (*ii*) No (*iii*) Yes
(*iv*) Yes (*v*) No

3. (*i*) $x = 3$ (*ii*) $p = 6$

4. (*i*) $x + 4 = 9$ (*ii*) $y - 3 = 9$ (*iii*) $10x = 50$
(*iv*) $9x + 6 = 87$ (*v*) $\frac{x}{5} - 6 = 3$

5. (*i*) 2 subtracted from x is 6 (*ii*) 2 subtracted from 3 times a number y is 10
(*iii*) one-sixth of a number x is 6 (*iv*) 15 subtracted from 7 times a number x is 34.
(*v*) add 2 to half of a number x to get 8.

6. (*i*) $5x + 4 = 54$ (*ii*) $2x + 7 = 87$
(*iii*) $4x = 180°$ (*iv*) $8x + 4 = 100$

EXERCISE 4.2

1. (*i*) Subtract 1 from both sides; $x = -1$
(*ii*) Add 1 to both sides, $x = 6$
(*iii*) Subtract 6 from both sides; $x = -4$
(*iv*) Subtract 4 from both sides; $y = 0$
(*v*) Add 3 to both sides; $y = 6$

2. (*i*) Divide both sides by 3 ; $x = 5$

(*ii*) Multiply both sides by 7 ; $p = 28$

(*iii*) Divide both side by 8 ; $y = \frac{9}{2}$

(*iv*) Divide both sides by 20 ; $x = -\frac{1}{2}$

3. (*i*) Step 1 : Subtract 7 from both sides

Step 2 : Divide both sides by 5, $x = 2$

(*ii*) Step 1 : Multiply both sides by 3

Step 2 : Divide both sides by 20, $x = 6$

(*iii*) Step 1 : Add 2 on both sides

Step 2 : Divide both sides by 3, $p = 16$

4. (*i*) $x = 9$ (*ii*) $p = -15$ (*iii*) $x = -4$ (*iv*) $q = 3$

(*v*) $p = 0$ (*vi*) $s = -3$

EXERCISE 4.3

1. (*i*) $x = -2$ (*ii*) $y = \frac{5}{2}$ (*iii*) $a = -5$ (*iv*) $x = \frac{4}{9}$

(*v*) $x = -2$ (*vi*) $x = 8$

2. (*i*) $x = 4$ (*ii*) $x = 2$ (*iii*) $x = 0$ (*iv*) $x = -4$

3. (*i*) $x = \frac{14}{5}$ (*ii*) $x = \frac{6}{5}$ (*iii*) $p = 7$ (*iv*) $y = \frac{1}{2}$

4. (*i*) Possible equations are : $10x + 2 = 22$; $\frac{x}{5} = \frac{2}{5}$; $5x - 3 = 7$

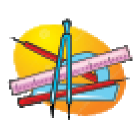
(*ii*) Possible equation are $3x = -6$; $3x + 7 = 1$, $3x + 10 = 4$

5. (*iii*) **6.** (*iv*) **7.** (*iii*) **8.** (*i*) **9.** (*iv*) **10.** (*iii*)

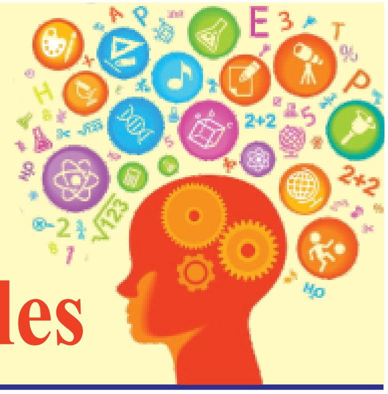
EXERCISE 4.4

1. 10 **2.** 13 **3.** 35 **4.** 10 **5.** 13 years

6. 70° each **7.** 6 **8.** 4 units, 7 units



CHAPTER 5



Lines and Angles

Learning Objectives :-

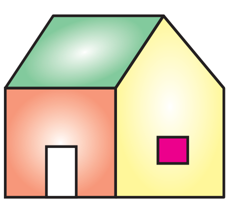
In this chapter you will learn :-

1. To identify lines, line segments, rays and angles.
2. To recognize, classify and describe various types of lines and angles.
3. To draw and name the angles.
4. To relate angles to shapes.
5. About various pairs of angles and their relations and using the same to find the missing angles.
6. To realize the importance of lines and angles in daily life.

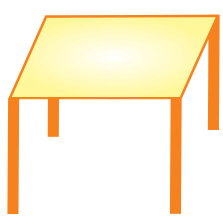
INTRODUCTION

Lines and angles, being two vital elements of geometry form the basis of all the shapes and structures. One can find them every where around, be it the corner of a table, the ramp of a building or an arch of a bridge. The knowledge about these elements helps the architects to design and the engineers to construct better structures ; the athletes to enhance their performance and the astronomers to study stars and plants. So, we can say that the study of lines and angles is important not only to understand some specific phenomena but also to uplift the quality of life.

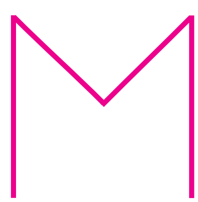
Can you identify the different line segments and angles formed in the following figures ?



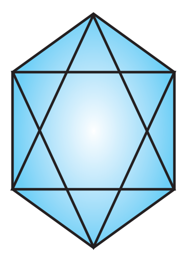
(*i*)



(*ii*)



(*iii*)



(*iv*)

Review :-

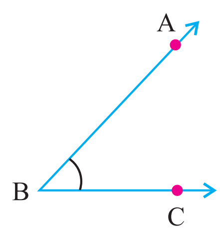
1. **Line :** A line can be defined as a set of continuous points that has an indefinite length. It has no thickness. It can be extended endlessly in both the directions. It is denoted by $\overleftrightarrow{AB}$.

2. Ray : A ray can be defined as a part of a line that has a fixed starting point but no end point. It can be extended infinitely in one direction. It is denoted by $\overrightarrow{AB}$.

3. Line segment : A line segment is a part of a line that has two end points and a fixed length. It can't be extended in any direction. It is denoted by $\overline{AB}$.

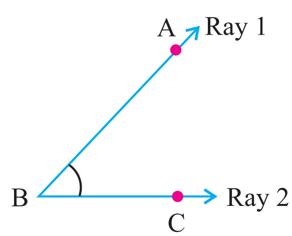
ANGLES AND ITS TYPES

Angle : An angle is formed by two rays having common starting initial point. Rays form the arms of the angle and the common point is called the vertex. Angle is measured in degrees (°) using a protractor.



It is denoted by the symbol ∠.

Naming of an angle : Name any point on Ray 1, then the vertex and then any point on Ray 2.



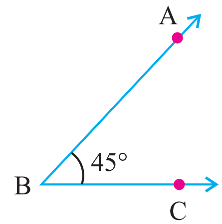
In the fig, Ray BA and BC form an ∠ABC or ∠CBA.

TYPES OF ANGLES

1. Zero angle : An angle whose measure is 0° is called a zero angle. When two arms of an angle lie on each other, 0° angle is formed. In figure, ∠ABC = 0°

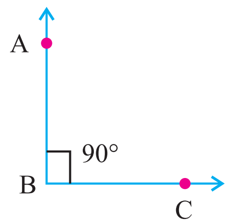
∴ ∠ABC is zero angle.

2. Acute angle : An angle that measures between 0° and 90° (0° < acute angle < 90°) is called an acute angle. In figure, ∠ABC = 45° (< 90°) (0° < ∠ABC < 90°)



∴ ∠ABC is an acute angle.

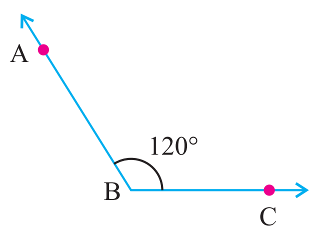
3. Right angle : An angle whose measure is 90° is called a right angle. The rays making a right angle are also called perpendicular rays. In figure, ∠ABC = 90°



∴ ∠ABC is a right angle.

4. Obtuse angle : An angle whose measure lies between 90° and 180° is called an obtuse angle. In given figure,

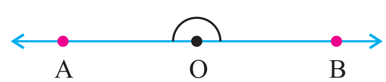
∠ABC = 120° (90° < ∠ABC < 180°)



∴ ∠ABC is an obtuse angle.

5. Straight angle : An angle that measures 180° is called a straight angle. It is called so as the two rays form a straight line.

In figure, ∠AOB = 180°

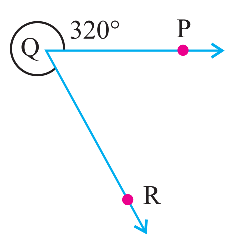


∴ ∠AOB is a straight angle.

6. Reflex angle : An angle whose measure lies between 180° and 360° is called a reflex angle.

In given figure, Reflex ∠ PQR = 320°

(180° < reflex ∠ PQR < 360°)



7. **Complete angle :** An angle whose measure is exactly 360° is called a complete angle. It forms a complete circle.

In figure, ∠PQR = 360°

∴ ∠PQR is a complete angle.

More about angles :-

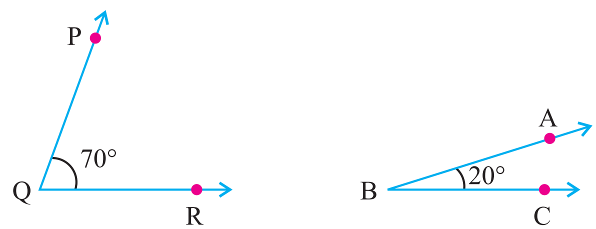
(*i*) **Complementary angles :** When the sum of measure of two angles is 90° then these angles are called complementary angles. For example, 70° + 20° = 90°, then 70° and 20° are complementary to each other.

In given figure,

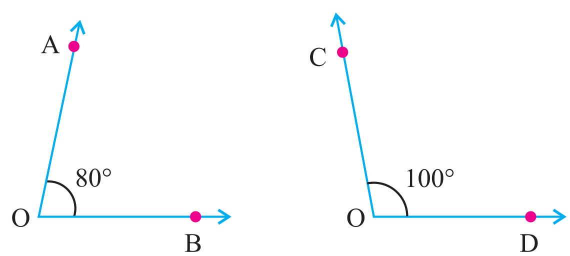
$$\angle PQR + \angle ABC = 70° + 20° = 90°$$

∠PQR and ∠ABC are complementary angles and

∠PQR is complement of ∠ABC and vice versa.



(*ii*) **Supplementary angles :** When the sum of measure of two angles is 180° Then these angles are called supplementary angles. For example 80° + 100° = 180°, then 80° and 100° are supplementary to each other.



In the given figures

$$\angle AOB + \angle COD = 80° + 100°$$
$$= 180°$$

∠AOB and ∠COD are supplementary angles and ∠AOB is called the supplement of ∠COD and vice versa.

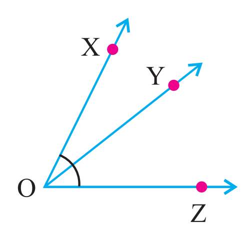
(*iii*) **Adjacent angles :** Two angles are said to be adjacent angles if.

(*a*) They have a common arm

(*b*) They have a common vertex

(*c*) Non-Common arms lie on either side of common arm.

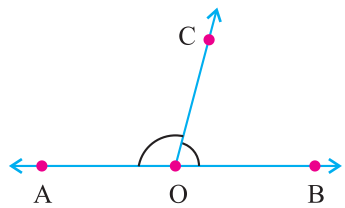
In fig. ∠XOY and ∠YOZ are adjacent angles with common vetex O having OY as common arm. OX and OZ as non common arms lying on either side of OY.



(*iv*) **Linear Pair :** Two adjacent angles whose sum is 180° are said to form a linear pair.

In figure $\angle AOC + \angle COB = 180°$

∴ These angles form a linear pair.

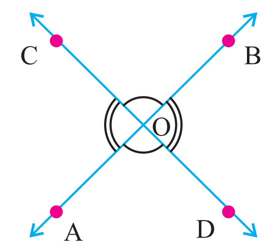


The angles in a linear pair are supplementary i.e, sum of angles is 180°.

(*v*) **Vertically Opposite Angles :** When two straight lines intersect each other, four angles are formed.

The pair of angles which lie on the opposite side of the point of intersection are called vertically opposite angles.

In the adjoining figure straight lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect each other at point O. ∠AOD and ∠BOC form one pair of vertically opposite angles, ∠AOC and ∠BOD form another pair of vertically opposite angles



Vertically opposite angles are always equal.

i.e, $\angle AOD = \angle BOC$

$\angle AOC = \angle BOD$

Example-1 : Find the complement of following angles

(*i*) 38° (*ii*) 63°

Sol. (*i*) Complement of 38° = (90° – 38°) = 52°

(*ii*) Complement of 63° = (90° – 63°) = 27°

Example-2 : Find the supplement of following angles

(*i*) 35° (*ii*) 62°

Sol. (*i*) Supplement of 35° = (180° – 35°) = 145°

(*ii*) Supplement of 62° = (180° – 62°) = 118°

Example-3 : Two complementary angles are in the ratio of 4 : 5, find the angles.

Sol. Let the required angles be $(4x)°$ and $(5x)°$

∴ $(4x)° + (5x)° = 90°$

$(9x)° = 90°$

$x = 10°$

Required angles are (4 × 10)° and (5 × 10)°

i.e, 40° and 50°

Example-4 : Two supplementary angles are in the rato of 2:7 find the angles

Sol. Let the required angles be $(2x)°$ and $(7x)°$

Angles are supplementary

$\therefore \quad (2x)^\circ + (7x)^\circ = 180^\circ$

$(9x)^\circ = 180^\circ$

$x = 20^\circ$

Hence Required angle are $(2 \times 20)^\circ$ and $(7 \times 20)^\circ$

i.e., 40° and 140°

Example-5 : Find the angle which is double of its supplement

Sol. Let the measure of one of the required angles be x.

Supplement of angle $x = 180^\circ - x$

According to the question, $x = 2\,(180^\circ - x)^\circ$

$x = 360^\circ - 2x$

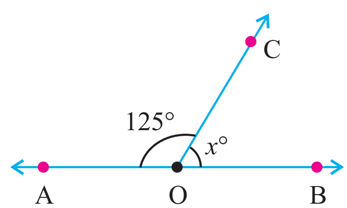
$x + 2x = 360^\circ$

$3x = 360^\circ$

$x = 120^\circ$

Required angle $= 120^\circ$

Example-6 : Find the measure of x in the following figure.



Sol. In fig $\angle AOC = 125^\circ$

$\angle AOC$ and $\angle COB$ form a linear pair

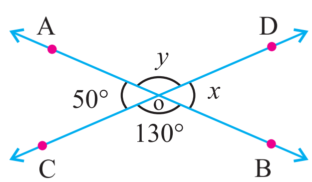
$\therefore \quad \angle AOC + \angle COB = 180^\circ$

$125^\circ + x = 180^\circ$

$x = 180^\circ - 125^\circ$

$x = 55^\circ$

Example-7 : Find the value of x and y in the following figure.



Sol. $\angle AOC$ and $\angle BOD$ are vertically opposite angles

$\therefore \quad \angle BOD = \angle AOC$

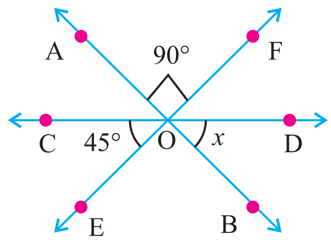
$x = 50^\circ$

$\angle AOD$ and $\angle BOC$ are vertically opposite angle

$\therefore \quad \angle AOD = \angle BOC$

$y = 130^\circ$

Example-8 : In the given figure AB, CD and EF are three straight lines intersecting each other at a point O. If ∠COE = 45° and ∠AOF = 90°, find ∠DOB.



Sol. Let ∠DOB = x

∠FOD and ∠COE are vertically opposite angles

∴ ∠FOD = ∠COE = 45°

AOB is a straight line

$$\angle AOF + \angle FOD + \angle DOB = 180^\circ$$

$$90^\circ + 45^\circ + x = 180^\circ$$

$$135^\circ + x = 180^\circ$$

$$x = 180^\circ - 135^\circ$$

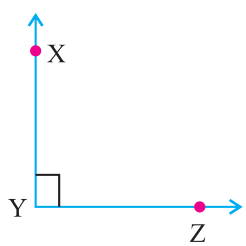
$$x = 45^\circ$$

Required angle ∠DOB = 45°

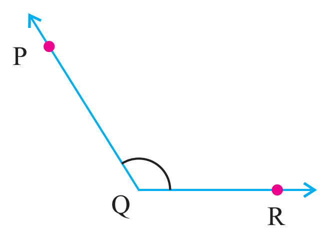
EXERCISE - 5.1

1. Name each of the following as acute, obtuse, right straight or a reflex angle.

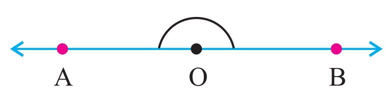
(*i*)



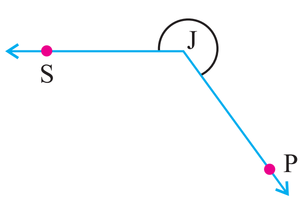
(*ii*)



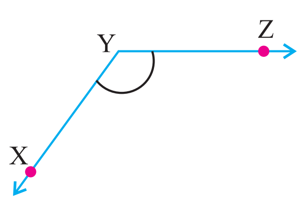
(*iii*)



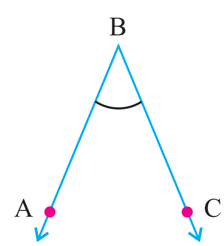
(*iv*)



(*v*)



(*vi*)



2. Write the complement of each of the following angles

(*i*) 53° (*ii*) 90° (*iii*) 85°

(*iv*) $\frac{4}{9}$ of a right angle (*v*) 0°

3. Write the supplement of each of the following angle

(*i*) 55° (*ii*) 105°

(*iii*) 100° (*iv*) $\frac{2}{3}$ of a right angle

(*v*) $\frac{1}{3}$ of 270°

4. Identify the following pairs of angles as complementary or supplementary.

(*i*) 65° and 115° (*ii*) 112° and 68°

(*iii*) 63° and 27° (*iv*) 45° and 45°

(*v*) 130° and 50°

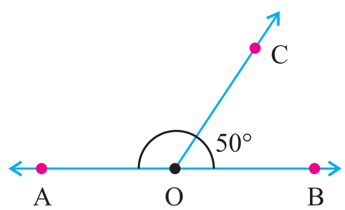
5. Two complementary angles are in the ratio of 4 : 5, find the angles.

6. Two supplementary angles are in the ratio of 5 : 13, find the angles.

7. Find the angle which is equal to its complement.

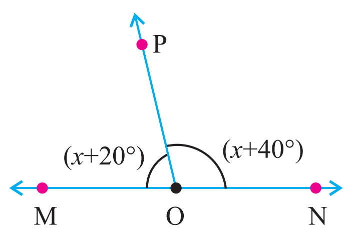
8. Find the angle which is equal to its supplement.

9. In the given figure, AOB is straight line. Find the measure of ∠AOC.



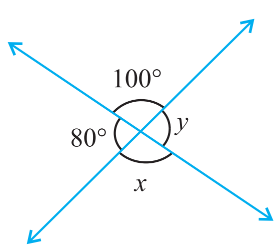
10. In the given figure, MON is straight line find

(*i*) ∠MOP (*ii*) ∠NOP

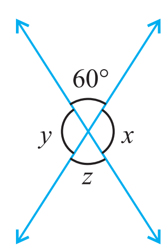


11. Find the value of *x*, *y* and *z* in each of following.

(*i*)

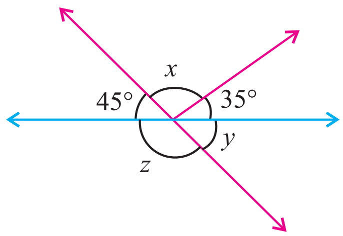


(*ii*)

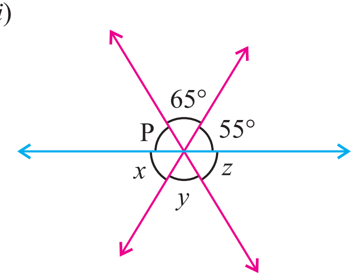


12. Find the value of x, y, z and p in each of following.

(*i*)



(*ii*)



13. Multiple Choice Question :-

(*i*) If two angles are complementary then the sum of their measure is ....

(*a*) 180° (*b*) 90°
(*c*) 360° (*d*) none of these

(*ii*) Two angles are called .... if the sum of their measures is 180°.

(*a*) supplementary (*b*) complementary
(*c*) right (*d*) none of these

(*iii*) If two adjacent angles are supplementary then, they form a ....

(*a*) right angle (*b*) vertically opposite angles
(*c*) linear pair (*d*) corresponding angles

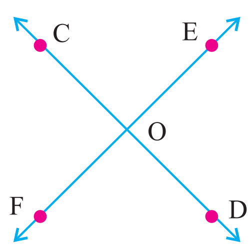
(*iv*) If two lines intersect at a point, the vertically opposite angles are always ....

(*a*) equal (*b*) zero
(*c*) 90° (*d*) none of these

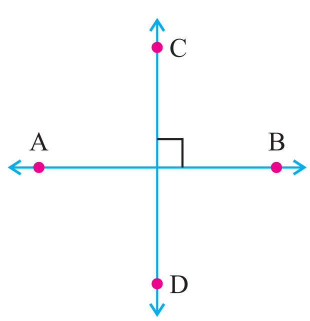
PAIRS OF LINES

Here, we shall discuss about intersecting lines, transversal, perpendicular lines, angles made by transversal with parallel lines and non-parallel lines.

1. **Intersecting lines :** When two lines intersect exactly at one point, they are called intersecting lines. In figure, CD and EF are intersecting lines and O is the point of intersection.

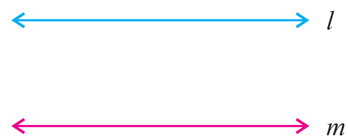


2. **Perpendicular lines :** Two lines are said to be perpendicular to each other if they meet or intersect at a right angle. In figure, CD is perpendicular to AB and is written as CD $\perp$ AB.



3. Parallel Lines : Two lines in the same plane are said to be parallel if they are at an equal distance from each other throughout and never meet.

In the given figure, line *l* and *m* are parallel to each other. It is represented as $l \parallel m$.



4. Transversal : A transversal is a line that intersects two or more lines in the same plane at distinct points. The lines may or may not be parallel. In fig. (*i*) *l* for *m* & *n* ; *m* for *l* & *n* and *n* for *l* & *m* is transversal line.

If fig. (*ii*) *p* is the transversal line for *l* & *m*.

ANGLES MADE BY A TRANSVERSAL (WITH NON PARALLEL LINES)

In figure, you see lines *l* and *m* cut by transversal *n*. The eight angles marked 1 to 8 have their special names :-

Interior angles	$\angle 3, \angle 4$ $\angle 5, \angle 6$
Exterior angles	$\angle 1, \angle 2$ $\angle 7, \angle 8$
Pairs of corresponding angles	$\angle 1$ and $\angle 5$ $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$ $\angle 4$ and $\angle 8$
Pairs of alternate interior angles	$\angle 3$ and $\angle 6$ $\angle 4$ and $\angle 5$
Pairs of alternate exterior angles	$\angle 1$ and $\angle 8$ $\angle 2$ and $\angle 7$
Pairs of interior angles on the same side of the transversal also called co-interior angles.	$\angle 3$ and $\angle 5$ $\angle 4$ and $\angle 6$

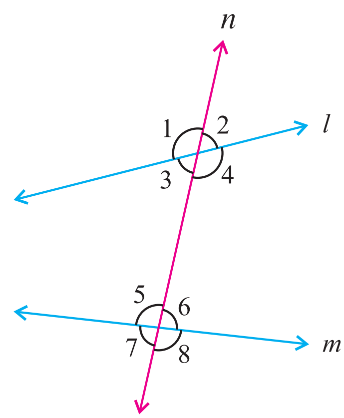


Fig. (*i*)

Transversal of parallel lines : Transversal of parallel lines gives rise to some interesting results.

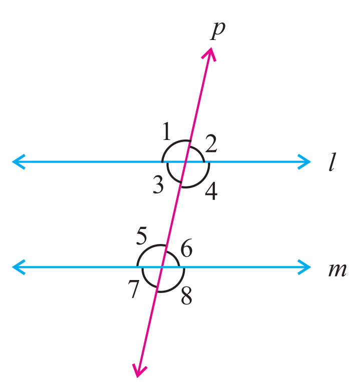
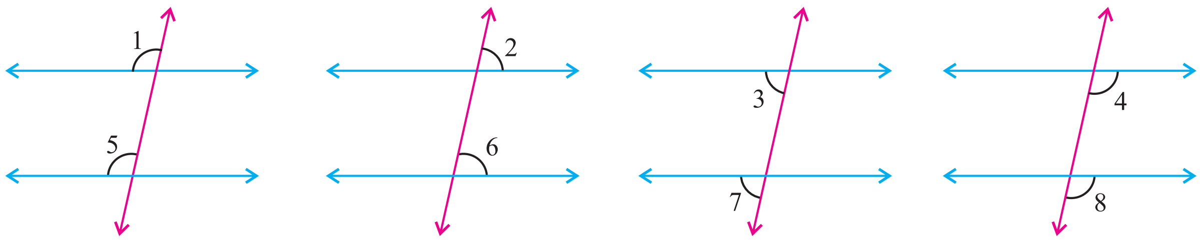
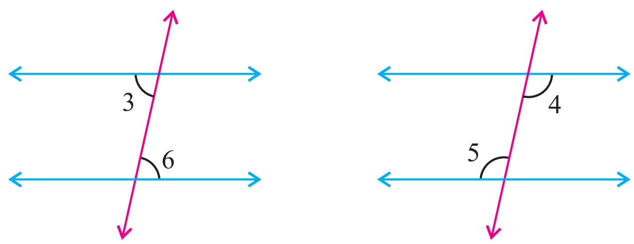


Fig. (*ii*)

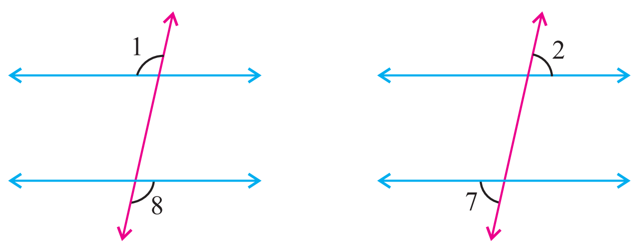
1. If two parallel lines are cut by a transversal. All the pairs of corresponding angles are equal in measure.
i.e., $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$.



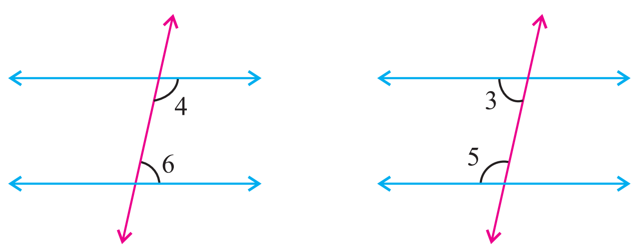
2. If two parallel lines are cut by a transversal, all pairs of alternate interior angles are equal
i.e $\angle 3 = \angle 6$ and $\angle 4 = \angle 5$



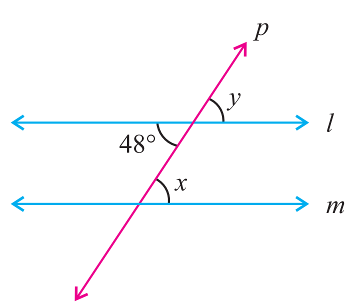
3. If two parallel lines are cut by a transversal, both the pairs of alternate exterior angles are equal *i.e* $\angle 1 = \angle 8$ and $\angle 2 = \angle 7$



4. If two parallel lines are cut by a transversal then both pair of interior angles on the same side of the transversal or co-interior angles are supplementary i.e. $\angle 4 + \angle 6 = 180°$ and $\angle 3 + \angle 5 = 180°$



Example-1 : In the figure $l \parallel m$ and p is a transversal then find the value of x and y.



Sol. Since $l \parallel m$ and p is a transversal

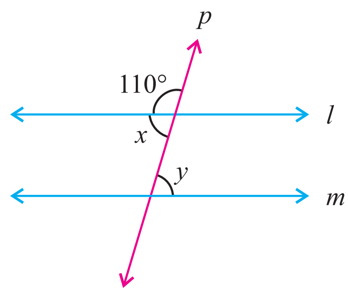
So 48° and $\angle x$ are alternate interior angles

$\therefore \quad \angle x = 48°$

also x and y are corresponding angles

$\therefore \quad \angle y = 48°$

Example-2 : In the figure $l \parallel m$ and p is a transversal then find the value of x and y.



Sol. p is straight line

$110° + \angle x = 180°$ [linear pair]

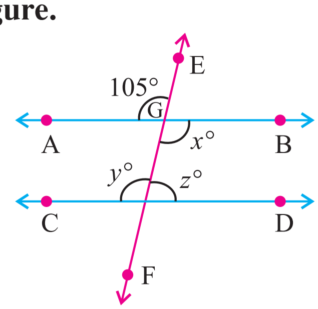
$\angle x = 180° - 110°$

$\angle x = 70°$

$\angle y = \angle x$ [alternate interior angles]

$y = 70°$

Example-3 : In the given figure AB || CD and EF is a transversal. If $\angle AGE = 105°$ find $\angle x$, $\angle y$ and $\angle z$ marked in the figure.



Sol. We have

$\angle x = \angle AGE = 105°$ [vertically opposite angles)

$\angle y = \angle x = 105°$ [alternate interior angles]

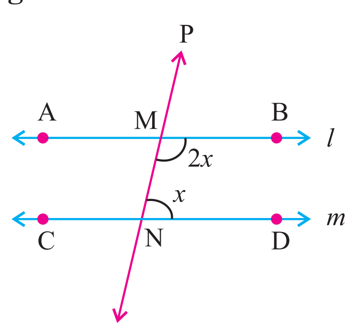
$y + z = 180°$ [linear pair]

$105° + \angle z = 180°$

$\angle z = 180° - 105°$

$z = 75°$

Example-4 : In the given figure find the value of x and also find the angles marked in the figure.

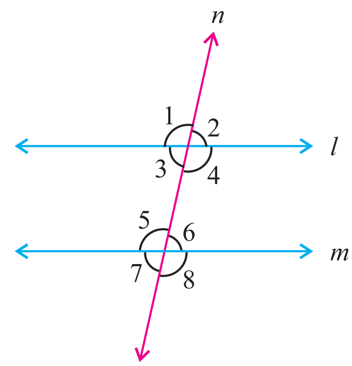


Sol.

$$\angle BMN + \angle DNM = 180^\circ \quad \text{(co-interior angles)}$$
$$2x + x = 180^\circ$$
$$3x = 180^\circ$$
$$x = 60^\circ$$
$$\angle BMN = 2x = 2 \times 60 = 120^\circ$$
$$\angle DNM = x = 60^\circ$$

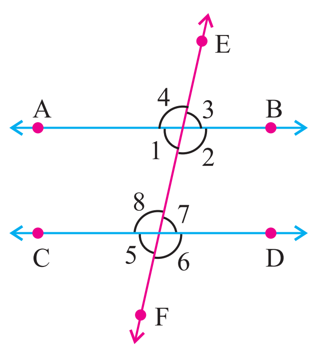
EXERCISE - 5.2

1. In the figure question identify the pair of angles as corresponding angles alternate interior angles, exterior alternate angles, adjacent angles, vertically opposite angles and co-interior angles, linear pairs.



(*i*) $\angle 3$ and $\angle 6$
(*ii*) $\angle 3$ and $\angle 7$
(*iii*) $\angle 2$ and $\angle 4$
(*iv*) $\angle 2$ and $\angle 7$
(*v*) $\angle 1$ and $\angle 8$
(*vi*) $\angle 4$ and $\angle 6$
(*vii*) $\angle 1$ and $\angle 5$
(*viii*) $\angle 1$ and $\angle 4$
(*ix*) $\angle 5$ and $\angle 7$

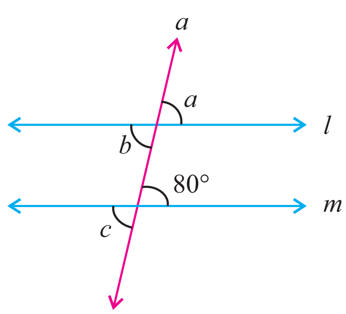
2. In the figure identify



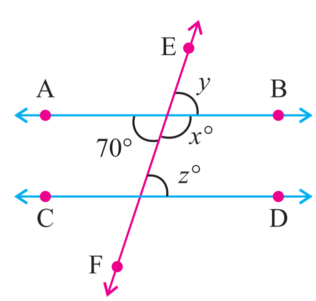
(*i*) The pairs of corresponding angle.
(*ii*) The pairs of alternate interior angles.
(*iii*) The pairs of interior angles on the same side of the transversal.
(*iv*) The pairs of vertically opposite angles

3. In the given figures, the intersected lines are parallel to each other. Find the unknown angles.

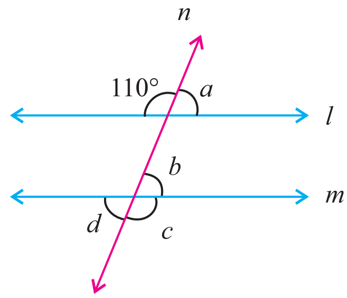
(*i*)



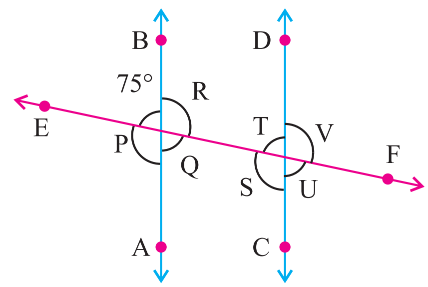
(*ii*)



(*iii*)

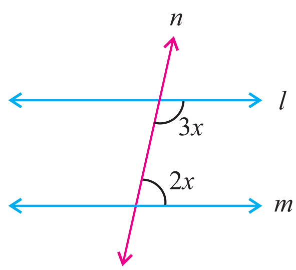


(*iv*)

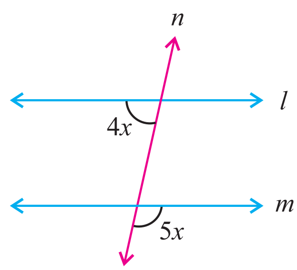


4. Find the value of x in the following figures if $l \parallel m$

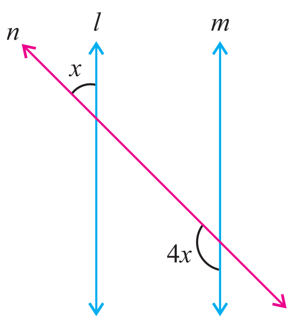
(*i*)



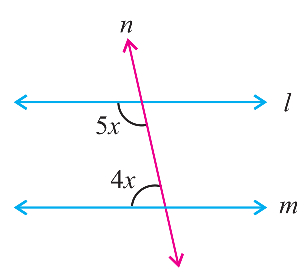
(*ii*)



(*iii*)



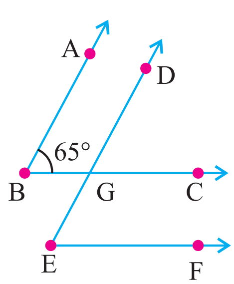
(*iv*)

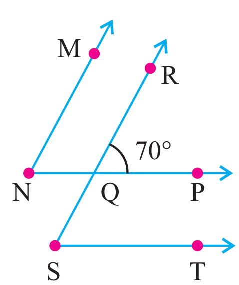


5. In the given figures arms of two angles are parallel find the following.

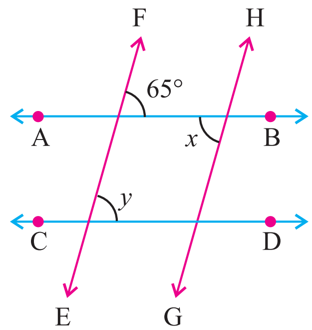
(*a*) (*i*) ∠DGC
(*ii*) ∠DEF

(*b*) (*i*) ∠MNP
(*ii*) ∠RST

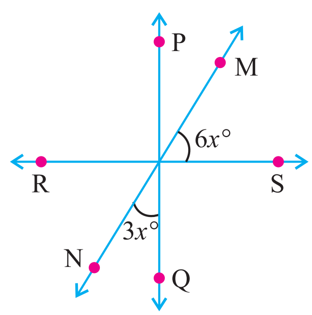




6. In the following figure AB || CD and EF || GH, find the measure of $\angle x$ and $\angle y$.

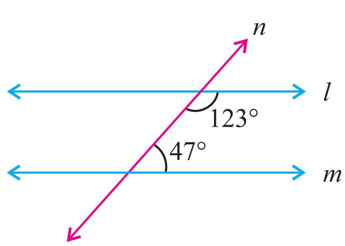


7. PQ ⊥ RS find the value of x in the following figure.

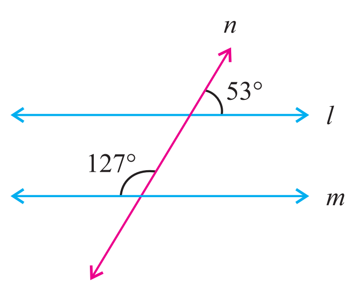


8. In the given figure below, decide whether l is parallel to m.

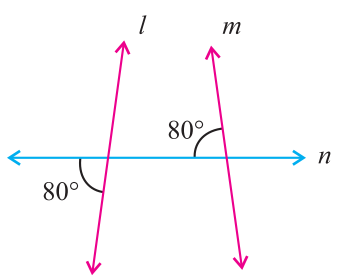
(*i*)



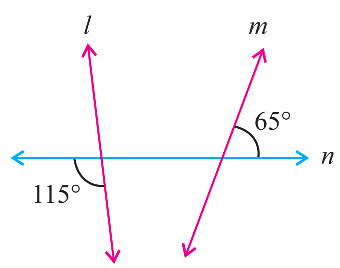
(*ii*)



(*iii*)



(*iv*)



9. Multiple Choice Questions

(*i*) A pair of complementary angles is

(*a*) 130°, 50° (*b*) 35°, 55°

(*c*) 25°, 75° (*d*) 27°, 53°

(*ii*) A pair of supplementary angles is

(*a*) 55°, 115° (*b*) 65°, 125°

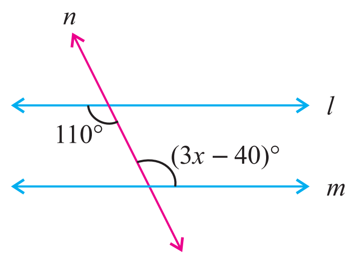
(*c*) 47°, 133° (*d*) 40°, 50°

(*iii*) If one angle of a linear pair is acute, then the other angle is

(*a*) acute (*b*) obtuse

(*c*) right (*d*) straight

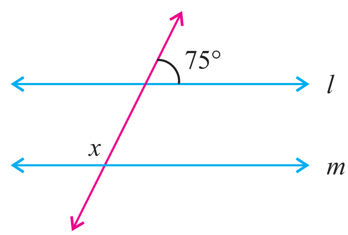
(*iv*) In the adjoining figure, if $l \parallel m$ then the value of x is.



(*a*) 50° (*b*) 60°

(*c*) 70° (*d*) 45°

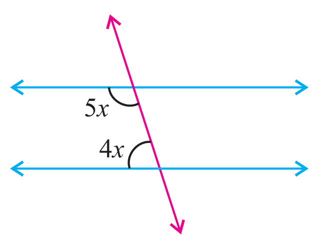
(*v*) In the adjoining figure, if $l \parallel m$, then



(*a*) 75° (*b*) 95°

(*c*) 105° (*d*) 115°

(*vi*) In the adjoining figure, the value of x that will make the lines l and m parallel is.



(*a*) 20 (*b*) 30

(*c*) 60 (*d*) 80

AIM : To introduce the properties of parallel lines cut by a transversal.

Objective : To verify the equality of alternate interior angles and corresponding angles made by a transversal on parallel lines through paper cutting and pasting.

Previous Knowledge :

(*i*) Concept of alternate interior, corresponding and vertically opposites angles.

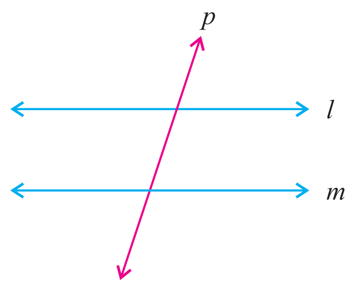
(*ii*) Concept of parallel lines

Material Required :

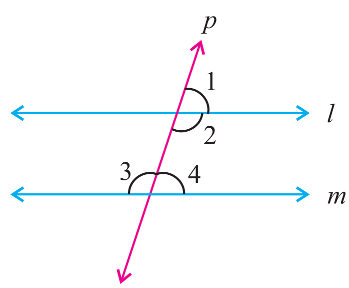
(*i*) White chart paper
(*ii*) Pair of Scissors
(*iii*) Geometry box
(*iv*) Coloured Sketch Pen
(*v*) Coloured Papers
(*vi*) Glue Sticks

Procedure :

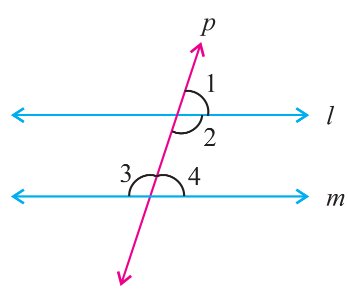
1. Take white chart paper and draw parallel lines *l* and *m* and *p* is a transversal line see figure and give them name.



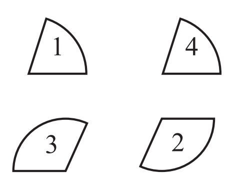
2. Mark the angles formed as ∠1, ∠2, ∠3 and ∠4 respectively as shown in figure.



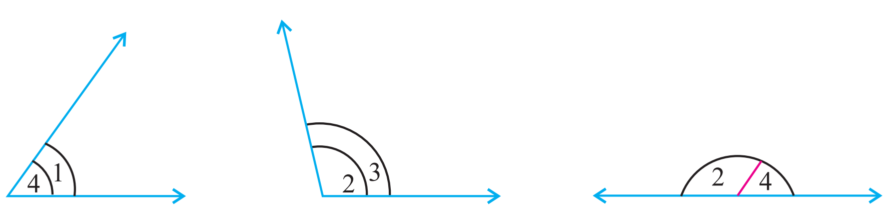
3. Fill different colours and cut it out.



4. Cut out the angles as shown in figure 4.



5. Now arrange the cut outs of ∠1 and ∠4, ∠2 and ∠3 and ∠2 and ∠4.



Observation : We observe that

(*i*) ∠1 and ∠4 completely over lap each other

(*ii*) ∠2 and ∠3 completely over lap each other

(*iii*) ∠2 and ∠4 combined and make straight line.

Result :

1 Corresponding angle : ∠1 and ∠4 are corresponding angles ; ∠1 = ∠4

2. Alternate angles : ∠2 and ∠3 are alternate angle ; ∠2 = ∠3

3. Interior angles : ∠2 and ∠4 are Interior angles on same side ∠2 + ∠4 = 180° (Showing straight line)

VIVA VOCE

Q.1. What are parallel lines ?

Ans. Lines which are at a constant distance throughout.

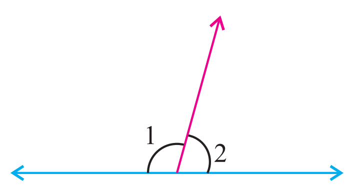
Q.2. What is the sum of interior angles on the same side of transversal ?

Ans. 180°

Q.3. When two lines intersect, what is the relation between the vertically opposite angles so formed ?

Ans. They are equal

Q.4. Identify the type of the pair of angles.



Ans. ∠1 and ∠2 are linear pair of angles.

WHAT HAVE WE DISCUSSED ?

1. An angle is formed by two rays having one common end point.

2. (*i*) Two angles are said to be complementary, If the sum of their measures is 90°

(*ii*) Two angles are said to be supplementary, If the sum of their measures is 180°

3. The sum of angles that form linear pair is 180°

4. The sum of all the angles at a point is 360°

5. A line which intersects two or more lines in a plane is called a transversal.

6. When a transversal intersects two parallel lines, then

(*i*) Pairs of alternate interior angles are equal

(*ii*) Pairs of alternate exterior angles are equal

(*iii*) Pairs of corresponding angles are equal

(*iv*) Pairs of co-interior angles are supplementary

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Identify lines, line segments, rays and angles.
2. Classify pairs of angles based on their properties as linear pair, supplementary, complementary, adjacent and vertically opposite and find value of the one when the other is given.
3. Draw and name the angles.
4. Verify the properties of various pairs of angles formed when a transversal cuts two lines.
5. Relate angles to the shapes, objects and structures they find in their surroundings.

ANSWERS

EXERCISE 5.1

1. (*i*) Right angle (*ii*) Obtuse angle
(*iii*) Straight angle (*iv*) Reflex angle
(*v*) Obtuse angle (*vi*) Acute angle

2. (*i*) 37° (*ii*) 0°
(*iii*) 5° (*iv*) 50°
(*v*) 90°

3. (*i*) 125° (*ii*) 75°
(*iii*) 80° (*iv*) 120°
(*v*) 90°

4. (*i*) Supplementary (*ii*) Supplementary
(*iii*) Complementary (*iv*) Complementary
(*v*) Supplementary

5. 40° and 50°

6. 50°, 130°

7. 45°

8. 90°

9. 130°

10. (*i*) 80° (*ii*) 100°

11. (*i*) $x = 100°, y = 80°$ (*ii*) $x = 120°, y = 120°, z = 60°$

12. (*i*) $x = 100°, y = 45°, z = 135°$ (*ii*) $x = 55°, y = 65°, z = 60°, p = 60°$

13. (*i*) 90° (*ii*) 180°
(*iii*) linear pair (*iv*) equal

EXERCISE 5.2

1. (*i*) Alternate interior angles (*ii*) Corresponding angles
(*iii*) Adjacent angles (*iv*) Alernate exterior angles
(*v*) Alternate exterior angle (*vi*) Co-interior angle

(*vii*) Corresponding angles (*viii*) Vertically opposite angles

(*ix*) Linear pair

2. (*i*) ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8

(*ii*) ∠1 and ∠7, ∠2 and ∠8,

(*iii*) ∠1 and ∠8, ∠2 and ∠7

(*iv*) ∠1 and ∠3, ∠2 and ∠4, ∠5 and ∠7, ∠6 and ∠8

3. (*i*) $a = 80°$ $b = 80°$ $c = 80°$

(*ii*) $x = 110°$ $y = 70°$ $z = 70°$

(*iii*) $a = 70$ $b = 70°$ $c = 110°$ $d = 70°$

(*iv*) P = 105° Q = 75° R = 105° S = 105° T = 75° U = 75° V = 105°

4. (*i*) $x = 36$ (*ii*) $x = 20$

(*iii*) $x = 36$ (*iv*) $x = 20$

5. (*a*) (*i*) 65° (*ii*) 65°

(*b*) (*i*) 70° (*ii*) 70°

6. $x = 65°$ $y = 65°$ 7. $x = 10$

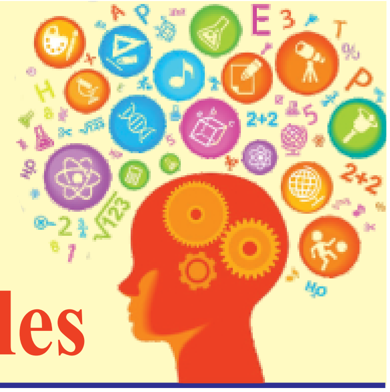
8. (*i*) Not (*ii*) Yes

(*iii*) Not (*iv*) Yes

9. (*i*) (*b*) (*ii*) (*c*) (*iii*) (*b*)

(*iv*) (*a*) (*v*) (*c*) (*vi*) (*a*)

CHAPTER 6



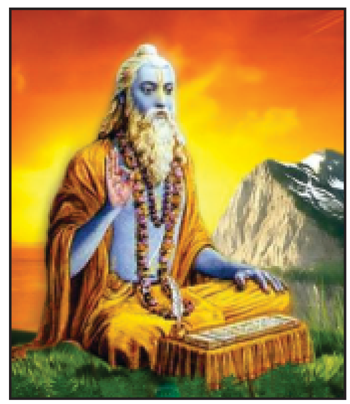
Triangles

Learning Objectives :-

In this chapter you will learn :-

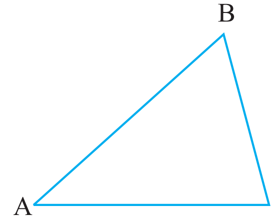
1. To identify various parts of a triangle.
2. To understand the relation between exterior angle of a triangle with interior opposite angles.
3. To understand the relation between interior angles of a triangle.
4. To understand the relation between sides of a right angled triangle.
5. To use the angle sum property, exterior angle property and Pythagoras theorem for triangles.

OUR NATIONS'S PRIDE



Baudhayan (800 BC - 740 BC approx.) Baudhayan is noted as the author of the earliest 'Sulabhsutra' called Baudhayan's sulabhsutra, which contained several important mathematical results. Baudhayan was the first one ever to arrive at several concepts in mathematics, which were later discovered by the western world. It is amazing to know, what is known as 'Pythagoras Theorem' today is already found in Baudhayan's Sulabhsutra, which was written several years before the age of Pythagoras. The value of pi was first calculated by him.

INTRODUCTION

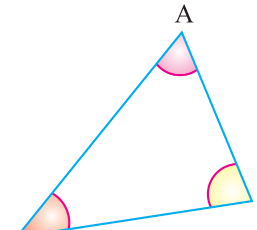


Triangle : A closed figure bounded by three line segments is called a 'triangle' and is usually denoted by the greek letter delta (Δ). It has three vertices, three sides and three angles. In given figure ABC is a triangle. It has

(*i*) Three sides namely AB, BC, CA

(*ii*) Three angles namely $\angle BCA$, $\angle BAC$ and $\angle ABC$ to be denoted by $\angle C$, $\angle A$, $\angle B$ respectively.

(*iii*) Three vertices A, B, C. Here A is the vertex opposite to the side BC. B is the vertex opposite to the side CA and C is the vertex opposite to the side AB.

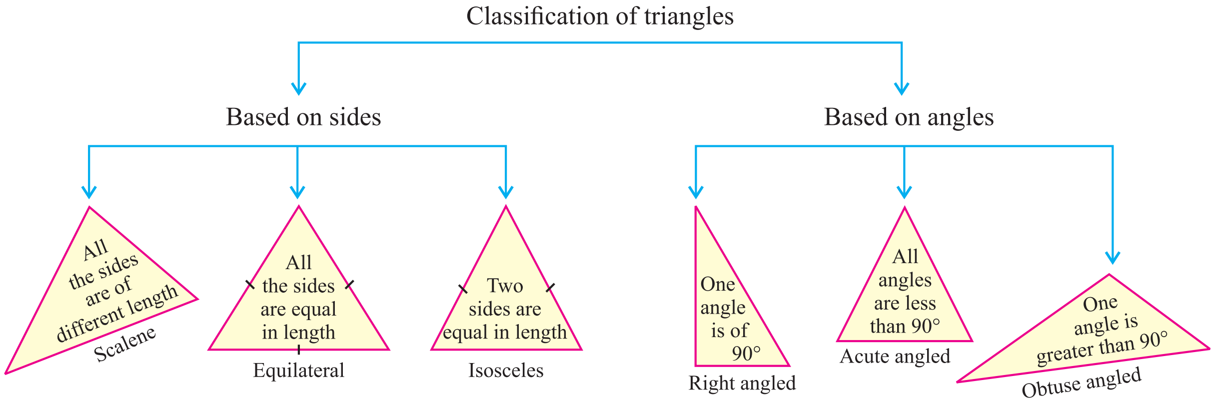


Interior Angles : In ΔABC; $\angle BAC$, $\angle ABC$ and $\angle ACB$ are called the interior angles as they lies inside the triangle. In the figure shown alongside shaded angles are interior angles.

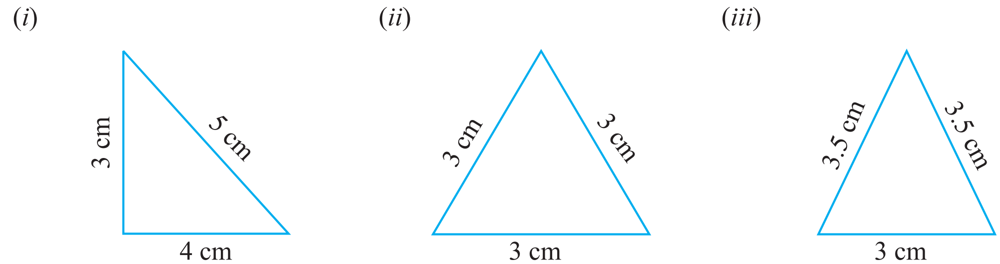
Classification of Triangles :

We know that the triangles are calssified on the basis of

(*i*) Sides (*ii*) Angles



Example-1 : Classify the following triangles on the basis of sides.

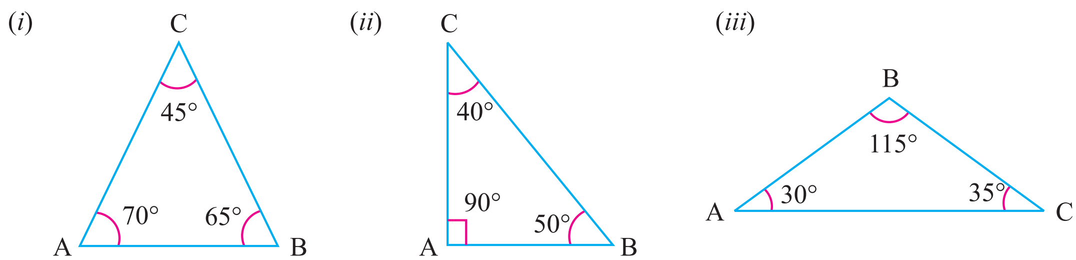


Sol. (*i*) All sides are of different length. So it is a scalene triangle.

(*ii*) All sides are equal, so it is an equilateral triangle.

(*iii*) Two sides are equal, so it is an isosceles triangle.

Example-2 : Classify the following triangles on the basis of angles.

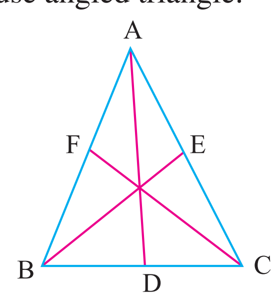


Sol. (*i*) In ΔABC, all angles are smaller than 90°, so it is an acute angled triangle.

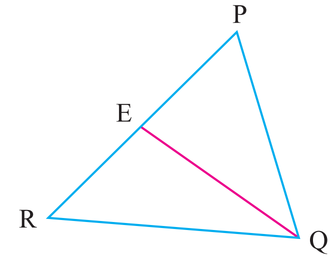
(*ii*) In ΔABC, ∠A = 90°, So it is a right angled triangle

(*iii*) In ΔABC ∠B = 115°, which is greater than 90°, so it is an obtuse angled triangle.

Median of a triangle : A median of a triangle is the line segment that joins any vertex of the triangle to the mid point of its opposite side. In the figure shown along side, the median from A meets the mid point of the opposite side BC at point D. Hence AD is a median of ΔABC and it bisects the side BC into two halves where BD = DC similarly, BE and CF are other two medians of the given triangle.



Example-3 : Draw a triangle PQR. Also draw a median QE of triangle PQR



Sol. In the given figure we have ΔPQR, on joining Q and E, the mid point of PR, We get the required median QE.

NOTE :

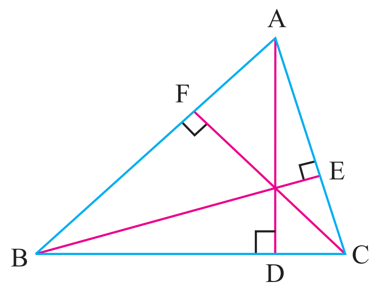
(*i*) A triangle has three medians, all of which intersect at a point called "CENTROID".

(*ii*) Medians lie completely in the interior of the triangle.

(*iii*) A median divides the area of a triangle in two equal parts

(*iv*) The length of all the medians in an equilateral triangle is always equal.

Altitudes of a triangle : The perpendicular line segment drawn from a vertex of a triangle to its opposite side is called an altitude.



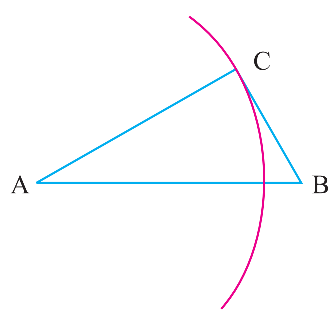
In the given figure AD, BE and CF are the altitudes of ΔABC drawn from the vertex A, B and C respectively.

ACTIVITY

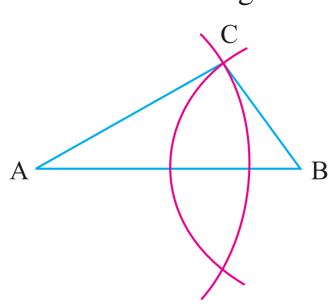
To draw an altitude by an activity.

Given a triangle ABC :-

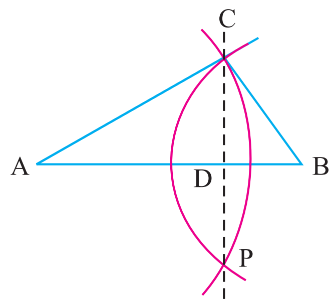
1. Draw an arc taking A as centre and line segment AC as radius.



2. Draw another arc taking B as centre and line segment BC as a radius.



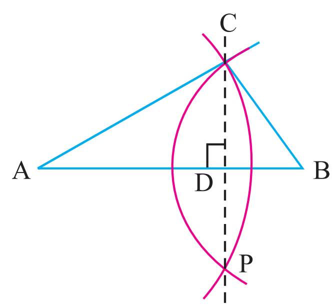
3. The two arcs intersect at two points. One of the point of intersection is the vertex C. Let the other point of intersection be P.



4. Join the point C and P.

5. PC intersects AB at a point, say D.

6. Now CD is an altitude of the given triangle.



NOTE :

(*i*) A triangle has three altitudes.

(*ii*) The altitude is also called the height of the triangle.

(*iii*) All the altitudes of an acute angled triangle lie inside the triangle.

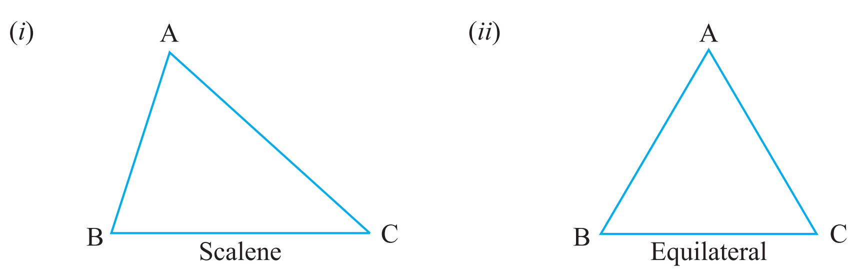
(*iv*) In an obtuse triangle, the altitude corresponding to the vertex with obtuse angle, lies inside the triangle, where as the other two altitudes lie outside the triangle.

(*v*) Two altitudes of a right angled triangle are actually the perpendicular legs of the triangle itself while the third lies inside the triangle.

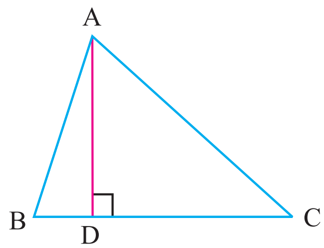
(*vi*) The equilateral triangle has all the altitudes of equal length

(*vii*) The altitudes of a triangle intersect at one point called "ORTHOCENTER"

Example-4 : **Draw altitudes from A to BC for the following triangles.**

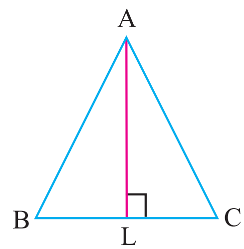


Sol. (*a*) In the given figure, altitude can be drawn as below.



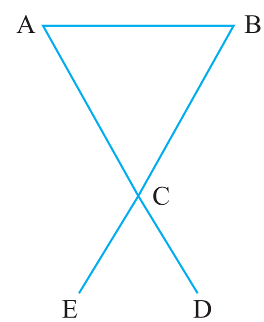
AD is the altitude from A to BC.

(*b*) In the given figure, altitude can be drawn as below.



AL is the altitude from A to BC.

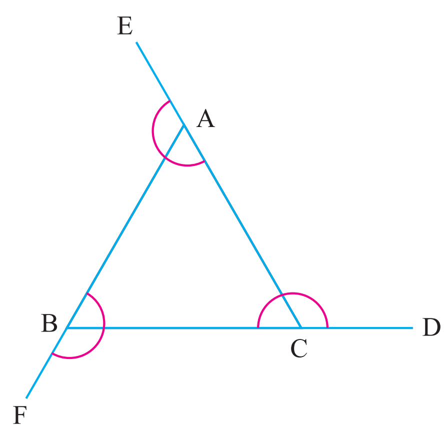
Exterior angle of a triangle : When a side of a triangle is produced, an exterior angle is formed. For a given ΔABC, if side AC is produced to the point D. Then ∠BCD is its exterior angle and if side BC is produced to the point E, then the exterior angle would be ∠ACE



The two interior angles of a triangle opposite to the exterior angle are called its interior opposite angles. While the third opposite interior is called the adjacent interior angle.

In above triangle if ∠ACE is an exterior angle then ∠BCA is adjacent to ∠ACE. The remaining two angles ∠A and ∠B are called opposite interior angles.

Note : For a triangle, sum of an exterior angle and adjacent interior angle is always equal to 180° becasue every pair of exterior angle and its adjacent interior angle forms a linear pair.



∴ For given ΔABC,

∠BAC + ∠BAE = 180°

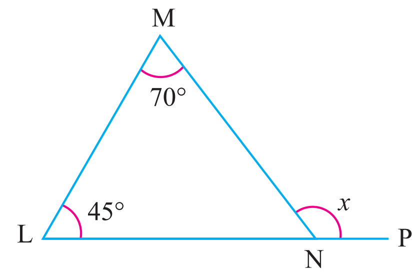
∠CBA + ∠CBF = 180°

∠ACB + ∠ACD = 180°

EXTERIOR ANGLE PROPERTY OF A TRIANGLE

An exterior angle of a triangle is equal to the sum of its opposite interior angles.

Example-5 : Find the value of x in the given figure.



Sol. In given figure $\angle LMN = 70°$

$\angle MLN = 45°$

By exterior angle property of a triangle

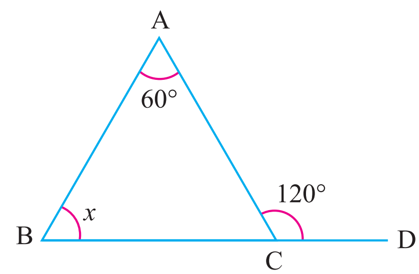
$\angle LMN + \angle MLN = \angle MNP$

$70° + 45° = x$

$115° = x$

i.e $x = 115°$

Example-6 : Find the angle x in ΔABC



Sol. In given ΔABC, $\angle A = 60°$, Exterior angle ACD = 120°

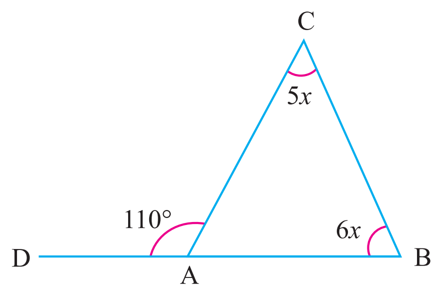
By exterior angle property of a triangle

$60° + x = 120°$

$x = 120° - 60°$

$x = 60°$

Example-7 : Find the value of $\angle ABC$ and $\angle BCA$ in the given figure.



Sol. In given figure $\angle ACB = 5x$, $\angle CBA = 6x$

$\angle CAD = 110°$

By exterior angle property of a triangle

$\angle ACB + \angle CBA = \angle CAD$

$5x + 6x = 110°$

$11x = 110°$

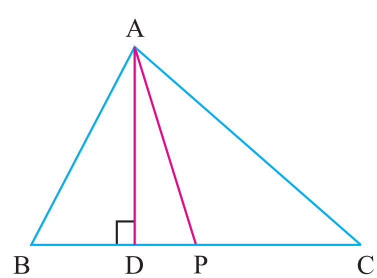
$$x = \frac{110°}{11}$$

$x = 10°$

$\therefore \angle CBA = 6 \times 10° = 60°$

$\angle ACB = 5 \times 10° = 50°$

EXERCISE - 6.1



1. In ΔABC, P is mid point of BC, then

(*i*) BP = ...

(*ii*) AP is a .... of ΔABC

(*iii*) $\angle ADC$ = ...

(*iv*) BD = BC (True/ False)

(*v*) AD is an .... of ΔABC

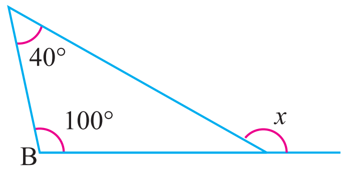
2. (*a*) Draw AD, BE, CF three medians in a ΔABC

(*b*) Draw an equilateral triangle and its medians. Also compare the lengths of the medians.

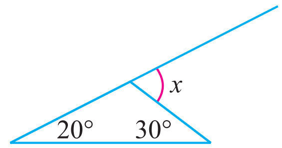
(*c*) Draw an isosceles triangle ABC in which AB = BC. Also draw its altitudes.

3. Find the value of the unknown exterior angles

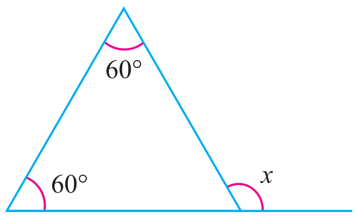
(*i*)



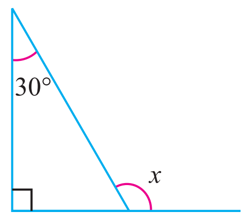
(*ii*)



(*iii*)

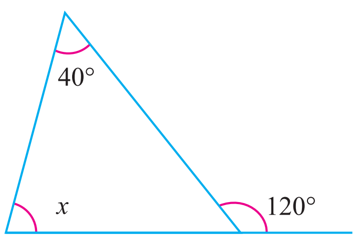


(*iv*)

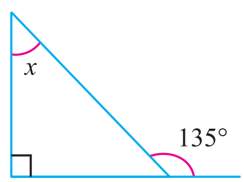


4. Find the value of x in the following figures.

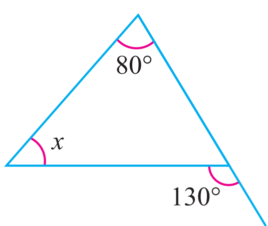
(*i*)



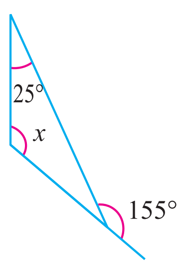
(*ii*)



(*iii*)

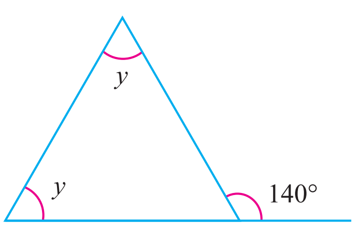


(*iv*)

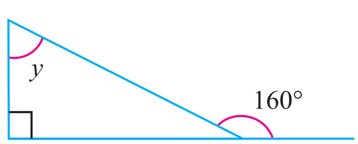


5. Find the value of y in following figures

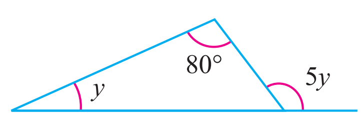
(*i*)



(*ii*)



(*iii*)



Angle sum property of a Triangle : In ΔABC, $\angle BAC + \angle ABC + \angle ACB = 180°$.

To justify this let us use the exterior angle property of a triangle.

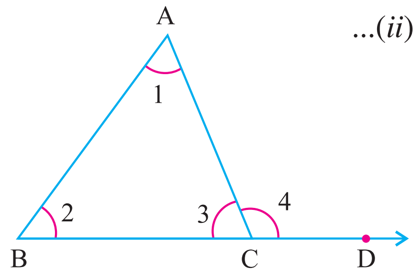
Here $\angle 1$, $\angle 2$, $\angle 3$ are the interior angles of ΔABC, and $\angle 4$ in the exterior angle.

We know that

$$\angle 1 + \angle 2 = \angle 4 \text{ [Exterior angle property of a triangle]} \quad ...(i)$$

Adding $\angle 3$ to both the side of (*i*)

$$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 3 \quad ...(ii)$$



But $\angle 4$ and $\angle 3$ from a linear pair

$$\therefore \quad \angle 4 + \angle 3 = 180°$$

So (*ii*) becomes

$$\angle 1 + \angle 2 + \angle 3 = 180°$$

Or $$\angle BAC + \angle ABC + \angle ACB = 180°$$

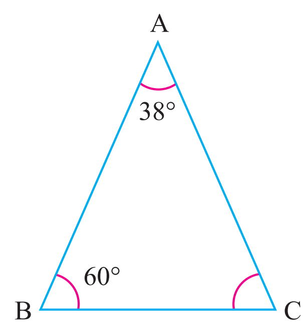
Example-1 : Can a triangle have angles 50°, 70°, 90° ?

Sol. $50° + 70° + 90° = 210°$

But, we know that sum of angles of a triangle is always 180° [Angle sum property]

$\therefore$ A triangle cannot have angles 50°, 70° and 90°

Example-2 : In the given figure find $\angle C$.



Sol. By angle sum property of a triangle

$$\angle A + \angle B + \angle C = 180°$$

or $$38° + 60° + \angle C = 180°$$

$$98° + \angle C = 180°$$

$$\angle C = 180° - 98°$$

$$\angle C = 82°$$

Example-3 : Three angles of a triangle are $(3x + 4)°$, $(2x + 8)°$ and $(3x + 8)°$ find the angles.

Sol. Since, we know that the sum of angles of a triangle is always 180°

$$(3x + 4)° + (2x + 8)° + (3x + 8)° = 180°$$

$$(8x + 20)° = 180°$$

$$(8x)° = 180° - 20°$$

$$(8x)° = 160°$$

$$x = \frac{160}{8} = 20$$

$$\therefore \quad x = 20$$

$$\therefore \quad \text{Required angles} = (3x + 4)°, (2x + 8)° \text{ and } (3x + 8)°$$

$$= (3 \times 20 + 4)°, (2 \times 20 + 8)° \text{ and } (3 \times 20 + 8)°$$

$$= 64°, 48°, 68°$$

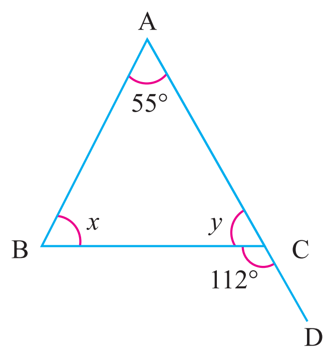
Example-4 : The angles of a triangle are in the ratio 3 : 4 : 5. Find the measure of each angle of the triangle.

Sol. Let the measure of the given angles be $(3x)°, (4x)°, (5x)°$

By angle sum property of a triangle

$$(3x)° + (4x)° + (5x)° = 180°$$
$$(12x)° = 180°$$
$$x = \frac{180}{12}$$
$$x = 15$$
$$\text{Required angles} = (3 \times 15)°, (4 \times 15)°, (5 \times 15)°$$
$$\therefore \quad = 45°, 60°, 75°$$

Example-5 : Find the value of x and y in the given figure.



Sol. Since in ΔABC, AC is produced to D

$\therefore \quad 55° + x = 112°$ [By exterior angle property]

or $\quad x = 112° - 55°$

$x = 57°$ ...(1)

Now in ΔABC

$55° + x + y = 180°$ (Angle sum property of triangle)

$55° + 57° + y = 180°$ (by using 1)

$112° + y = 180°$

$y = 180° - 112°$

$y = 68°$

EXERCISE - 6.2

1. State, if a triangle is possible with the following angles.

(*a*) 35°, 70°, 65°

(*b*) 70°, 50°, 60°

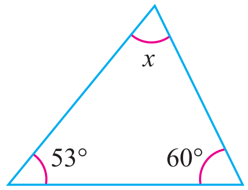
(*c*) 90°, 80°, 20°

(*d*) 60°, 60°, 60°

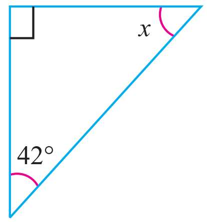
(*e*) 90°, 90°, 90°

2. Find the value of x in the following figures

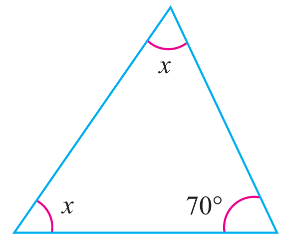
(*i*)



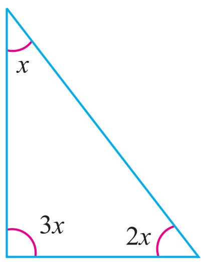
(*ii*)



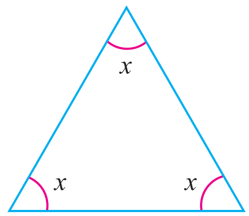
(*iii*)



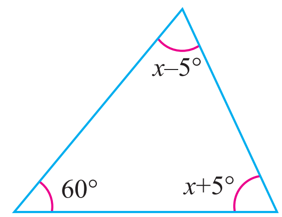
(*iv*)



(*v*)

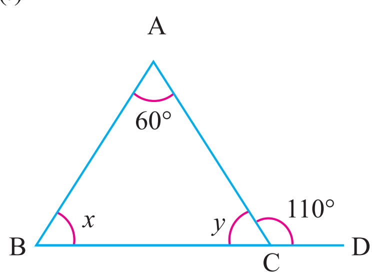


(*vi*)

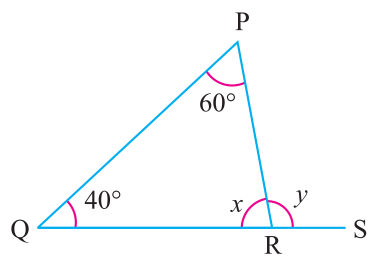


3. Find the values of x and y in the following figures.

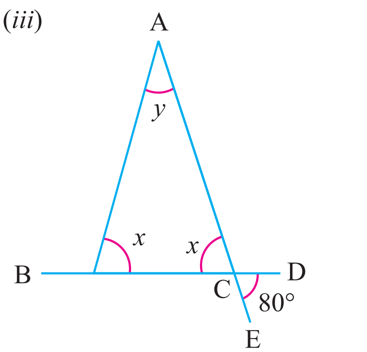
(*i*)



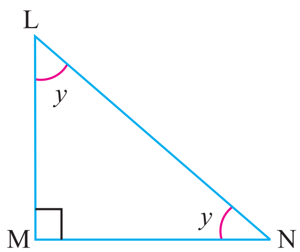
(*ii*)



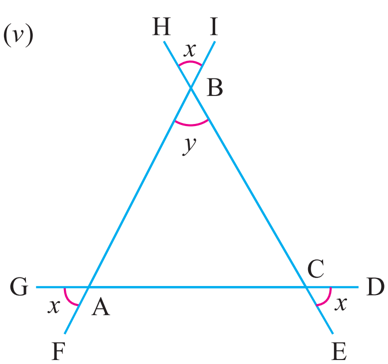
(*iii*)



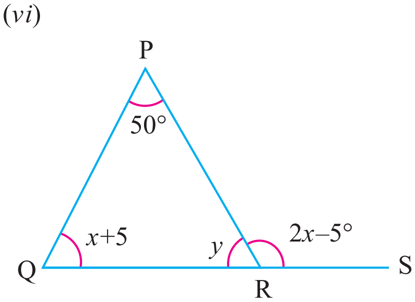
(*iv*)



(*v*)



(*vi*)



4. The angles of a triangle are in the ratio 5 : 6 : 7. Find the measure of each of the angles.

5. One angle of a triangle is 60°. The other two angles are in the ratio 4 : 8. Find the angles.

6. In a triangle ABC, $\angle B = 50°$, $\angle C = 62°$ Find $\angle A$.

7. In a right angled triangle two acute angles are in the ratio 2 : 3. Find the angles.

8. Three angles of a triangle are $(2x + 20)°$, $(x + 30)°$ and $(2x – 10)°$. Find the angles.

9. Multiple choice questions :

(*i*) A triangle can have two ....

(*a*) Acute angles (*b*) Obtuse angles

(*c*) Right angles (*d*) None of these

(*ii*) A triangle is possible with measure of angles

(*a*) 30°, 40°, 100° (*b*) 60°, 60°, 70°

(*c*) 60°, 50°, 70° (d) 90°, 89°, 92°

(*iii*) One of the equal angles of an isosceles triangle is 45° then its third angle is

(*a*) 45° (*b*) 60°

(*c*) 100° (*d*) 90°

(*iv*) The number of obtuse angles that a triangle can have

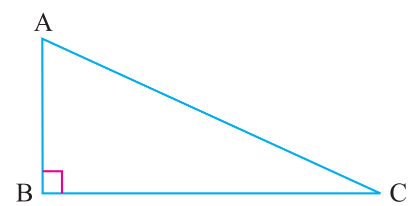
(*a*) 2 (*b*) 1

(*c*) 3 (*d*) 4

PYTHAGORAS THEOREM

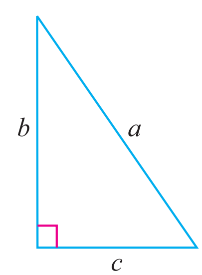
Right-angled Triangle and Pythagoras Property

Pythagoras, a Greek philosopher of sixth century BC (Before Christ), is said to have found a very important and useful property of right-angled triangles. The property is, hence named after him. Infact this property was known to people of many other countries too. The Indian mathematician Baudhayan has also given an equivalent form of this property. We now try to explain the pythagoras property.

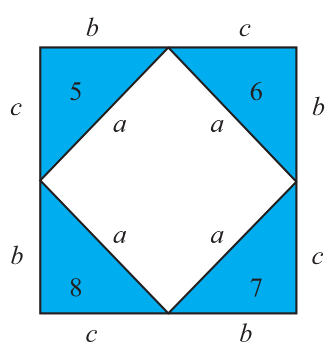


In a right-angled triangle, the sides have some special names. The side opposite to the right angle is called the hypotenuse ; the other two sides are known as the legs of the right-angled triangle.

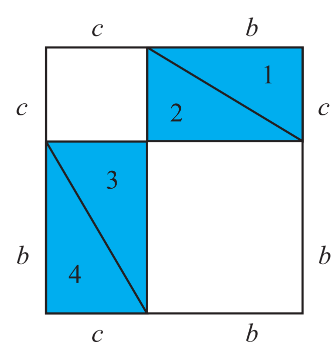
In ΔABC, the right angle is at B. so AC is the hypotenuse. AB and BC are the legs of ΔABC.



Make eight identical copies of right angled triangle of any size you prefer. For example, you make a right-angled triangle whose hypotenuse is a units long and the legs are of lengths b units and c units. Draw two identical squares on a sheet with sides of lengths $b + c$ you are to place four triangles in one square and the remainig four triangles in the other square, as show in the following diagram.



Square A

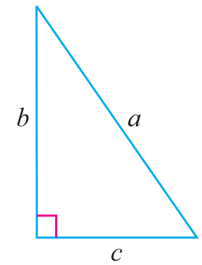


Square B

The square are identical ; the eight triangles inserted are also identical. Hence, the uncovered area of square

A = unovered area of square B

i.e., Area of inner square of square A = The total area of two covered squares in square B.



$$a^2 = b^2 + c^2$$

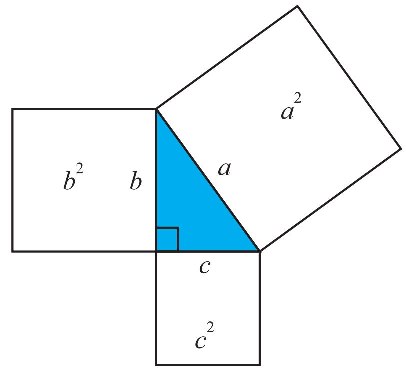
This is pythagoras property. It may be stated as follows.

In a right-angled triangle.

The square on the hypotenuse = sum of squares on the legs.

It says that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sun of the areas of the squares on the legs.

Draw a right angled triangle, preferably on a square sheet, construct squares on its sides, compute the area of these squares and verify the theorem practically.



If you have a right angled triangle, the Pythagoras property holds. If the pythagoras property holds for some triangle, will the triangle be right angled ?

Now, we will show that, if there is a triangle in which sum of the squares on two of its sides is equal to the square of the third side, it must be a right-angle triangle.

PYTHAGORAS THEOREM :-

In a right triangle, the square of hypotenuse equals the sum of the squares of its remaining two sides.

Thus, in a right ΔABC, in which $\angle C = 90°$

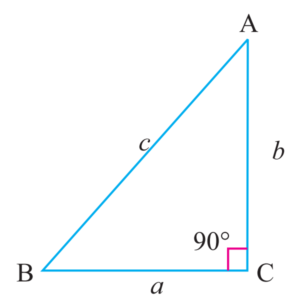
We have

$$AB^2 = BC^2 + AC^2$$

If $AB = c$, $BC = a$, $AC = b$

Then we have

$$c^2 = a^2 + b^2$$



The hypotenuse AB is the longest side of the right angled triangle.

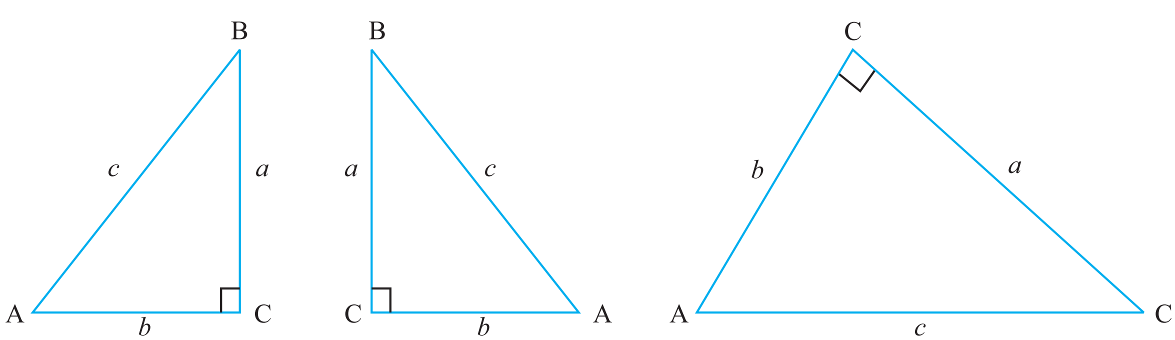
The other two sides are called the legs of the right triangle.

- The side opposite to the right angle is called the hypotenuse.
- The hypotenuse is the longest side in the right angled triangle.

Verification of Pythagoras Theorem

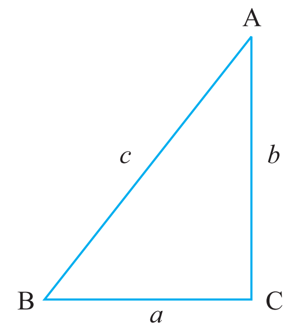
The above result can be verifed by the following activity.

Activity : Consider three right triangles T_1, T_2 and T_3 as shown below Label each of them as ΔABC such that $\angle C = 90°$ in each case



In each case, measure the sides a, b and the hypotenuse c of the triangle, compute a^2, b^2 and c^2 and tabulate the observations as under.

Right triangles	Measurements			Computations				
T_1	a	b	c	a^2	b^2	c^2	a^2+b^2	$c^2-(a^2+b)$
T_2								
T_3								



You will find that in each case, $c^2-(a^2+b^2)=0$

Hence $c^2 = a^2 + b^2$.

In a right triangle, The hypotenuse is the longest side.

If :- In right ΔABC, We have

$c^2 = a^2 + b^2$ [By Pythagoras theorem]

$\Rightarrow$ $c^2 > a^2$ and $c^2 > b^2$

$\Rightarrow$ $c > a$ and $c > b$

Thus, in a right triangle, the hypotenuse is greater than each of the remaining two sides. Hence in a right triangle, the hypotenuse is the longest side.

> If the square of one side of a triangle is equal to sum of the squares of the other two sides, then the triangle is right angled.
> Thus in a ΔABC if $AB^2 = BC^2 + AC^2$, then ΔABC is right angled at C.

Pythagorean triplets :

Three positive integers *a, b, c* in the very same order are said to form a pythagorean triplet if $c^2 = a^2 + b^2$

For example consider three numbers 3, 4 and 5

Take $a = 3$, $b = 4$, $c = 5$

$$a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$$
$$c^2 = 5^2$$

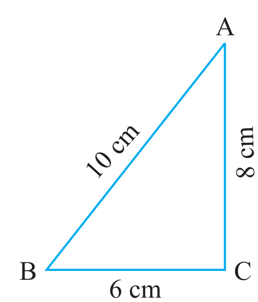
As $c^2 = a^2 + b^2$

$\therefore$ 3, 4, 5 from a Phythagoras triples.

The various examples of pythagorean triplets are (5, 12, 13) (6, 8, 10) (7, 24, 25), (8, 15, 17) etc.

Example-1 : The lengths of sides of two triangles are given below. Check, whether the triangles are right traingles.

(*i*) 6*cm*, 8*cm*, 10*cm*   (*ii*) 5*cm*, 8*cm*, 11*cm*.



Sol. (*i*) Let in ΔABC, the longest side is AB = 10cm

$\therefore$ $(BC)^2 + (AC)^2 = 6^2 + 8^2$
$= 36 + 64$
$= 100 = 10^2$

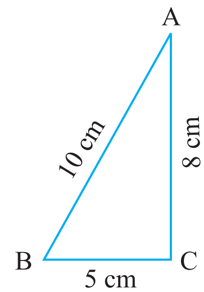
$\Rightarrow$ $(BC^2) + (AC^2) = (10)^2$ ....(1)

Also $(AB)^2 = (10)^2$ ....(2)

From (1) & (2) $(AB)^2 = (BC)^2 + (AC)^2$

Therefore, the triangle whose sides are 10*cm*, 8*cm* and 6*cm* is a right triangle.

(*ii*) Here, the longer side is AB = 11cm



We know $(AB) = (11)^2 = 121$

$(BC)^2 + (AC)^2 = 5^2 + 78^2 = 25 + 64$

or $(BC)^2 + (AC)^2 = 89$

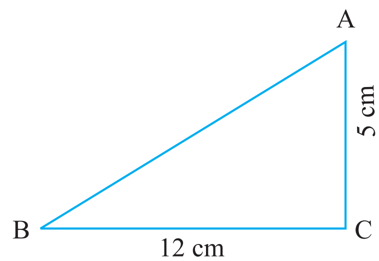
Since $89 \neq 121$

$\therefore$ The triangles with the given sides is not a right triangle.

Example-2 : ΔABC is right angled at C. If AC = 5*cm* and BC = 12*cm*. Find the length of AB.

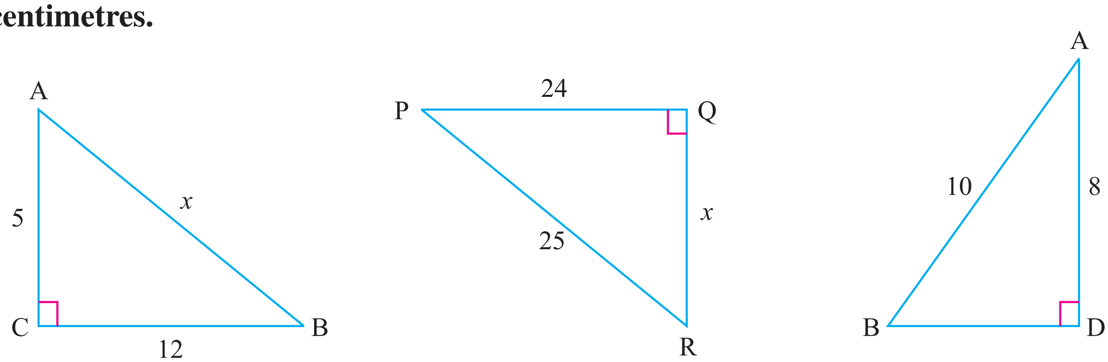
Sol. AC = 5*cm*, BC = 12*cm*

By Pythagoras theorem,



$$AB^2 = AC^2 + BC^2$$
$$= 5^2 + 12^2$$
$$= 25 + 144$$
$$= 169 = 13^2$$
$$\Rightarrow \quad AB = 13 \text{ cm}$$

Example-3 : Find the value of *x* in each of the following figures. All measurements are in centimetres.



Sol. (*i*) In ΔABC, ∠C = 90°, by Pythagoras theorem,

$$AB^2 = BC^2 + CA^2$$
$$x^2 = 12^2 + 5^2$$
$$x^2 = 144 + 25 = 169$$
$$\Rightarrow \quad x = 13 \text{ cm}$$

(*ii*) In PQR, ∠Q = 90°, by Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$
$$25^2 = 24^2 + x^2$$
$$625 = 576 + x^2 \Rightarrow x^2 = 625 - 576$$
$$x^2 = 49 \Rightarrow x = 7 \text{ cm}$$

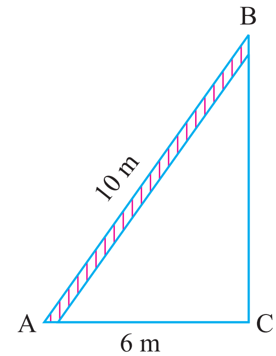
(*iii*) In ΔABD, ∠ADB = 90°, by Pythagoras theorem,

$$AB^2 = BD^2 + AD^2$$
$$10^2 = BD^2 + 8^2$$
$$100 = BD^2 + 64 \Rightarrow BD^2 = 100 - 64$$
$$\Rightarrow \quad BD^2 = 36 \Rightarrow BD = 6 \text{ cm}$$

Example-4 : A 10*m* long ladder is placed against a wall in such a way that the foot of the ladder is 6*m* away from the wall. Find the height of the wall.

Sol. Let AB be the ladder and BC be the height then AB = 10*m* and AC = 6*m*.

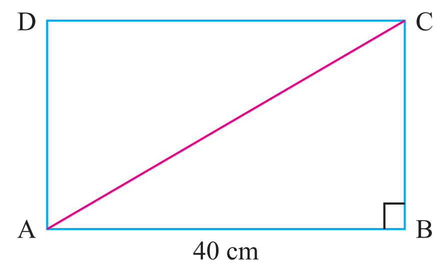
By Pythagoras theorem,



$$AB^2 = AC^2 + BC^2$$
$$BC^2 = AB^2 - AC^2$$
$$= 10^2 - 6^2$$
$$= 100 - 36$$
$$= 64$$
$$BC^2 = 64$$
$$BC = 8$$

Hence the required height is 8 m.

Example-5 : Find the perimeter of the rectangle whose length is 40*cm* and length of its diagonal is 41cm.



Sol. Let ABCD be a rectangle with length AB = 40*cm* and diagonal AC = 41*cm*.

In ΔABC, ∠B = 90° (each angle of a rectangle)

By pythagoras theorem,

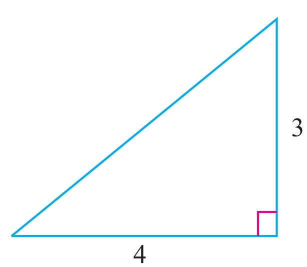
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad 41^2 = 40^2 + BC^2$$
$$BC^2 = 41^2 - 40^2$$
$$= 1681 - 1600 = 81$$
$$BC = 9cm$$

Perimeter of rectangle ABCD $= 2(AB + BC) = 2\,(40 + 9)cm = (2 \times 49) = 98cm$.

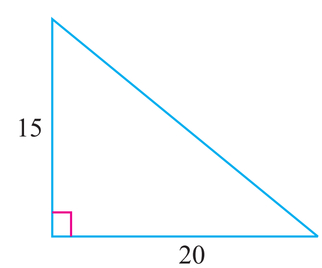
EXERCISE - 6.3

1. Find the length of the unknown side in each of following figures

(*i*)



(*ii*)



2. Which of the following can be the sides of a right triangle ?

(*i*) 4*cm*, 5*cm*, 7*cm*

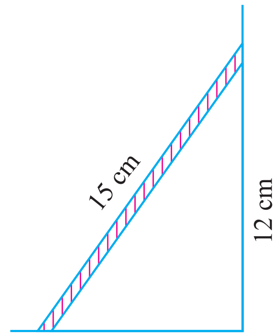
(*ii*) 1.5*cm*, 2*cm*, 2.5*cm*

(*iii*) 2*cm*, 2*cm*, 5*cm*

In the case of right angled triangles, identify the right angles.

3. Find the area and the perimeter of the rectangle whose length is 15*cm* and the length of one diagonal is 17*cm*.

4. A 15*m* long ladder reached a window 12*m* high from the ground on placing it against a wall at a distance, find the distance of the foot of the ladder from the wall.



a

5. The side of a rhombus is 5*cm*. If the length of one of the diagonals of the rhombus is 8*cm*, then find the length of the other diagonal.

6. A right triangle is isosceles. If the square of the hypotenuse is 50*m*, what is length of each of its sides ?

7. ΔABC is a triangle right angled at C if AC = 8*cm* and BC = 6*cm*, find AB.

8. State whether the following triplets are pythagorean or not.

(*i*) (5, 7, 12) (*ii*) (3, 4, 5)

(*iii*) (8, 9, 10) (*iv*) (5, 12, 13)

9. Multiple Choice Questions :

(*i*) In a ΔABC, if $\angle$A = 40° and $\angle$B = 55° then $\angle$C is

(*a*) 75° (*b*) 80°

(c) 95° (*d*) 85°

(*ii*) If the angles of *a* triangle are 35°, 35° and 110°, then it is

(*a*) an isosceles triangle (*b*) an equilateral triangle

(*c*) a scalene triangle (*d*) right angled triangle

(*iii*) A triangle can have two

(*a*) right angles (*b*) obtuse angles

(*c*) acute angles (*d*) striaght angles

(*iv*) A triangle whose angles measure 35°, 55° and 90° is

(*a*) acute angled (*b*) right angled

(*d*) obtuse angled (*d*) isosceles

(*v*) A triangle is not possible whose angles measure

(*a*) 40°, 65°, 75° (*b*) 50°, 56°, 74°

(*c*) 72°, 63°, 45° (*d*) 67°, 42°, 81°

(*vi*) A triangle is not possible with sides of lengths (in *cm*)

(*a*) 6, 4, 10 (*b*) 5, 3, 7

(*c*) 7, 8, 9 (*d*) 3.6, 5.4, 8

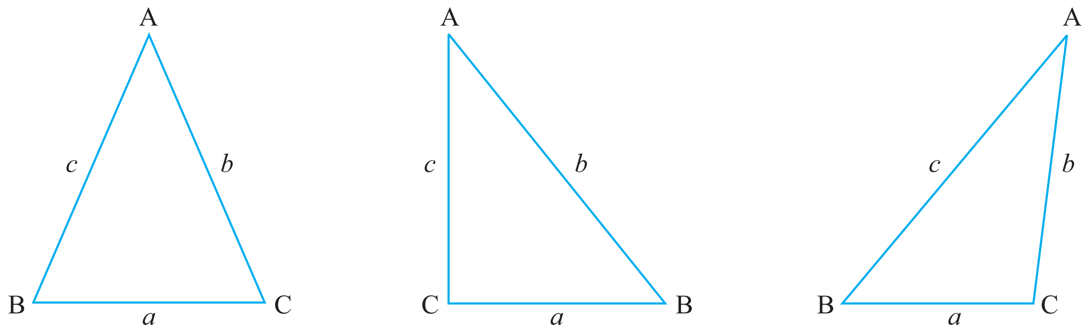
(*vii*) In a right angled triangle, the length of two legs are 6*cm* and 8*cm*. The length of the hypotenuse is.

(*a*) 14*cm* (*b*) 10*cm*

(*c*) 11*cm* (*d*) 12*cm*

TRIANGLE INEQUALITY

Draw three triangles – One acute angled, one right angled and one obtuse angle as shown below.



Let BC = a, AC = b and AB = C

Now, measure each sides of these triangles.

You will observe

$$a + b > c,\ b + c > a,\ c + a > b$$

Thus, we conclude that

The sum of the lengths of any two side of a triangle is greater than the length of third side.

Example-1 : Which of the following can be possible lengths (in cm) of triangle

(*a*) 8, 10, 15 **(*b*) 18, 10, 6**

(*c*) 6, 2, 8

Sol. (*a*) Since 8 + 10 > 15, 10 + 15 > 8, 15 + 8 > 10

So 8, 10, 15 can be lengths of the sides

(*b*) Since 10 + 6 < 18 so 10, 6, 18 can not be the lengths of sides of a triangle

(*c*) Since 6 + 2 is equal to 8 which is the third length. So 6, 2, 8 cannot be the lengths of the sides of triangle.

Example-2 : O is a point in the exteior of ΔABC show that

(*i*) OA + OB > AB **(*ii*) OB + OC > BC**

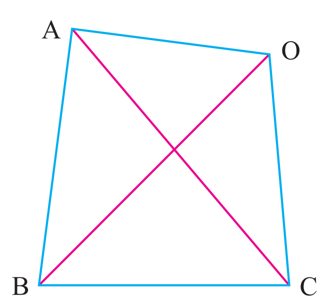
(*iii*) OC + OA > AC **(*iv*) 2 (OA + OB + OC) > AB + BC + CA**

Sol. (*i*) OA, OB and AB are the side of ΔOAB

∴ OA + OB > AB (Triangle inequality)

(*ii*) OB, OC and BC are sides of ΔOBC

∴ OB + OC > BC (Triangle inequality)



(*iii*) OC, OA & AC are the sides of ΔOAC,

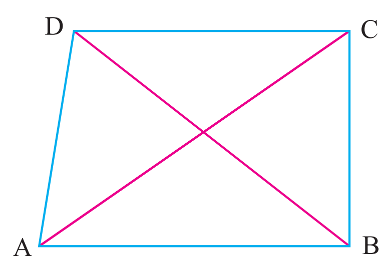
$\therefore$ OC + OA > AC (Triangle in equality)

Adding (*i*), (*ii*) & (*iii*)

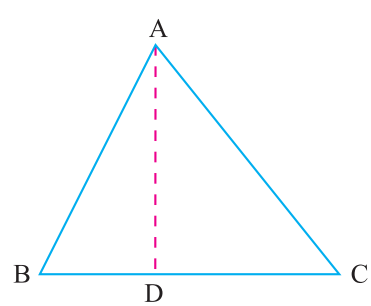
$$OA + OB + OB + OC + OC + OA > AB + BC + AC$$
$$2OA + 2OB + 2OC > AB + BC + AC$$
$$2(OA + OB + OC) > AB + BC + AC$$

EXERCISE - 6.4

1. Which of the following can be the sides of a triangle ?
   (*a*) 8*cm*, 10*cm*, 18*cm*
   (*b*) 6*cm*, 4*cm*, 8*cm*
   (*c*) 35*cm*, 38*cm*, 40*cm*
   (*d*) 3*cm*, 4*cm*, 10*cm*
2. A point O is in interior of *a* ΔABC use symbols >, < or = to make the following statements true.
   (*a*) OA + OB ☐ AB
   (*b*) OB + OC ☐ BC
   (*c*) OA + OC ☐ AC
3. ABCD is a quadrilateral
   Is AB + BC + CD + DA > AC + BD ?



4. AD is a median of $\angle ABC$
   Is AB + BC + CA > 2AD ?



5. The length of two sides of a triangle are 4cm and 6cm. Between what two measures should the length of the third side fall ?

ACTIVITY-1

To prove that the medians of a triangle are concurrent.

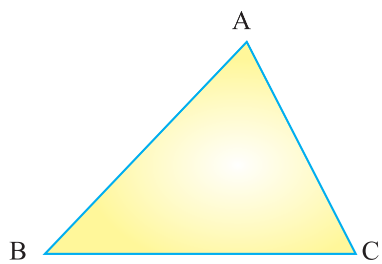
Objective : To explain the concept of median and centroid.

Previous knowledge required : Vertices, angles and sides of a triangle, skill of paper folding, concept of mid point and median.

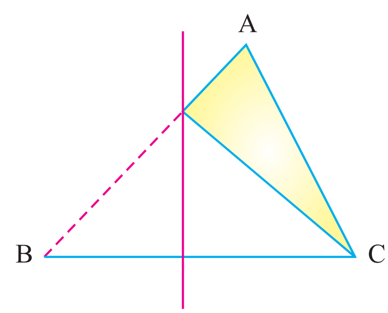
Material required : A white chart paper, a pair of scissors coloured pencil, a ruler.

Procedure :

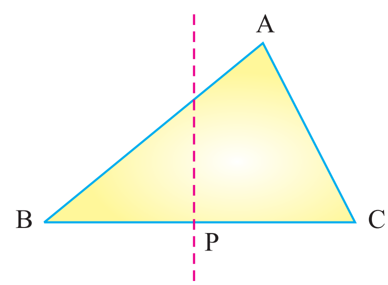
1. Draw a triangle on white paper and cut out of the triangle from the sheet. Fill a colour of your choice.



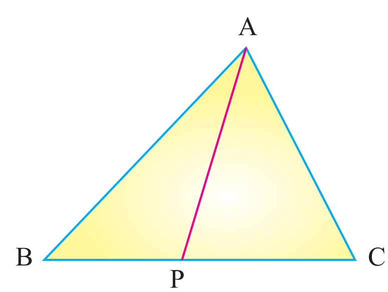
2. Fold the triangle ΔABC in such a way that vertex B falls on the vertex C and two parts of the side BC overlap each other.



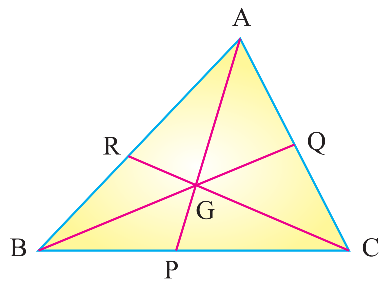
3. Mark the point of intersection of the line of fold with BC as P.



4. Join AP.



5. Line segment AP is the median from vertex A to side BC.
6. Similarly get medians from vertices B and C as BQ, CR.



Observation : Medians AP, BQ and CR intersect each other at a common point G.

Result : All medians passes through the same point G is called the controid of the triangle.

VIVA VOCE

Q.1. How many medians can be obtained in a triangle ?

Ans. 3 medians

Q.2. What do we call the point of concurrence of the medians of a triangle ?

Ans. Centroid

Q.3. Does a median bisect the line segment in which it fall ?

Ans. Yes

ACTIVITY-2

Angle sum property of a triangle.

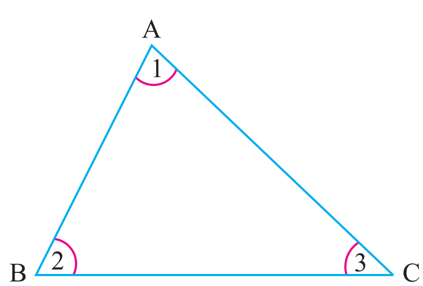
Objective : To Verify that sum of three angles of a triangle is 180°

Previous knowledge required : knowledge of straight angle and interior angles.

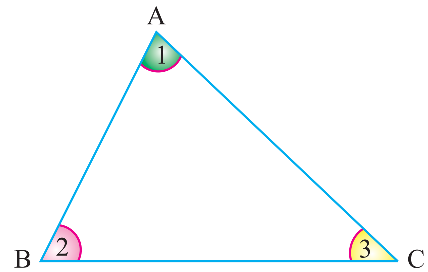
Materials required : White sheet of paper, pair of scissors, glue stick, coloured sketch pen, A ruler

Procedure :

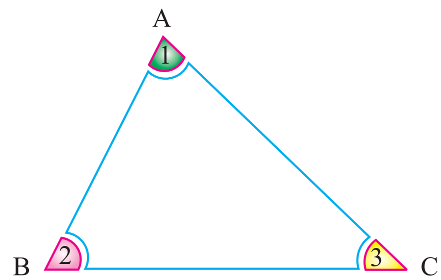
1. Draw a triangle say ΔABC on white sheet of paper. Mark the angle as 1, 2 and 3



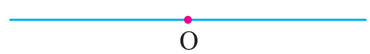
2. Cut out the triangle and fill different colour in three corners as shown in the figure.



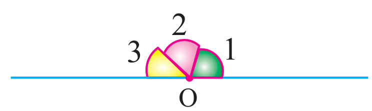
3. Cut out the three angles.



4. Draw a straight line and mark a point 'O' on it.



5. Paste the cut out angles so that their vertices lie on the point 'O' as shown in figure.



Observation : Three angles $\angle 1$, $\angle 2$ and $\angle 3$ in the figure are forming a straight line.

Result : Sum of interior angles of a triangle is 180°

Q.1. What is the angle sum property ?

Ans. The sum of all the interior angles of a triangle is 180°

Q.2. Is a triangle possible with angle 60°, 70°, 80°.

Ans. No

Exterior angle of a triangle is equal to the sum of two interior opposite angles.

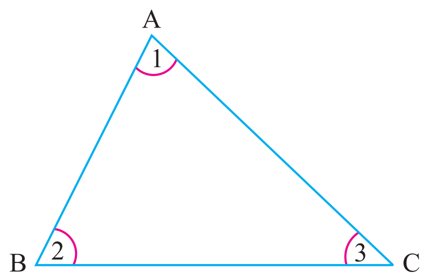
Objectives : To verify that the exterior angle in a triangle is equal to the sum of the interior opposite angles.

Previous knowledge required : Knowledge of exterior angles and interior opposite angles.

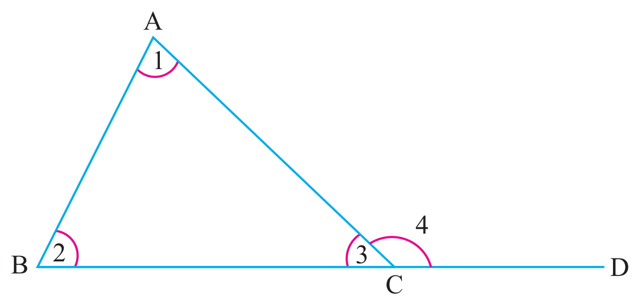
Material required : White sheet of paper, pair of scissors, glue stick, colared sketch pen.

Procedure :

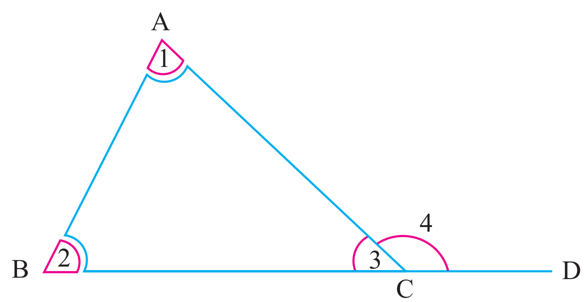
1. Take white sheet of paper and draw a triangle ABC. Name its angles as 1, 2 and 3 as shown in the figure.



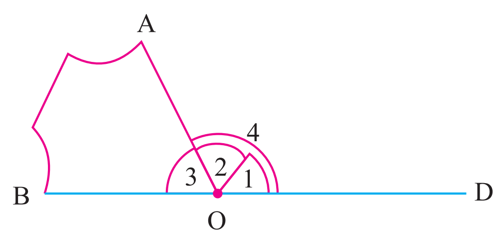
2. Extend the base of the triangle as shown in the figure. Name the exterior angle ∠ACD as 4.



3. Cut out the angle 1 and 2.



4. Paste angles 1 and 2 along the base of ∠ACD.



Observation : The angle 1 and 2 which are cut out completly fit into ∠ACD. This shows that ∠4 = ∠1 + ∠2

Result : The exterior angle is equal to the sum of two opposite interior angles of a triangle.

VIVA VOCE

Q.1. What is the exterior angle property of a triangle ?

Ans. In a triangle, exterior angle is equal to the sum of its two opposite interior angles.

Q.2. What is the measure of exterior angle formed in an equilateral triangle ?

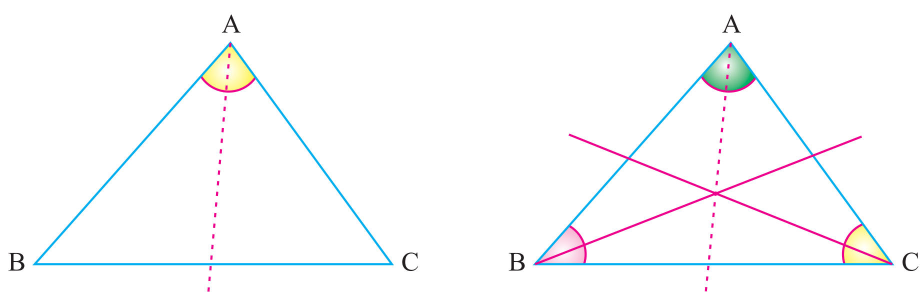
Ans. 120°

ACTIVITY-4

To illustrate that the internal bisectors of angles of a triangle concur at a point.

Pre-Requisite : The students must have skill to bisect an angle.

Material required : Cut a triangle (ABC) from a paper, Bisect the vertex A of the triangle by folding the paper. The crease formed is the angle bisector of angle A and so on as shown in figures below :



Observations : We see that the three angle bisectors are concurrent and the point, where these are meet, is called the incentre and the incentre of triangle always lies inside the triangle.

Learning Outcomes : We learn that the internal bisectors of angles of a triangle are concurrent and intersect inside than triangle at a common point, called incentre of triangle.

VIVA VOCE

Q.1. What is an angle bisector ?

Ans. A line which divides an angle into two equal parts is said to be the angle bisector of the given angle.

Q.2. How many angle bisectors are there in a triangle ?

Ans. There are three angle bisectors in a triangle.

WHAT HAVE WE DISCUSSED ?

1. A triangle is a closed figure with three sides. It has three angles and three vertices.
2. Sum of the angles of a triangle is 180° (Angle sum property of a triangle)
3. An exterior angle of a triangle is equal to the sum of its opposite interior angles.
4. The sum of any two sides of a triangle is always greater than the third side (Triangle inequality)

5. In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides (Pythagoras theorem).
6. An altitude of a triangle is the perpendicular line segment drawn to a side of a triangle from the opposite vertex.
7. A median is a line segment joining a vertex to the mid point of the opposite side of a triangle.
8. The medians of a triangle intersect at one point called "Centroid"
9. The altitudes of a triangle intersect at one point called "Orthocenter"

LEARNING OUTCOMES

After completion of the chapter the students are now able to :

1. Define and draw medians and altitudes of a triangle.
2. Understand the relation between exterior and interior angles of a triangle.
3. Understand the relation between the sides of a right angled triangle.
4. Use exterior angle property, angle sum property and Pythagoras theorem to solve various problems.
5. Find unknown the angle of a triangle when its two angles are known.

ANSWERS

EXERCISE 6.1

1. (*i*) PC (*ii*) Median
(*iii*) 90° (*iv*) False
(*v*) Altitude.

2. (*b*) lengths of all medians are equal.

3. (*i*) 140° (*ii*) 50°
(*iii*) 120° (*iv*) 120°

4. (*i*) 80° (*ii*) 45°
(*iii*) 50° (*iv*) 130°

5. (*i*) 70° (*ii*) 70°
(*iii*) 20°

EXERCISE 6.2

1. (*i*) No (*ii*) Yes
(*iii*) No (*iv*) Yes
(*v*) No

2. (*i*) 67° (*ii*) 48
(*iii*) 55° (*iv*) 30°
(*v*) 60° (*vi*) 60°

3. (*i*) $x = 50°, y = 70°$ (*ii*) $x = 80°, y = 100°$
(*iii*) $x = 80°, y = 20°$ (*iv*) $y = 45°$
(*v*) $x = 60°, y = 60°$ (*vi*) $x = 60°, y = 65°$

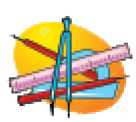
4. 50°, 60°, 70°
5. 40°, 80°
6. 68°
7. 36°, 54°
8. 76°, 58°, 46°
9. (*i*) *a* (*ii*) *c*
(*iii*) *d* (*iv*) *b*

EXERCISE 6.3

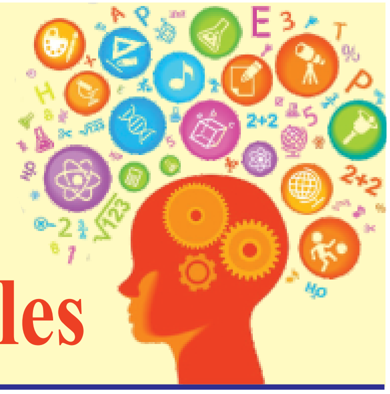
1. (*i*) 5*cm* (*ii*) 25*cm*
2. (*i*) Not a right angled triangle
(*ii*) right angled triangle, angle opposite to side of length 2.5*cm*
(*iii*) Not a right angled triangle.
3. 9*m*
4. 120cm^2, 46*cm*
5. 6*cm*
6. 5*m* each
7. 10cm
8. (*i*) No (*ii*) Yes
(*iii*) No (*iv*) Yes
9. (*i*) *d* (ii) *a*
(*iii*) *c* (iv) *b*
(v) *d* (vi) *a*
(*vii*) *b*

EXERCISE 6.4

1. (*b*) and (*c*)
2. (*a*) > (*b*) > (c) >
3. Yes
4. Yes
5. Between 3cm and 9cm



CHAPTER 7



Y4M2M3

Congruence of Triangles

Learning Objectives :-

In this chapter, you will learn :-

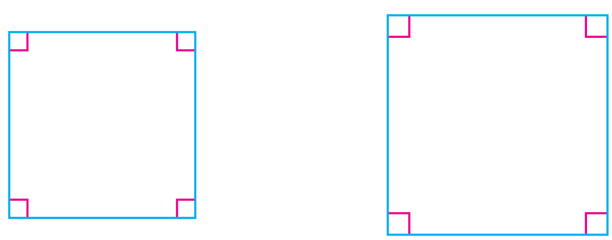
1. To recognise whether two figures are congruent.
2. To check whether two triangles are congruent by using SSS, SAS, ASA and RHS congruence rules.
3. To list the corresponding parts of congruent triangles.

INTRODUCTION

In previous chapter you have learnt about the properties of triangles such as angle sum property of a triangle, exterior angle property of a triangle, pythagoras property etc. Now you are ready to learn a very important geometrical idea called "Congruence". In particular, you will study about congruence of triangles.

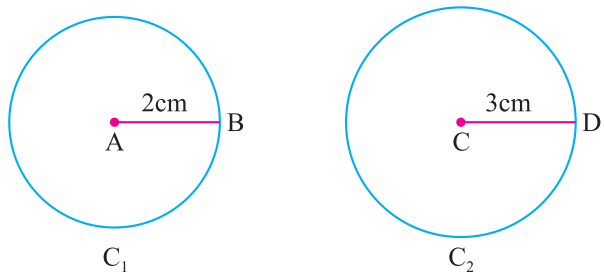
Congruence : In our daily life we deal with many shapes and figures. We find that some of the shapes are exactly same to each other. Let us check whether the following figures have the same size and shape ?

(*a*) In this case, these two squares are of different size but same shape.



(*b*) Here, the two postal stamps have same size and same shape.

(*c*) These circles are of same shape but different size.



So, we observe that two postal stamps have same shape and same size. Therefore, they are congruent. In general, two objects are called congruent if and only if they have exactly the same shape and the same size. The relation of two objects being congruent is called "congruence". In this chapter, we will deal with plane figures only, although congruence is a general idea applicable to three dimensional shapes also.

CONGRUENCE OF PLANE FIGURES

Consider the two figures F_1 and F_2 (Fig 7.1) of same shape. Are they congruent ?

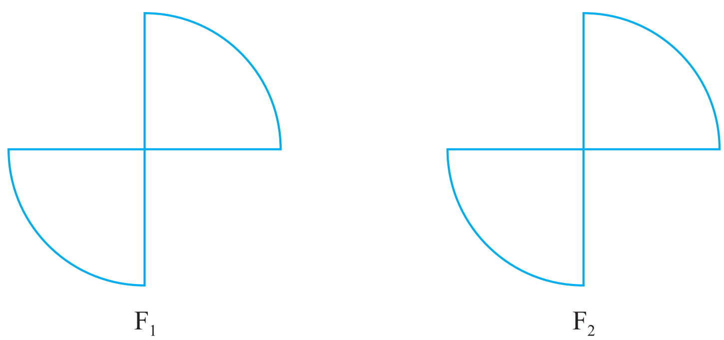


Figure : 7.1

You can check it by method of superimposition, In order to decide whether they are of the same size or not, lets take a traced copy of figure F_1 and place it over the figure F_2. If the figures cover each other completely they are congruent.

If figure F_1 is congruent to figure F_2 we write $F_1 \cong F_2$.

'≅' symbol is used to denote the congruence between the figures.

CONGRUENCE AMONG LINE SEGMENTS

Two line segments are congruent, if they are of equal length, Also if two line segments are congruent then they are of equal length.

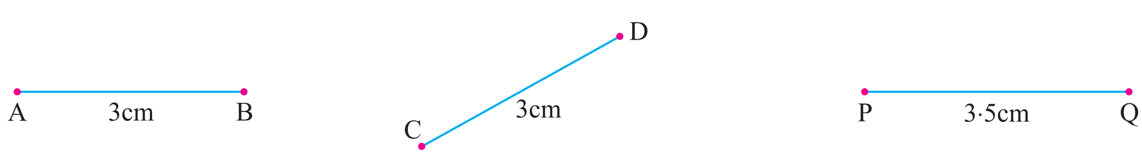


Figure : 7.2

In fig 7.2 length of AB = length of CD = 3 cm

∴ AB is congruent to CD *i.e* AB ≅ CD

But length of AB ≠ length of PQ as $3cm \neq 3.5cm$.

∴ Line segment AB is not congruent to line segment PQ.

We can also check it by superimposition method. Take a trace copy of AB and place it on CD. You find that AB covers CD with A on C and B on D. Hence, the line segments are congruent. Try this method to check congruence of AB and PQ.

CONGRUENCE OF ANGLES

Two angles are congruent, if they have equal measure conversely. If two angles are congruent, then their measure are same.

Look at the following figures (figure 7.3)

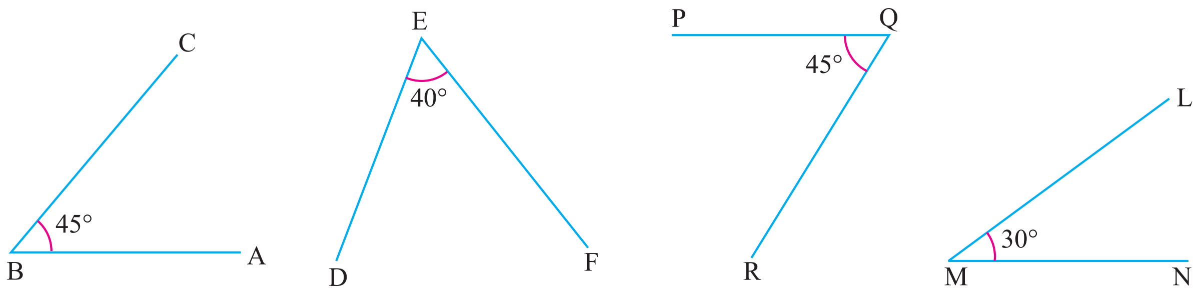


Figure : 7.3

Take a trace copy of ∠ABC and try to superpose it on ∠DEF such that B falls on E and BA falls along ED. You will find that arm BC does not fall on EF. Therefore, ∠ABC and ∠DEF are not congruent. Now check whether the following pairs of angles are congruent or not ?

(*i*) ∠PQR and ∠LMN (fig 7.3) (*ii*) ∠ABC and ∠PQR (fig 7.3)

CONGRUENCE OF SOME MORE PLANE FIGURES

(*i*) Two circles are congruent, if they have equal radii. Also if raddi of two circles are equal they are congruent. In figure 7.4 Circle $C_1 \cong$ Circle C_2.

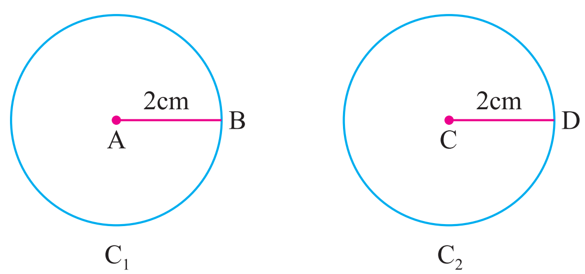


Figure : 7.4

(*ii*) Two squares are congruent, if they have equal sides. Conversly, two congruent squares have equal sides. In the following figure square ABCD ≅ square PQRS but square ABCD is not congruent to square LMNO.

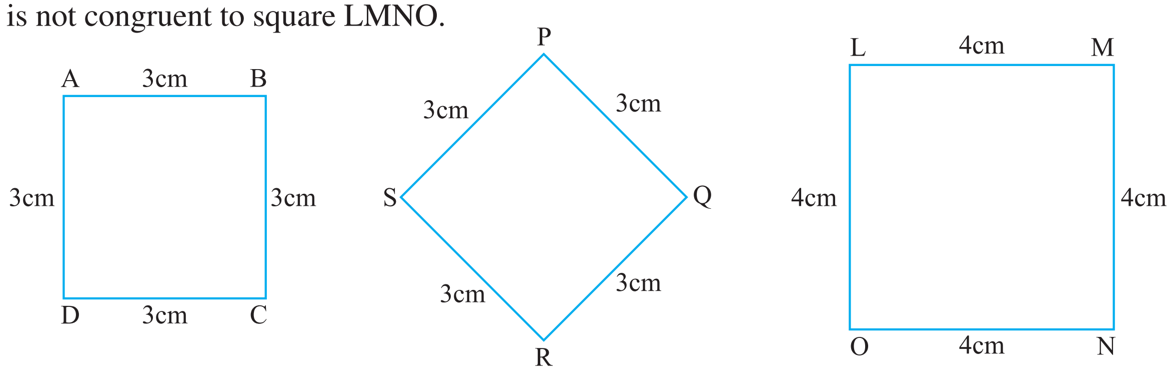


Figure : 7.5

(*iii*) Two rectangles are congruent if they have equal length and breadth. Conversely two congruent rectangles have equal length and breadth. Here rectangle ABCD ≅ rectangle LMNO.

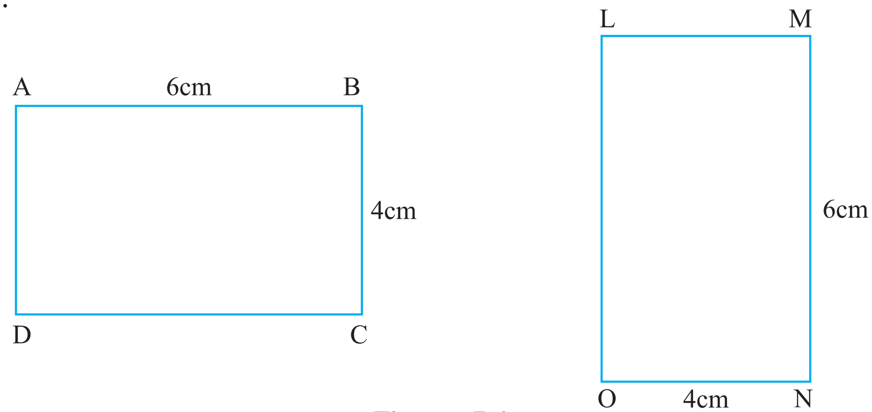


Figure : 7.6

CONGRUENCE OF TRIANGLES

Two triangle are said to be congruent if corresponding sides and angles of two triangles are equal and when superposed they cover each other exactly. Thus congruent triangles are exactly identical.

i.e

$$AB = PQ$$
$$CA = RP, BC = QR$$
$$\angle BAC = \angle QPR$$
$$\angle ABC = \angle PQR$$
$$\angle ACB = \angle PRQ$$

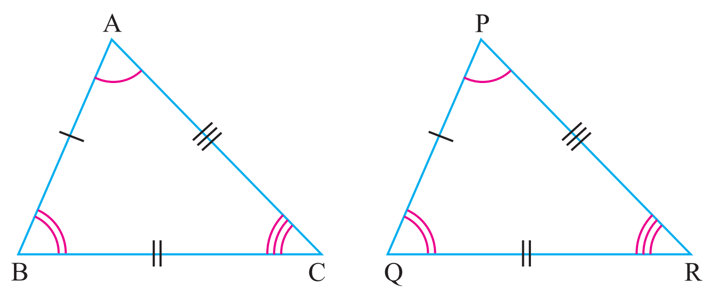


Figure : 7.7

Here ΔABC ≅ ΔPQR. *i.e* when superimposed A falls on P, B falls on Q, C falls on R, BC falls on QR, CA falls on RP and AB falls on PQ

This relationship between the parts of two triangles is known as correspondence denoted by ↔. Thus in these two congruent triangle we have

Corresponding vertices : A ↔ P, B ↔ Q, C ↔ R

Corresponding sides : AB = PQ, BC = QR, CA = RP

Corresponding Angles : ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R

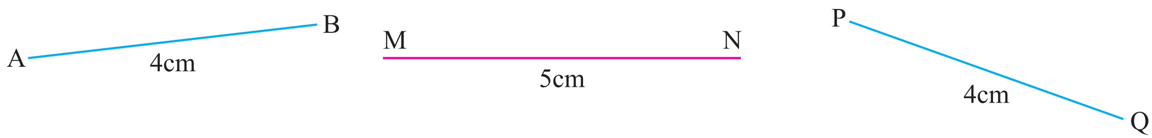
For the above congruence, we can also write ΔBCA ≅ ΔQRP or ΔCAB ≅ ΔRPQ. But it will not be correct to write ΔABC ≅ ΔRPQ or ΔCAB ≅ ΔPQR.

Thus, while talking about congruence of triangles not only the measures of angles and lengths of sides matter but also the matching of vertices is important.

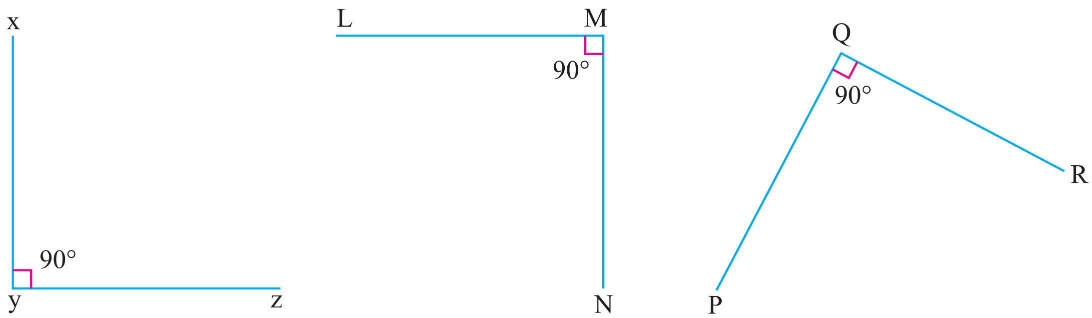
Note : The corresponding parts of congruent triangles are always equal. In short, it is written as (c.p.c.t.) (corresponding parts of congruent triangles)

Example-1 : The measure of lengths and angles are given in the following figures. Find which are congruent.

(*a*)



(*b*)



Sol. (*a*) Length of line segment AB = 4*cm*

Length of line segment MN = 5*cm*.

Length of line segment PQ = 4*cm*

as length of line segment AB = length of line segment PQ = 4*cm*

∴ AB ≅ PQ

(*b*) In figure

$\angle XYZ = 90°$

$\angle LMN = 90°$

$\angle PQR = 90°$

Now $\angle XYZ = \angle LMN = \angle PQR = 90°$

∴ $\angle XYZ \cong \angle LMN \cong \angle PQR$

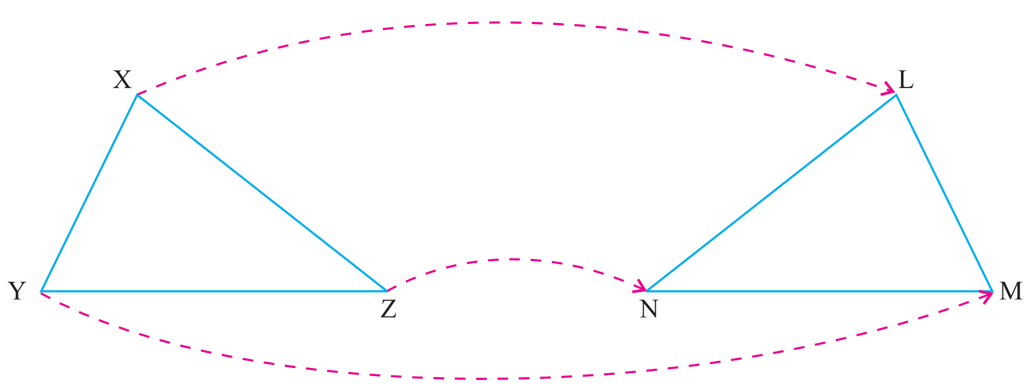
Example-2 : ΔXYZ and ΔLMN are congruent under the correspondence :

XYZ ↔ LMN *i.e* ΔXYZ ≅ ΔLMN

Write the parts of ΔXYZ that correspond to

(*i*) YZ (*ii*) ∠Y (*iii*) ZX

Sol. First of all we draw a diagram of given correspondence.



The correspondence is XYZ $\leftrightarrow$ LMN.

This means X $\leftrightarrow$ L, Y $\leftrightarrow$ M, Z $\leftrightarrow$ N

Therefore

(*i*) YZ = MN

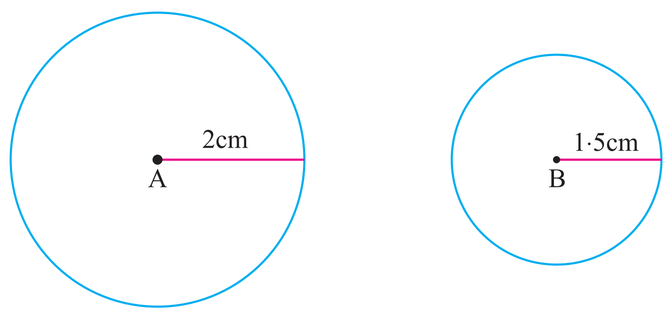
(*ii*) $\angle Y = \angle M$

(*iii*) ZX = NL

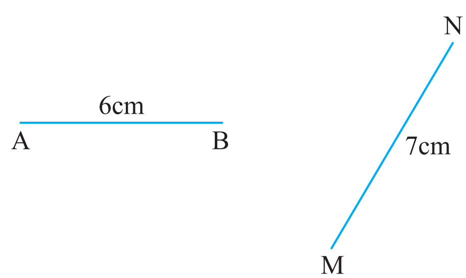
EXERCISE - 7.1

1. Identify the pairs of congruent figures and write the congruence in symbolic form.

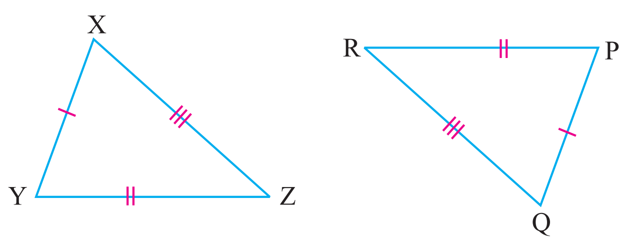
(*i*)



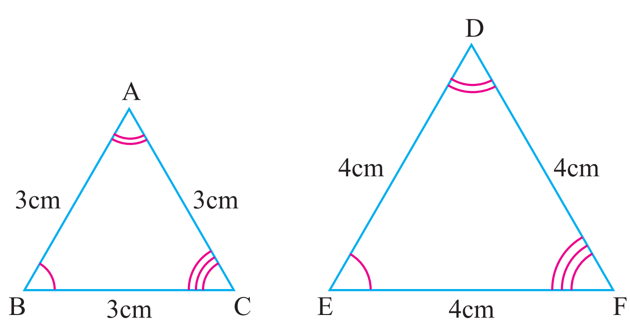
(*ii*)



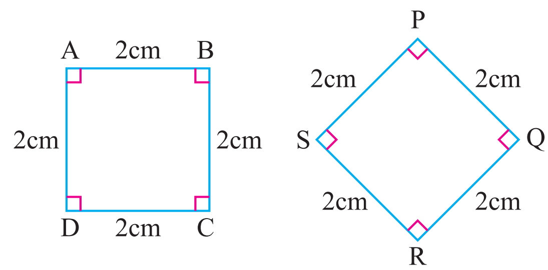
(*iii*)



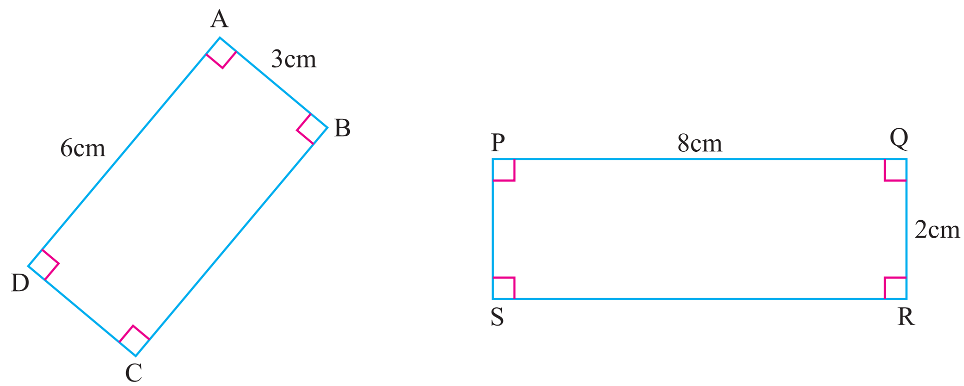
(*iv*)



(*v*)



(*vi*)



2. If ΔPQR ≅ ΔOMN under the correspondence PQR ↔ OMN, write all the corresponding congruent parts of the triangle.

3. Draw any two pairs of congruent triangles.

4. If ΔABC ≅ ΔZYX, write the parts of ΔZYX that correspond to.

(*i*) ∠B (*ii*) CA

(*iii*) AB (*iv*) ∠C

5. **Multiple choice questions :**

(*i*) If ΔABC ≅ ΔXYZ under the correspondence ABC ↔ XYZ, Then

(*a*) ∠A = ∠Z (*b*) ∠X = ∠B

(*c*) ∠A = ∠X (*d*) ∠C = ∠X

(*ii*) Two line segments are congruent if.

(*a*) They are parallel

(*b*) They intersect each other

(*c*) They are part of same line

(*d*) They are of equal length

(*iii*) Two triangles ΔABC and ΔLMN are congruent

AB = LM, BC = MN. If AC = 5*cm* then LN is :

(*a*) 3*cm* (*b*) 15*cm*

(*c*) 5*cm* (*d*) Can't find

6. Two right angles are always congruent. **(True/ False)**

7. Two opposite sides of a rectangle are always congruent. **(True/ False)**

CRITERIA FOR CONGRUENCE OF TRIANGLES

We know that the three sides and the three angles are the six matching parts for a congruence of triangles.

To determine congruence of two triangles, it is sufficient to compare only three pairs of corresponding parts of the given triangles. We can check it with the help of following criteria.

SSS congruence criterion : SSS stands for side-side-side.

This Criterion states that if all the pairs of the corresponding sides of the two triangles are equal.

Draw a ABC where AB = 5*cm*, BC = 4*cm* and CA = 6*cm*

Draw another PQR in which PQ = 6*cm*, QR = 5*cm* and RP = 4*cm* as shown in the following figure.

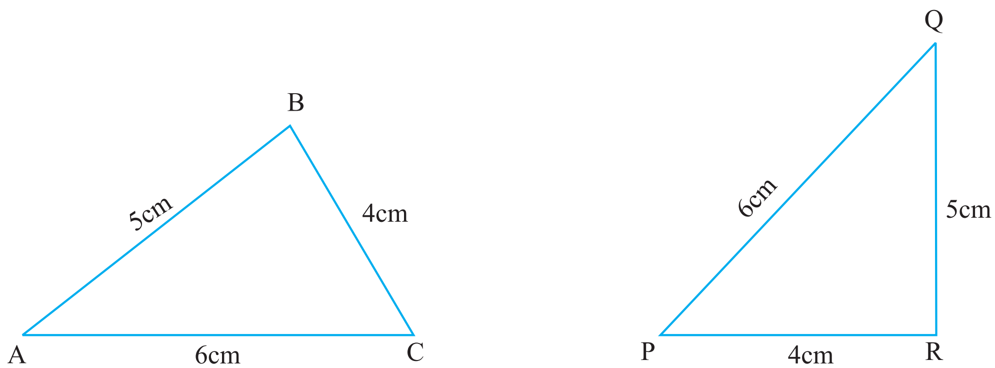


Figure : 7.10

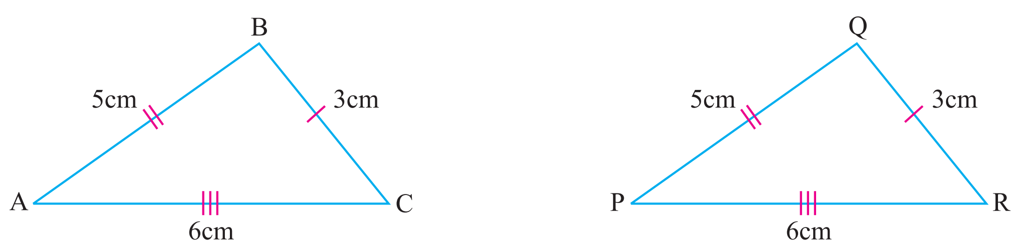
Make a trace copy of ΔABC using a tracing paper and superimpose it on ΔPQR where C falls on P, A falls on Q and B falls on R. We observe that ΔABC will cover ΔPQR

(Note that matching of vertices is also important)

∴ ΔABC ≅ ΔQRP

Example-1 : In triangle ABC and PQR, AB = 5*cm*, BC = 3*cm*, CA = 6*cm* and QR = 3*cm*, RP = 6*cm* and PQ = 5*cm*. Check whether the two triangles are congruent or not. If congruent, write the congruence rule.

Sol. Here, in ΔABC and ΔPQR



AB = PQ = 5cm

BC = QR = 3cm

CA = RP = 6cm

So by SSS congruence rule, the two triangles are congruent. From the above figure it is clear that A $\leftrightarrow$ P, B $\leftrightarrow$ Q and C $\leftrightarrow$ R

$\therefore$ $\Delta ABC \cong \Delta PQR$.

Example-2 : In (fig 7.11) AD = CD and AB = CB is $\Delta ABD \cong \Delta CBD$?

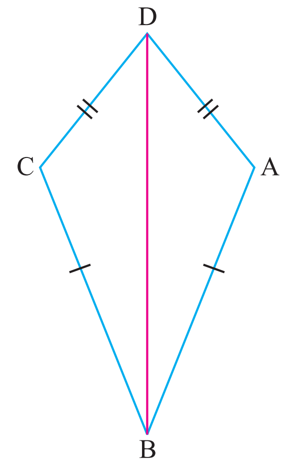


Figure : 7.11

Sol. In ΔABD and ΔCBD

AD = CD (given)

AB = CB (given)

DB = DB (common side)

$\therefore$ By SSS congruence rule, the two triangles are congruent

i.e. $\Delta ABD \cong \Delta CBD$

Example-3 : In the given figure 7.12 PQ = PR and S is the mid point of QR.

(*i*) Write the three pairs of equal sides in ΔPSQ and ΔPSR

(*ii*) Is $\Delta PSQ \cong \Delta PSR$? Give reason.

(*iii*) Is $\angle Q = \angle R$? Why ?

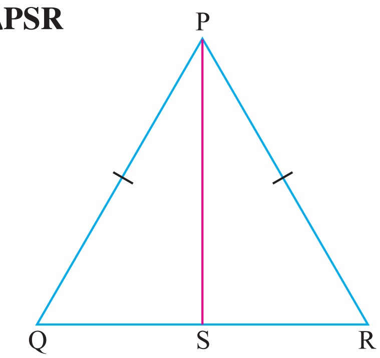


Figure : 7.12

Sol. In ΔPQR

PQ = PR

and S is the mid point of QR

(*i*) The three pairs of equal sides in ΔPSQ and ΔPSR are

PQ = PR (Given)

PS = PS (Common Side)

QS = RS [Since, S is mid point of QR]

(*ii*) Yes, From (*i*) part it is clear that

$\Delta PSQ \cong \Delta PSR$ (by SSS congruence rule)

(*iii*) Yes,

$\because$ $\Delta PSQ \cong \Delta PSR$

$\therefore$ $\angle Q = \angle R$ (By c.p.c.t)

SAS Congruence criteria : SAS stands for side-angle-side. SAS criterion states that two triangles are congruent if two sides and the angle-between them of one triangle are equal to the corresponding sides and angle of the other triangle.

ACTIVITY

With the help of a ruler and protractor construct a ΔPQR where PQ = 3*cm*, QR = 4*cm* and $\angle Q = 70°$.

Construct another ΔXYZ in which XY = 3 cm, YZ = 4 cm and Y = 70° as shown in figure 7.13.

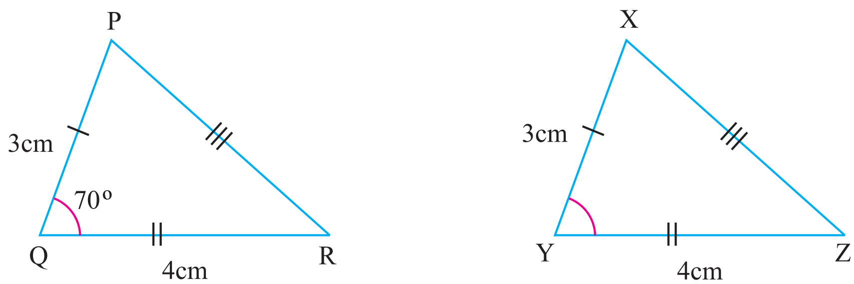
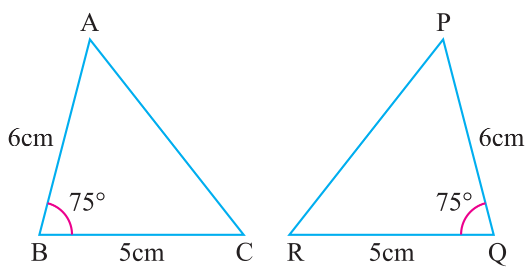


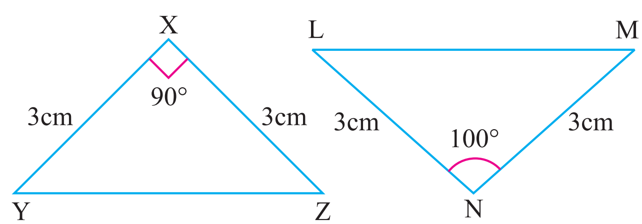
Figure : 7.12

Make a trace copy of ΔPQR using a tracing paper and superimpose it on ΔXYZ with PQ coinciding with XY and QR coinciding with YZ also angle Q lies on angle Y. We observe that PR coincide with XZ. This shows that ΔPQR ≅ ΔXYZ.

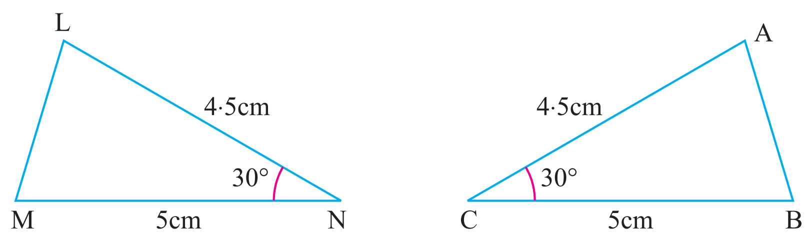
Example-4 : In the following figures, measures of some parts of the triangles are indicated. By applying SAS congruence rule, write the pairs of congruent triangles, if any. In case of congruent triangles write them in symbolic form.



(*i*)



(*ii*)



(*iii*)

Sol. (*i*) In ΔABC and ΔPQR, we have

$$AB = PQ = 6cm$$
$$BC = QR = 5cm$$
$$\angle ABC = \angle PQR = 75^\circ$$

So, by SAS congruence rule, two triangles are congruent

The correspondence is A ↔ P, B ↔ Q and C ↔ R.

So ΔABC ≅ ΔPQR

(*ii*) In ΔXYZ and ΔLMN, we have

$$XY = NL$$
$$XZ = NM$$

But $\angle YXZ \neq \angle LNM$ ($\because 90° \neq 100°$)

So SAS congruence rule cannot be applied

Therefore ΔXYZ is not congruent to ΔLMN

(*iii*) In ΔLMN and ΔABC, we have

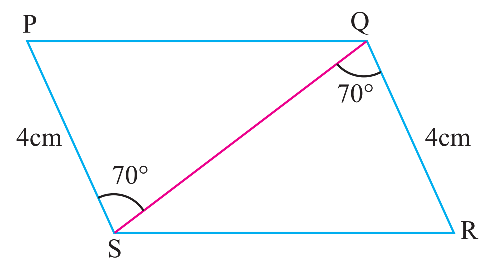
$$MN = BC = 5cm$$
$$LN = AC = 4.5cm$$
$$\angle LNM = \angle ACB = 30°$$

So, by SAS congruence rule, two triangles are congruent

The correspondence is $L \leftrightarrow A$, $N \leftrightarrow C$, $M \leftrightarrow B$

So $\Delta LNM \cong \Delta ACB$

Example-5 : In given quadrilateral PQRS, PS = 4*cm*, QR = 4*cm*, ∠PSQ = 70°, ∠RQS = 70°. Show that ΔPSQ ≅ ΔRQS. Where SQ is a diagonal of quadrilateral PQRS.



Sol. Clearly SQ divides quadrilateral PQRS into two triangles. ΔPSQ and ΔRQS

Now in ΔPSQ and ΔRQS

$PS = RQ = 4cm$ (Given)

$SQ = QS$ (Common Side)

$\angle PSQ = \angle RQS = 70°$ (Given)

So, by SAS congruence rule, two triangles are congruent, Here $P \leftrightarrow R$, $S \leftrightarrow Q$ and $Q \leftrightarrow S$

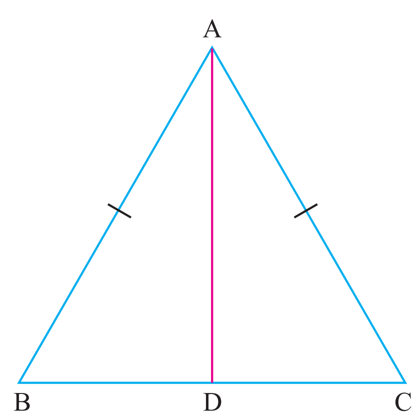
$\therefore \Delta PSQ \cong \Delta RQS$

Example-6 : In the adjoining figure, AB = AC and AD is the bisector of ∠BAC.

(*i*) Find three pairs of equal parts in triangles ΔADB and ΔADC

(*ii*) Is ΔADB ≅ ΔADC ? Give reasons.

(*iii*) Is ∠B = ∠C ? Give reasons.



Sol. (*i*) In ADB and ADC, three pairs of equal parts are

$AB = AC$ (Given)

$AD = AD$ (Common Side)

$\angle BAD = \angle CAD$ ($\because$ AD is bisector of $\angle BAC$)

(*ii*) Yes, from (*i*) by using SAS congruence rule, we can conclude that $\Delta BAD \cong \Delta CAD$

i.e $\Delta ADB \cong \Delta ADC$

(*iii*) From part (*ii*) note that $A \leftrightarrow A$, $D \leftrightarrow D$ and $B \leftrightarrow C$

$\therefore$ $\angle B = \angle C$ (corresponding parts of congruent triangles)

ASA congruence criterion : ASA stands for Angle-Side-Angle. ASA criterion states that two triangles are congruent if two angles and side between two angles of one triangle are respectively equal to the two angles and side between two angles of the other triangle.

ACTIVITY

Draw a triangle ΔCAR where AR = 4*cm*, $\angle A = 50°$ and $\angle R = 60°$

Draw another triangle ΔBUS in which US = 4cm $\angle U = 50°$ and $\angle S = 60°$ as shown in the following figure.

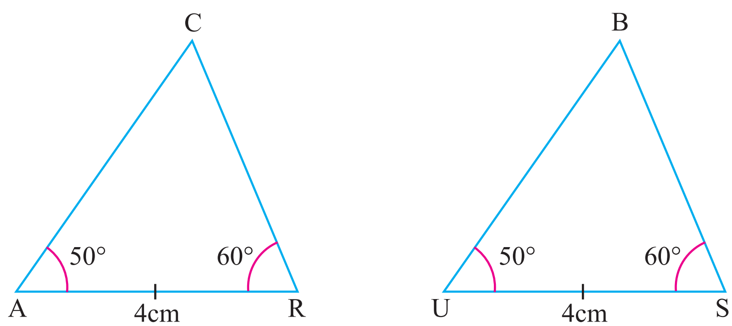


Figure : 7.14

Now make a trace copy of ΔCAR using tracing paper and place the traced copy of ΔCAR on the ΔBUS such that AR coincides with US and $\angle A$ falls on $\angle U$ and $\angle R$ falls on $\angle S$. We observe that ΔCAR will cover ΔBUS

$\therefore$ $\Delta CAR \cong \Delta BUS$

Example-7 : In the following figure $\angle B = 30°$, $\angle C = 45°$, $\angle Y = 30°$ and $\angle X = 105°$. Also BC = YZ. Prove that $\Delta ABC \cong \Delta XYZ$.

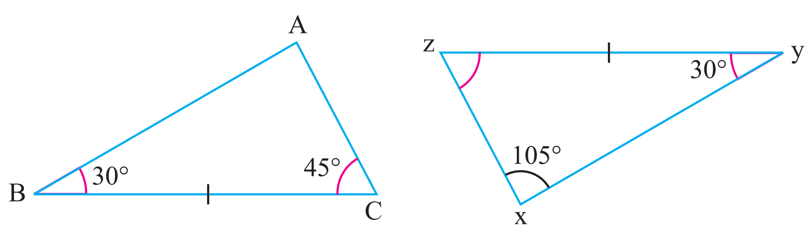


Figure : 7.15

Sol. Here $\angle Y = 30°, \angle X = 105°$

In ΔXYZ

$$\angle X + \angle Y + \angle Z = 180° \quad \text{(Angle sum property)}$$

$$105° + 30° + \angle Z = 180°$$

$$135° + \angle Z = 180°$$

$\angle Z = 180° - 135°$

$\angle Z = 45°$

Now, In ΔABC and ΔXYZ

$\angle B = \angle Y = 30°$

$\angle C = \angle Z = 45°$

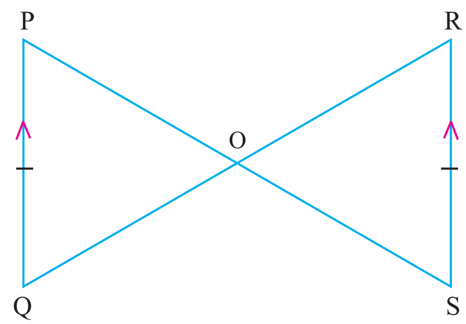
$BC = YZ$ (Given, Included sides)

$\therefore$ By ASA congruence rule

$\Delta ABC \cong \Delta XYZ$

Example-8 : In the adjoining figure, PQ || RS and PQ = RS. Prove that :

(*i*) $\Delta POQ \cong \Delta SOR$ (*ii*) PO = OS and QO = RO



Sol. (*i*) In ΔPOQ and ΔSOR

$PQ = RS$ (Given)

$\angle PQO = \angle SRO$ (Alternate angles)

$\angle QPO = \angle RSO$ (Alternate angles)

$\therefore$ By ASA congruence rule

$\Delta POQ \cong \Delta SOR$

(*ii*) From (*i*) part, $\Delta POQ \cong \Delta SOR$

$\therefore$ $PO = OS$ (corresponding sides of congruent triangles)

$QO = RO$ (corresponding sides of congruent triangles)

Example-9 : In ΔABC, the bisector AD of $\angle A$ is perpendicular to the side BC. (figure 7.16)

(*i*) Find three pairs of equal parts in triangles ADB and ADC

(*ii*) Is $\Delta ADB \cong \Delta ADC$? Give reasons

(*iii*) Is AB = AC ? Why ?

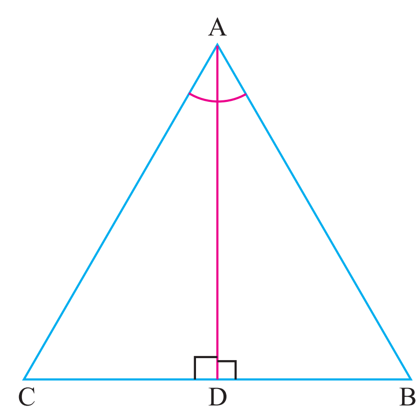


Figure : 7.16

Sol. (*i*) In ΔADB and ΔADC, three pairs of equal parts are

$\angle ADB = \angle ADC$ (90° each)

$\angle BAD = \angle CAD$ (As AD is bisector of $\angle A$)

$AD = AD$ (common)

(*ii*) Yes, from (*i*) by using ASA congruence rule, we conclude that $\Delta ADB \cong \Delta ADC$

(*iii*) From part (*ii*) note that $A \leftrightarrow A$, $D \leftrightarrow D$, $B \leftrightarrow C$

$\therefore$ AB = AC (corresponding parts of congruent triangles)

Note : When two angles of a triangle are known we can find the third angle by using angle sum property of triangle. So whenever, two angles and one side of a triangle are equal to the corresponding two angles and one side of another triangle, we can convert it into two angles and side between two angles form of congruence and then apply the ASA congruence rule.

Congruence among Right angled triangles–RHS congruence criteria.

RHS stands for right angle, hypotenuse, side. RHS criterion states that two right angled triangles are congruent, if the hypotenuse and one side of the first triangle are equal to the hypotenuse and one side of the second triangle.

ACTIVITY

With the help of a ruler and protractor, construct a ΔABC with $\angle C = 90°$, hypotenuse AC = 5cm and side AB = 4cm

Construct another LMN in which $\angle N = 90°$, Hypotenuse LM = 5cm and side LN = 4cm as shown in the following figure.

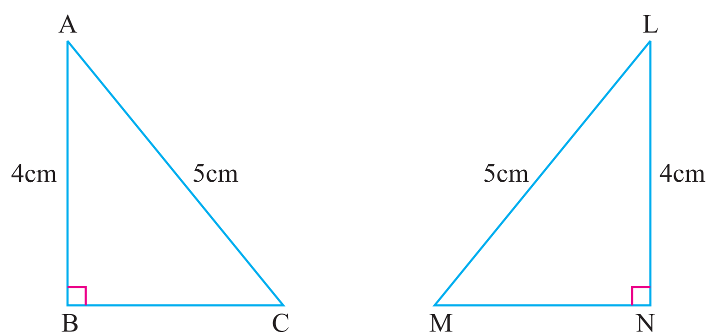


Figure : 7.17

Make a copy of ΔABC on a tracing paper and superimpose it on ΔLMN with AB coincides with LN and AC coincide with LM. We observe that BC coincide with MN. This shows that

$\Delta ABC \cong \Delta LNM$

Example-10 : The measures of some parts of two triangles ΔABC and ΔPQR are given below. Examine, whether the two triangles are congruent or not. In case of congruent triangles, write the result in symbolic form

(*i*) $\angle B = 90°$, $AC = 8cm$, $BC = 6cm$

$\angle Q = 90°$, $PR = 8cm$, $QR = 6cm$

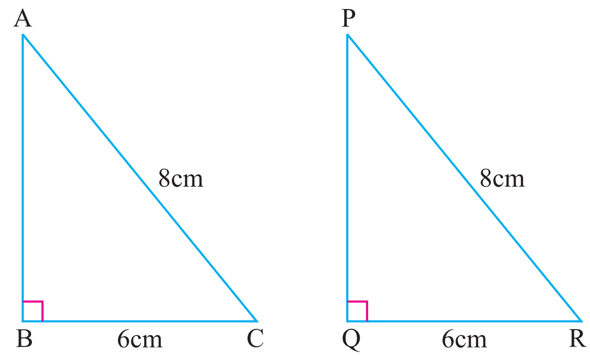
(*ii*) $\angle A = 90°$, $AC = 4cm$, $BC = 5cm$

$\angle Q = 90°$, $QP = 4cm$, $RP = 6cm$

Sol. (*i*) In ΔABC and ΔPQR

$\angle B = \angle Q$ (90° each)

Hypotenuse AC = Hypotenuse PR = 8cm

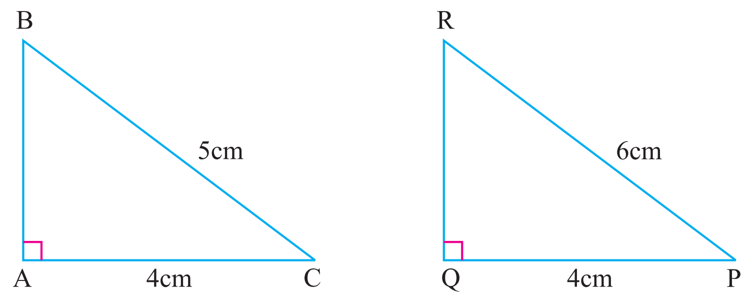


Side BC = Side QR = 6cm

∴ By RHS congruence rule

ΔABC ≅ ΔPQR

(*ii*) In ΔABC and ΔPQR ∠A = ∠Q = 90° (Given)



Side AC = Side QP = 4*cm*

But Hypotenuse BC ≠ Hypotenuse RP

as 5*cm* ≠ 6*cm*

So, the triangles are not congruent.

Example-11 : In figure 7.19, ΔPQR and ΔPQS are right angled at R and S respectively. Also PR = QS. Prove that ΔPQR and ΔPQS are congruent. Also show that ∠PQR = ∠QPS.

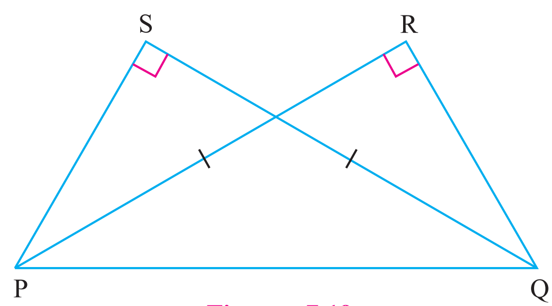


Figure : 7.19

Sol. In given triangles ΔPQR and ΔPQS

∠R = ∠S (90° each)

Side PR = Side QS (given)

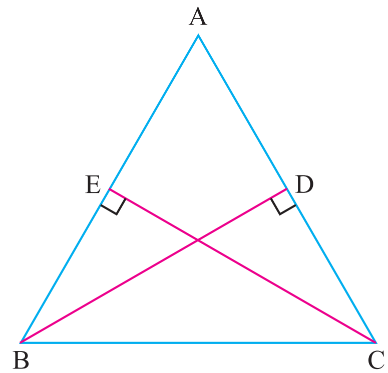
Hypotenuse PQ = Hypotenuse PQ (common)

So, By RHS congruence rule

ΔPQR ≅ ΔQPS

∴ ∠PQR = ∠QPS(corresponding parts of congruent triangles)

Example-12 : In adjoining figure BD and CE are altitudes of ΔABC such that BD = CE.



(*i*) Find three pairs of equal parts in ΔBCE and ΔCBD

(*ii*) Is ΔBCE ≅ ΔCBD ? Give reasons

(*iii*) Is ∠EBC = ∠DCB ? Give reasons

Sol. In ΔBCE and ΔCBD, three pairs of equal parts are

∠BEC = ∠BDC (90° each)

Hypotenuse BC = Hypotenuse CB (common side)

Side CE = side BD (given)

(*ii*) Yes, from part (*i*)by using RHS congruence rule, we conclude that ΔBCE ≅ ΔCBD with correspondence B↔C, C↔B, E↔D

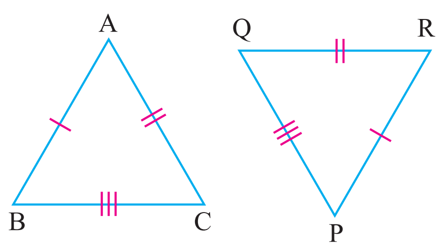
(*iii*) Yes, from part (*ii*) ΔBCE ≅ ΔCBD, we know that, the corresponding parts of congruent triangles are equal

Therefore ∠EBC = ∠DCB

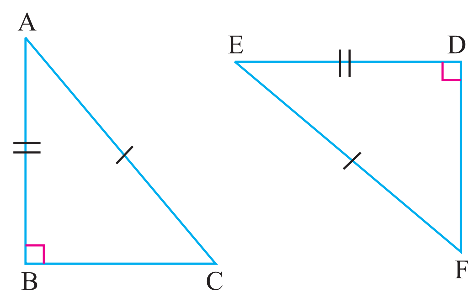
EXERCISE - 7.2

1. In the following pair of triangles examine whether the triangles are congruent or not. Write the rule of congruence if triangles are congruent.

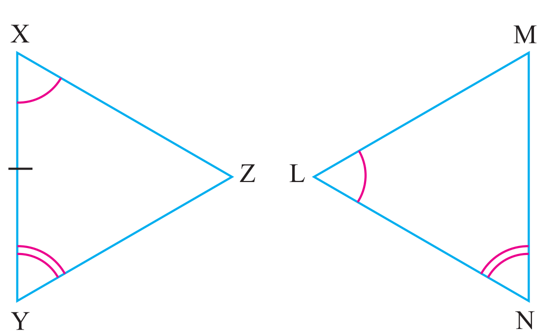
(*i*)



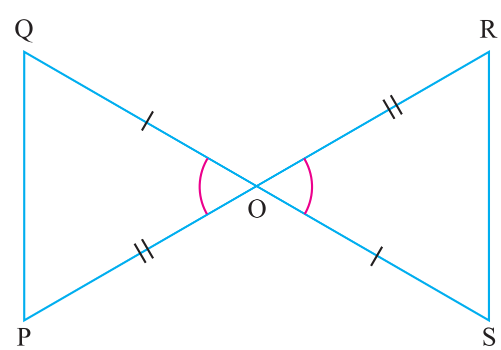
(*ii*)



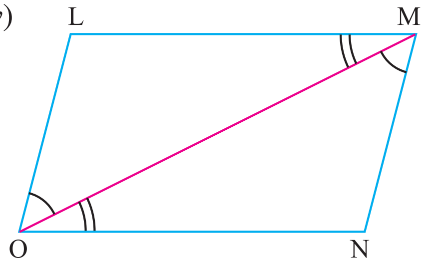
(*iii*)



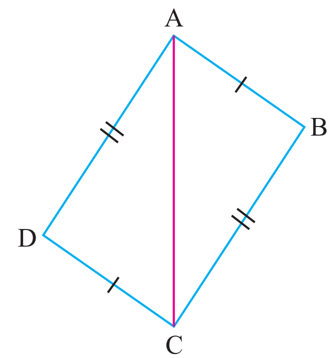
(*iv*)



(*v*)



(*vi*)



2. In fig 7.19 $\Delta AMP \cong \Delta AMQ$. Give reason for the following steps.

Steps	Reasons
(*i*) PM = QM	....
(*ii*) $\angle PMA = \angle QMA$	....
(*iii*) AM = AM	....
(*iv*) $\Delta AMP \cong \Delta AMQ$	....

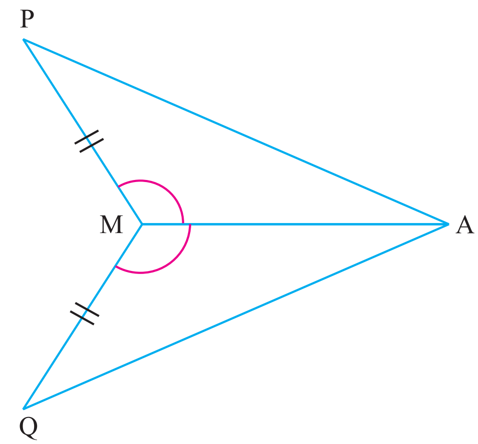
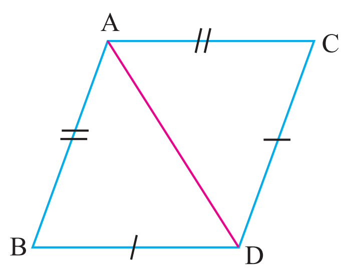


Figure : 7.19

3. In given figure AB = AC and BD = DC. Prove that

(*i*) $\Delta ABD \cong \Delta ACD$

(*ii*) $\angle B = \angle C$



4. In the given (figure 7.20), AC = CE and BC = CD. Prove that $\Delta ACB \cong \Delta ECD$.

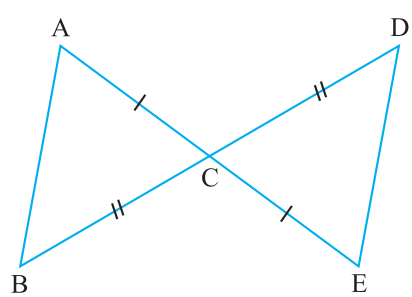
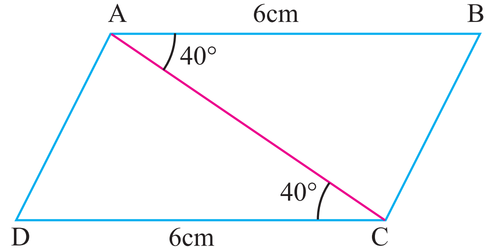


Figure : 7.20

5. In the adjoining figure.



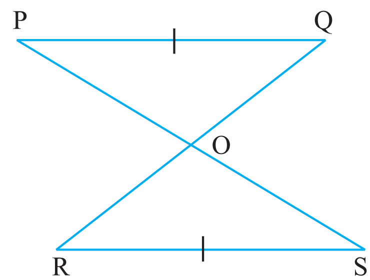
(*i*) Write three pairs of equal parts in ΔADC and ΔCBA

(*ii*) Is ΔADC ≅ ΔCBA ? Give reasons.

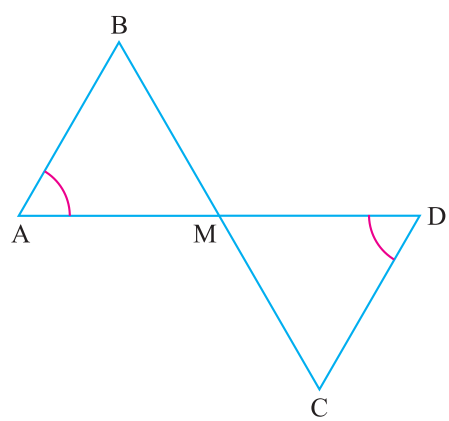
(*iii*) Is AD = CB ? Give reasons

6. In the given figure PQ || RS and PQ = RS. Prove that

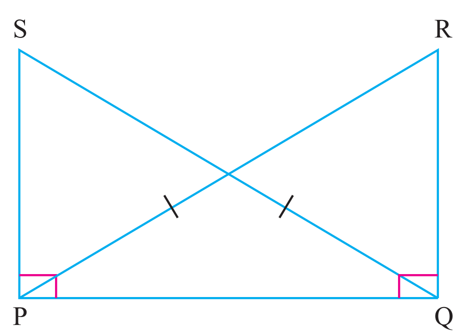
(*i*) ΔPOQ ≅ ΔSOR (*ii*) ∠POQ = ∠SOR



7. In the adjoining figure, M is mid point of AD and ∠A = ∠D. Show that ΔAMB ≅ ΔDMC



8. In the adjoining figure SP ⊥ PQ, RQ ⊥ PQ and PR = QS



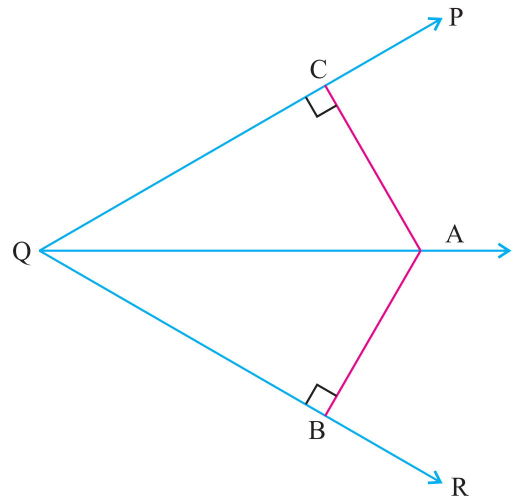
(*i*) Write three parts of equal parts in ΔPQR and ΔSPQ

(*ii*) Prove that ΔPQR ≅ ΔQPS

9. In given figure AB ⊥ QR, AC ⊥ QP and QC = QB. Prove that

(*i*) ΔQAB ≅ ΔQAC

(*ii*) ∠AQB = ∠AQC



10. Multiple choice questions :-

(*i*) Which of the following is not a congruence rule

(*a*) ASA (*b*) SAS

(*c*) SSS (*d*) AAA

(*ii*) If ΔABC ≅ ΔPQR, then the correct statement is

(*a*) ∠A = ∠Q (*b*) ∠A = ∠R

(*c*) ∠A = ∠P (*d*) AB = QR

(*iii*) If ∠A = ∠D, ∠B = ∠E and AB = DE, then ΔABC ≅ ΔDEF, by the congruence rule :

(*a*) SSS (*b*) ASA

(*c*) SAS (*d*) RHS

11. ASA congruence criterion is same as SAS congruence criterion. **(True/ False)**

12. Two right angled triangles are always congruent. **(True/ False)**

13. '=' symbol used for congruence of triangles. **(True/ False)**

Objective : In an isosceles triangle, the angle opposite to equal sides are equal.

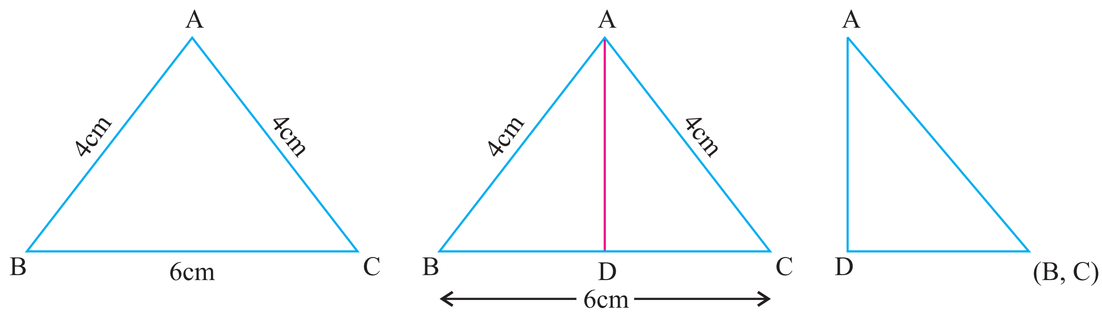
Previous knowledge : Students must have knowledge of isosceles triangle (Triangle having two sides equal)

Material Required : Coloured paper, Geometry box, Coloured pencils.

Procedure :

1. Draw a ΔABC on coloured sheet in which AB = AC = 4 cm, BC = 6 cm.
2. Cut ΔABC and fold it in such a way that AB coincides AC. Press it to get a crease.

3. We observe that vertex C lies on B and the parts of sides BC lie on over the other.



Observation : We will find that ∠B and ∠C fit on each other exactly, side AB completely overlaps the side AC.

Result : $m\angle B = m\angle C$

∴ In an isoscles Δ, the angle opposite to equal sides are equal.

VIVA VOCE

Q.1. How many sides of an isosceles triangle are equal ?

Ans. 2

Q.2. How many angles does a triangle have ?

Ans. 3

Q.3. How many angles of an isosceles triangle are equal ?

Ans. 2

WHAT HAVE WE DISCUSSED ?

1. The figures having same shape and size are called congruent figures.

2. Symbol used to denote the congruence between two figures is '≅'

3. The method of superimposition examines the congueence of plane figures.

4. Two lines segment are congruent if and only if they have equal length.

5. Two angles are congruent if and only if they have equal measurement.

6. Two triangles are congruent if three parts of a triangle are equal to the corresponding parts of another triangle satisfying certain congruence criterion.

7. **SSS :** Stands for side-side-side SSS criterian states that two triangles are congruent, if three sides of one triangle are respectively equal to the three sides of other triangle.

8. SAS : Stands for side-angle-side. SAS criterion states that two triangles are congruent, if two sides and the included angle (angle between these sides) of one triangle are respectively equal to the two sides and the included angle (angle between two sides) of the other triangle.

9. **ASA :** Stands for angle-side-angle. ASA criterion states that two triangles are congruent, if two angles and the included side. (Side between these angles) of one triangle are respectively equal to the two angles and the included side (side between two angles) of the other triangle.

10. **RHS :** Stands for Right angle-Hypotenuse-side. RHS criterion states that two right angled triangles are congruent, if the hypotenuse and one side of the first triangle are respectively equal to the hypotenuse and one side of the second triangle.

11. **AAA** and **SSA** do not work as congruence criterions.

LEARNING OUTCOMES

After completion of the chapter, students are now able to :

1. Check congruence through superimposition.
2. Explain the congruency of triangles on the basis of the information given about them like. SSS, SAS, ASA, RHS.
3. List the corresponding parts of congruent triangles.

ANSWERS

EXERCISE 7.1

1. (*i*) Not congruent (*ii*) Not congruent

(*iii*) $\Delta XYZ \cong \Delta QPR$ (*iv*) Not congruent

(*v*) $\square$ ABCD $\cong$ $\square$ PQRS (*vi*) Not congruent

2. Vertices : P $\leftrightarrow$ O, Q $\leftrightarrow$ M, R $\leftrightarrow$ N

Sides : PQ $\leftrightarrow$ OM, QR $\leftrightarrow$ MN, RP $\leftrightarrow$ NO

Angles : $\angle$PQR $\leftrightarrow$ $\angle$OMN, $\angle$QRP $\leftrightarrow$ $\angle$MNO, $\angle$RPQ $\leftrightarrow$ $\angle$NOM

4. (*i*) $\angle$Y (*ii*) XZ

(*iii*) ZY (*iv*) $\angle$X

5. (*i*) *c* (*ii*) *d*

(*iii*) *c*

6. True **7.** True

EXERCISE 7.2

1 (*i*) SSS (*ii*) RHS

(*iii*) ASA (*iv*) SAS

(*v*) ASA (*vi*) SSS

2. (*i*) Given (*ii*) Given

(*iii*) Common (*iv*) SAS congruence criterion

5. (*i*) AC = AC, DC = AB, $\angle$DCA = $\angle$BAC

(*ii*) Yes, SAS congruence criterion

(*iii*) Yes, c.p.c.t.

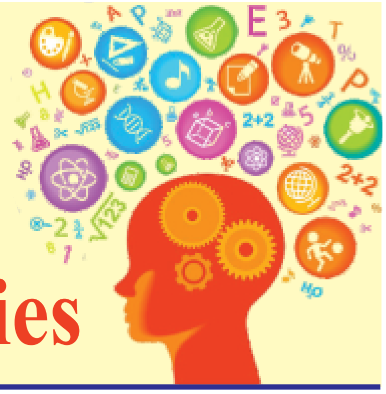
8. (*i*) PQ = QP, QR = SP, $\angle$PQR = $\angle$QPS

10. (*i*) *d* (*ii*) *c* (*iii*) *b*

11. False **12.** False **13.** False

CHAPTER 8

E2H6W4



Comparing Quantities

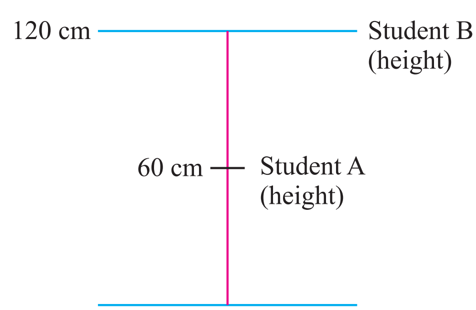
Learning Objectives :-

In this chapter, you will learn :-

1. To compare two quantities in your daily life.
2. To compare two ratios by converting them into like fraction.
3. About the concept of equivalent ratios.
4. About the concept of proportion i.e. a proportion is the equivalence of two ratios.
5. About the meaning of percentage.
6. To convert percentage into fraction, fraction into percentage, percentage into decimal, decimal into percentage, percentage into ratio and ratio into percentage.
7. To find the percentage of a given quantity.
8. Some new terms like cost price, selling price, profit, loss, profit% and loss%.
9. The concept of borrowing money at a particular rate of interest for a particular time period.
10. To solve the problems involving simple interest and amount.

INTRODUCTION

Suppose the interest rate provided by your bank on your savings increases, and you want to know the increase in your savings after the change. Or suppose you need to monitor how some change in the price of a particular model of a product, say a car or a washing machine, affects its sales. To do this, you should be well versed in concepts that involve comparisons such as ratios, proportions and percentages. Thus, to broaden your horizons as a banker or an economist, you need to possess a strong and in-depth knowledge and understanding of mathematics. Mathematics helps you to be a logical thinker. To be an economist, you need to be equipped with mathematical language and tools. As economists have to work in terms of percentages, ratios and proportions while comparing and measuring quantities such as prices, sales, wages, productivity, etc. Percentages are also used to calculate discounts, taxes, economic growth, etc. So let's understand the concepts of comparison.



In many situations, we need to compare two or more quantities and values. Suppose we are comparing heights of two students A and B. If the height of student A is 60*cm* and that of student B is 120cm then, we can say that, the height of student B is double to the height of student A or the height of student A is half of the height of student B.

Note : To compare two quantities, the units must be the same.

RATIO AND PROPORTION

Ratio : The ratio of two quantities of the same kind and in the same unit is the fraction that one quantitiy is of the other.

The ratio of a to b is the fraction $\frac{a}{b}$ and it is written as $a : b$

Where a is called the first term and b is called the second term.

A ratio has no units.

Example-1 : Find the ratio of 4*km* to 300*m*

Sol. To find the ratio of 4*km* to 300*m*.

Firstly, convert both the quantities into same unit.

$\because \quad 1km = 1000m \Rightarrow 4km = 4 \times 1000 = 4000m$

So, ratio of 4000*m* to 300*m* $= \dfrac{4000}{300} = 40 : 3$

Hence, required ratio is 40 : 3

SIMPLEST FORM

The ratio $(a : b)$ is in the simplest form, if H.C.F of a and b is 1, *i.e.* there is no common factor between a and b other than 1.

EQUIVALENT RATIO

To compare different ratios, first write the given ratios in the form of fraction. Convert fractions into like fractions. If the resulting fractions are equal, then the given ratios are said to be equivalent.

A ratio remains unchanged, if both of its terms are multiplied or divided by the same non-zero quantity.

Note : The ratio of two numbers is usually expressed in its simplest form.

For example if we have 1 : 3 and 2 : 9, we can compare these as follows :

Fraction of 1 : 3 is $\frac{1}{3}$, fraction of 2 : 9 is $\frac{2}{9}$

Now, change these fractions into like fractions $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$ and $\frac{2}{9} \times \frac{1}{1} = \frac{2}{9}$

Like fraction are $\frac{3}{9}$ and $\frac{2}{9}$, where 3 > 2, so $\frac{3}{9} > \frac{2}{9}$.

Hence 1 : 3 is greater than 2 : 9.

Example-2 : Are the ratios 1 : 5 and 2 : 15 equivalent ?

Sol. To check this, we need to know whether 1 : 5 and 2 : 15 are equal.

First convert the ratios into fractions. 1 : 5 is written as $\frac{1}{5}$ 2 : 15 is written as $\frac{2}{15}$. For converting these into like fraction, make denominator of both the fractions same.

$$\frac{1}{5} \times \frac{3}{3} = \frac{3}{15} \text{ and } \frac{2}{15} \times \frac{1}{1} = \frac{2}{15}$$

so $$\frac{3}{15} > \frac{2}{15}$$

Hence 1 : 5 and 2 : 15 are not equivalent.

Example-3 : Following is the performance of a circket team in the matches it played:

Year	Wins	Losses
Last year	8	2
This Year	4	2

in which year, was the record better ?

Sol. Last year, wins : losses = 8 : 2 = 4 : 1

This year, wins : losses = 4 : 2 = 2 : 1

Obviously, 4 : 1 > 2 : 1 (in fractional form, $\frac{4}{1} > \frac{2}{1}$)

Hence, we can say that the team performed better, last year

Example-4 : There are 10 soffa sets, 8 double beds and 16 dining tables in a furniture showroom. Find the ratio of :

(*i*) Number of dining tables to double beds

(*ii*) Number of double beds to sofa sets.

Sol. (*i*) Number of dining tables = 16

Number of double beds = 8

∴ Ratio of dining tables to double beds = 16 : 8 $= \frac{16}{8} = \frac{2}{1}$ = 2 : 1

(*ii*) Number of double beds = 8

Number of sofa sets = 10

Ratio of double beds to sofa sets = 8 : 10 $= \frac{8}{10} = \frac{4}{5}$ = 4 : 5

PROPORTION

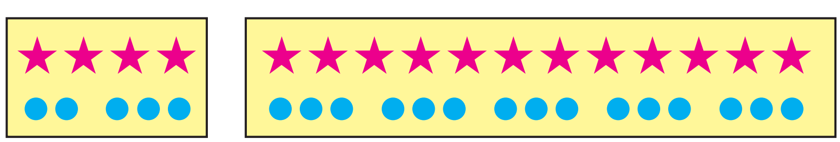
We know that proportion is the equivalence of two ratios, therefore, a proportion involves four quantities. For example, when we say 4, 5, 12 and 15 are in proportion, we mean

4 : 5 : : 12 : 15 (: : denotes the symbol for proportion)

i.e. $$\frac{4}{5} = \frac{12}{15}$$

The first and the last terms (4 and 15) are called extremes, the second and third terms (5 and 12) are called middle terms.

× 3 (4 : 5 / 12 : 15) × 3



4 : 5 12 : 15

Product of middle terms = product of exterme terms.

Given three terms of a proportion, it is easy to find the missing term. Let's see how this helps us solve problems.

USE OF PROPORTION IN SOLVING PROBLEMS

Before you start solving a problem using proportion, you need to determine whether the type of proportion is direct or indirect Let us analyse some problems to identify the type of proportion and solve them.

Proportion can be direct or indirect

Direct Proportion : The given quantities are said to be in direct proportion if with increase or decrease in one quantity leads to increase or decrease respectively in the other quantity.

Indirect Proportion : The given quantities are said to be in indirect proportion if with increase in one quantity the other quantity decreases and vice versa.

Example-5 : If 2 pens cost Rs 15, how many pens can you purchase with Rs 90 ?

Sol. The more the number of pens one purchases, the more is the amout to be paid.

More Pens → More is the amount to be paid therefore, there is a direct proportion between the number of pens and the amount to be paid. Let x be number of pens to be purchased.

$2 : 15 : : x : 90$

$$\frac{2}{15} = \frac{x}{90}$$

So, $$x = \frac{2 \times 90}{15} = 12$$

Thus, we can buy 12 pens for Rs 90.

Example-6 : If it takes 6 days for 4 men to repair a road, how long will it take for 7 men to do the same job if they work at the same rate ?

Sol. The more the number of men, the lesser the number of days they will take to complete a job.

More Men → Less days to complete a job

Therefore, there is an indirect proportion between the number of men and number of days they take to complete a job. Let the number of days required be x. using the formula for indirect proportion, we get.

$4 : 7 : : x : 6$ or

$$4 \times 6 = 7 \times x$$

So, $$x = \frac{4 \times 6}{7} = \frac{24}{7} = 3\frac{3}{7}$$

Therefore, it would take $3\frac{3}{7}$ days for 7 men to repair the road.

EXERCISE - 8.1

1. Find the ratio of

(i) ₹ 5 to 50 paise

(ii) 15kg to 210g

(iii) 4m to 400cm

(iv) 30 days to 36 hours

2. Are the ratios 1 : 2 and 2 : 3 equivalent ?
3. If the cost of 6 toys is ₹ 240, find the cost of 21 toys.
4. The car that I own can go 150 km with 25 litres of petrol. How far can it go with 30 litres of petrol ?
5. In a computer lab, there are 3 computers for every 6 students. How many computers will be needed for 24 students ?

PERCENTAGE – ANOTHER WAY OF COMPARING QUANTITIES

Do you remember percentage ?

- A fraction with denominator 100 is called percent.
- Symbol used for percent is %

Example of percent are $\frac{19}{100} = 19\%$, $\frac{7}{100} = 7\%$

Remember : Percent can be

(*i*) Converted into fraction.
(*ii*) Expressed as a ratio.
(*iii*) Converted into a decimal.

Do you know ?

The word 'percent' is an abbreviation of the latin word percentum which means per hundred or hundredths.

For example

Aman got 88 marks out of hundred marks; it means she got 88 percent marks. conversely, when a student got 65 percent marks; it means that the student scored 65 marks out of hundred marks.

Thus 25% means 25 out of hundred $= \frac{25}{100}$,

62% means 62 out of hundred $= \frac{62}{100}$

So, the symbol % stands for one hundredth *i.e* $\frac{1}{100}$

To understand percentage, consider the following example.

Suman made table top of 100 different coloured tiles. She counted blue, red, yellow and green tiles separately and filled the table given below.

Colour	Number of tiles	Fraction	Percentage	Written as
Blue	16	$\frac{16}{100}$	16	16%
Red	33	$\frac{33}{100}$	33	33%
Yellow	23	$\frac{23}{100}$	23	23%
Green	28	$\frac{28}{100}$	28	28%
Total	**100**			

This leads to :

In the fraction $\frac{r}{100}$, percentage = r it is written as r %. The numerator of the fraction i.e. r is also called rate percent. Thus, percentage = rate percent.

PERCENTAGE WHEN THE TOTAL IS NOT HUNDRED

If the total is not hundred, then convert it into an equivalent fraction with denominator 100. Consider the following example :

Rina has a necklace with 20 beads in two different colours.

Colour	Number of beads	fraction	Fraction with denominator 100	Percentage
Red	12	$\frac{12}{20}$	$\frac{12}{20} \times \frac{5}{5} = \frac{60}{100}$	60%
Green	8	$\frac{8}{20}$	$\frac{8}{20} \times \frac{5}{5} = \frac{40}{100}$	40%
Total	**20**			

Note : In practice, the words percent and percentage both are used synonymously.

Example-1 : Out of 25 students of a class, 16 are girls. What is the percentage of girls ?

Sol. Out of 25 students, there are 16 girls.

$$\therefore \quad \text{Percentage of girls} = \left(\frac{16}{25} \times 100\right)\% = 64\%$$

Example-2 : Teena scored 320 marks out of 400 marks and Reena scored 300 marks out of 360 marks. Whose performance is better ?

Sol. Teena scored 320 marks out of 400 marks.

$$\therefore \quad \text{Percentage of marks scored by Teena} = \left(\frac{320}{400} \times 100\right)\% = 80\%$$

Reena scored 300 marks out of 360 marks

$\therefore$ Percentage of marks scored by Reena

$$= \left(\frac{300}{360} \times 100\right)\%$$

$$= \frac{250}{3}\% = 83\frac{1}{3}\%$$

As $83\frac{1}{3} > 80$, therefore, performance of Reena is better than that of Teena.

Example-3 : Radhika spends Rs 350 every month. If this is 70% of her pocket money find her pocket money.

Sol. Let Radhika's pocket money = Rs x

Money spent = Rs 350

Also money spent is = 70% of x

$\therefore$ 70% of x = 350

$$\Rightarrow \quad \frac{70}{100} \times x = 350$$

$$\Rightarrow \quad x = \frac{350 \times 100}{70} = 500, \ x = 500$$

Hence Radhika's pocket money is Rs 500.

CONVERTING A PERCENTAGE INTO A FRACTION

Rule : To convert a percentage into a fraction, replace the % sign with $\frac{1}{100}$ and reduce the fraction to simplest form.

Example-4 : Express the following percentages as fractions :

(*i*) 20% (*ii*) 6.5% (*iii*) $3\frac{1}{8}$ % (*iv*) 135%

Sol. (*i*) $20\% = \frac{200}{100} = \frac{1}{5}$

(*ii*) $6.5\% = \frac{65}{1000} = \frac{13}{200}$

(*iii*) $3\frac{1}{8}\% = \frac{25}{8}\% = \frac{\frac{25}{8}}{100} = \frac{25}{8} \times \frac{1}{100} = \frac{1}{32}$

(*iv*) $135\% = \frac{135}{100} = \frac{27}{20} = 1\frac{7}{20}$

(Replace % sign with $\frac{1}{100}$ and simplify)

CONVERTING A FRACTION INTO PERCENTAGE

Rule : To convert a fraction into percentage, multiply the given fraction by 100, put the sign of %.

Example-5 : Converting the following fractions into percentages.

(*i*) $\frac{1}{2}$ (*ii*) $\frac{2}{3}$ (*iii*) $1\frac{5}{8}$

Sol. (*i*) $\frac{1}{2} = \frac{1}{2} \times 100 = 50$

Thus, $\frac{1}{2} = 50\%$

(*ii*) $\frac{2}{3} \times 100 = 66.67\%$

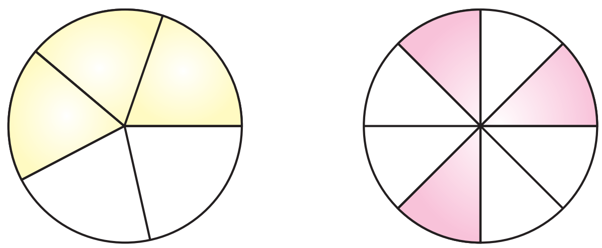
Thus, $\frac{2}{3} = 66.67\%$

(Multiply by 100 and put % sign)

(*iii*) $1\frac{5}{8} = \frac{13}{8} \times 100 = 13 \times \frac{25}{2} = \frac{325}{2} = 162.5$

Thus, $1\frac{5}{8} = 162.5\%$

Example-6 : Write the part of the circle which is shaded and hence find percentage of part which is shaded.



(*i*) (*ii*)

Sol.

(*i*) Shaded part $= \frac{3}{5}$

Percentage of shaded part $= \left(\frac{3}{5} \times 100\right)\% = 60\%$

(*ii*) Shaded part $= \frac{3}{8}$

Percentage of shaded part $= \left(\frac{3}{8} \times 100\right)\%$

$= \frac{75}{2}\% = 37.5\%$

CONVERTING A PERCENTAGE INTO A RATIO

Rule : To convert a percentage into a ratio, first convert the given percentage into a fraction in simplest form and then to a ratio.

Example-7 : Convert the following percentages as ratios in simplest form.

(*i*) 28% (*ii*) 17.5% (*iii*) $66\frac{2}{3}\%$

Sol. (*i*) $28\% = \frac{28}{100} = \frac{7}{25} = 7 : 25$

(*ii*) $17.5\% = \frac{17.5}{100} = \frac{175}{1000} = \frac{7}{40} = 7 : 40$

(*iii*) $66\frac{2}{3}\% = \frac{200}{3}\% = \frac{200}{3} \times \frac{1}{100} = \frac{2}{3} = 2 : 3$

(First convert a fraction in simplest form)

CONVERTING A RATIO INTO A PERCENTAGE

To convert a ratio into percentage, first convert the given ratio into a fraction and then to a percentage.

Example-8 : Express the following ratios as percentages :

(*i*) 1 : 2 (*ii*) 7 : 6

(First convert ratio into a fraction)

Sol. (*i*) $1:2 = \frac{1}{2} = \left(\frac{1}{2}\times 100\right)\% = 50\%$

(ii) $7:6 = \frac{7}{6} = \left(\frac{7}{6}\times 100\right)\% = \frac{350}{3}\%$

$= 116\frac{2}{3}\%$

Example-9 : Ritu's mother said, to make idlis, you must take 5 parts rice and 3 parts urad dal. What percentage of such a mixture would be rice and what percent would be urad dal ?

Sol. In terms of ratio we write this as rice : urad dal = 5 : 3

Total number of parts = 5 + 3 = 8

This means $\frac{5}{8}$ part in rice and $\frac{3}{8}$ part in urad dal.

Then, Percentage of rice $= \left(\frac{5}{8}\times 100\right)\% = \frac{125}{2}\% = 62.5\%$

Percentage of urad dal $= \left(\frac{3}{8}\times 100\right)\% = \frac{75}{2}\% = 37.5\%$

CONVERTING A PERCENTAGE INTO A DECIMAL

Rule : To convert a percentage into a decimal, first convert the percentage into a fraction by replacing the % with $\frac{1}{100}$. Then convert the fraction to decimal.

Example-10 : Convert the following percentages into decimals.

(*i*) 25% (*ii*) 78.5% (*iii*) 150%

Sol. (*i*) $25\% = 25 \times \frac{1}{100} = \frac{25}{100} = 0.25$

(*ii*) $78.5\% = 78.5 \times \frac{1}{100} = \frac{78.5}{100} = 0.785$

(*iii*) $150\% = 150 \times \frac{1}{100} = \frac{15}{10} = 1.5$

Note : From the above example, we observe that to convert a percentage into a decimal, remove the sign % and move the decimal point two places to the left.

CONVERTING A DECIMAL INTO A PERCENTAGE

Rule : To convert a decimal into a percentage, multiply the decimal by 100 and put the sign %

EXample-11 : Convert the following decimals to percent

(*i*) 0.75 (*ii*) 0.025 (*iii*) 0.4

Sol. (*i*) $0.75 = (0.75 \times 100)\% = 75\%$

(*ii*) $0.025 = (0.025 \times 100)\% = 2.5\%$

(*iii*) $0.4 = (0.4 \times 100)\% = 40\%$

Note : From the above example, we observe that to convert a decimal into a percentage, move the decimal point two places to the right (adding zeros if necessary) and put the % sign.

FIND A PERCENTAGE OF A GIVEN QUANTITY

Rule : To find a percentage of a given quantity, change the percentage into fraction and multiply by the given quantity.

EXample-12 : Find the value of

(*i*) 75% of 12 (*ii*) $12\frac{1}{2}$ % of 64

Sol. (*i*) $75\% \text{ of } 12 = \frac{75}{100} \times 12 = \frac{3}{4} \times 12 = 9$

(*ii*) $12\frac{1}{2}\% \text{ of } 64 = \frac{\frac{25}{2}}{100} \times 64 = \frac{25}{2} \times \frac{16}{25} = 8$

EXample-13 : A survey of 50 children showed that 20% like playing cricket. How many children liked playing cricket ?

Sol. Total number of children = 50

Out of these, 20% liked playing cricket.

∴ Number of children who liked playing circket

$$= 20\% \text{ of } 50$$

$$= \frac{20}{100} \times 50$$

$$= \frac{1}{5} \times 50 = 10$$

EXPRESSING ONE QUANTITY AS PERCENTAGE OF ANOTHER QUANTITY

Rule : To express one quantity as a percentage of another quantity,

$$\text{Percentage} = \left(\frac{\text{one quantity}}{\text{other quantity}} \times 100\right)\%$$

Note that both quantities must be of same kind (in same units)

Example-14 : A person ate 3 icecream cups out of 5 kept in the fridge what percent did he eat ?

Sol. Required percentage = $\left(\frac{3}{5}\times 100\right)\% = 60\%$

Example-15 : Express

(*i*) **15 as a percentage of 45.**
(*ii*) **20 paise as a percentage of ₹ 5.**

Sol. (*i*) Required percentage = $\left(\frac{15}{45}\times 100\right)\% = \frac{100}{3}\% = 33\frac{1}{3}\%$

(*ii*) ₹ 5 = 500 paise

Required percentage = $\left(\frac{20}{500}\times 100\right)\% = 4\%$

FINDING INCREASE / DECREASE PERCENTAGE

Rule : Percentage increase = $\left(\frac{\text{increase in value}}{\text{original value}}\times 100\right)\%$

Percentage decrease = $\left(\frac{\text{decrease in value}}{\text{orignal value}}\times 100\right)\%$

Example-16 : Apples were selling at ₹ 50 per kg last season. This season they are selling at ₹ 55 per kg. Find the percentage increase or decrease in price.

Sol. Obviously the price has increased from ₹ 50 to ₹ 55.

Original price = ₹ 50

Increase in Price = ₹ 55 – ₹ 50
= ₹ 5

∴ Percentage increase = $\left(\frac{\text{increase in value}}{\text{original value}}\times 100\right)\%$

$= \left(\frac{5}{50}\times 100\right)\%$

Hence, the price of apples has increased by 10%.

Example-17 : A computer costing ₹ 60000 one year ago now costs ₹ 40000. Find the percentage increase or decrease in the price.

Sol. The price has decreased from ₹ 60000 to ₹ 40000.

Original price = ₹ 60000,

Decrease in price = ₹ 60000 – ₹40000
= ₹ 20000

$\therefore$ Percentage decrease $= \left(\dfrac{\text{decrease in value}}{\text{original value}} \times 100\right)\%$

$$= \left(\frac{20000}{60,000} \times 100\right)\% = \frac{100}{3}\% = 33\frac{1}{3}\%$$

Hence the price has decreased by $33\frac{1}{3}\%$

USE OF PERCENTAGE

We shall now solve some real life problems on percentages.

Example-18 : In a class of 50 students, 20% students wear spectacles. How many students do not wear spectacles ?

Sol. Since 20% wear spectacles.

$\therefore$ Percentage of students who do not wear spectacles.

$$= (100 - 20)\% = 80\%$$

Hence, the number of students who do not wear spectacles

$$= 80\% \text{ of } 50 = \frac{80}{100} \times 50 = 40$$

Example-19 : On a rainy day, only 36 students out of 48 came to the class. What percentage were absent ?

Sol. Total number of students in class = 48

Number of students absent = 48 – 36 = 12

$\therefore$ Percentage of absent students $= \left(\dfrac{12}{48} \times 100\right)\% = 25\%$

EXERCISE - 8.2

1. Convert the following fractions into percents

(*i*) $\frac{1}{8}$ (*ii*) $\frac{49}{50}$ (*iii*) $\frac{5}{4}$ (*iv*) $1\frac{3}{8}$

2. Convert the following percents into fractions in simplest form :

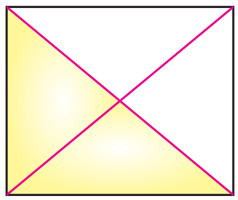
(*i*) 25% (*ii*) 150% (*iii*) $7\frac{1}{2}\%$

3. (*i*) Anita secured 324 marks out of 400 marks. Find the percentage of marks secured by Anita.

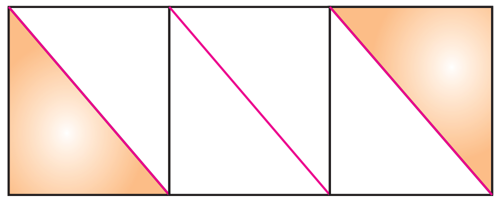
(*ii*) Out of 32 students, 8 are absent from the class. What is the percentage of students who are absent.

(*iii*) There are 120 voters, 90 out of them voted. What percent did not vote ?

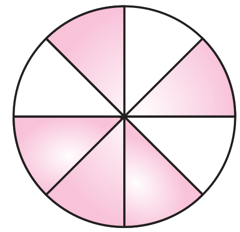
4. Estimate the part of figure which is shaded and hence find the percentage of the part which is shaded.



(*i*)



(*ii*)



(*iii*)

5. Convert the following percentages into ratios in simplest form :

(*i*) 14% (*ii*) $1\frac{3}{4}\%$ (*iii*) $33\frac{1}{3}\%$

6. Express the following ratios as percentages :

(*i*) 5 : 4 (*ii*) 1 : 1 (*iii*) 2 : 3 (*iv*) 9 : 16

7. Chalk contains calcium, carbon and sand in the ratio 12 : 3 : 10. Find the percentage of carbon in the chalk.

8. Convert each part of the following ratios into percentage :

(*i*) 3 : 1 (*ii*) 1 : 4 (*iii*) 4 : 5 : 6

9. Convert the following percentages to decimals :

(*i*) 28% (*ii*) 3% (*iii*) $37\frac{1}{2}\%$

10. Convert the following decimals to percentage :

(*i*) 0.65 (*ii*) 0.9

(*iii*) 2.1

11. (*i*) If 65% of students in a class have a bicycle, then what percent of the students do not have a bicycle ?

(*ii*) We have a basket full of apples, oranges and mangoes. If 50% are apples, 30% are oranges, then what percent are mangoes ?

12. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.

13. Arun bought a plot for ₹ 3,50,000. The next year, the price went upto ₹ 3,70,000. What was the percentage of price increase ?

14. **Find :**

(*i*) 15% of 250 (*ii*) 25% of 120 litres

(*iii*) 4% of 12.5 (*iv*) 12% of ₹ 250

15. **Multiple Choice Questions :**

(*i*) The ratio 2 : 3 expressed as percentage is

(*a*) 40% (*b*) 60% (*c*) $66\frac{2}{3}\%$ (*d*) $33\frac{1}{3}\%$

(*ii*) If 30% of x is 72, then x is equal to

(*a*) 120 (*b*) 240 (*c*) 360 (*d*) 480

(*iii*) 0.025 when expressed as a percent is

(*a*) 250% (*b*) 25% (*c*) 4% (*d*) 2.5%

(*iv*) In a class, 45% of students are girls. If there are 22 boys in the class, then the total number of students in the class is

(*a*) 30 (*b*) 36 (*c*) 40 (*d*) 44

(*v*) What percent of $\frac{1}{7}$ is $\frac{2}{35}$?

(*a*) 20% (*b*) 25% (*c*) 30% (*d*) 40%

PROFIT AND LOSS

A shopkeeper (dealer or retailer) buys his goods from a manufacturer or a wholesale dealer and then he sells them to a customer. If he sells his goods at a higher price than he paid for them, he makes a profit (gain). If for some reasons, he sells his goods at a lower price than he paid for them, then he suffers a loss.

Cost price ⟶ The price at which an article is purchased, is called its cost price (abbreviated C.P)

Selling price ⟶ The price at which an article is sold, is called its selling price (abbreviated S.P)

Profit ⟶ If the selling price of an article is more than its cost price, then there is a profit.

Profit = selling price – cost price

If SP > CP, then there is gain or profit

Loss ⟶ If the selling price of an article is less than its cost price, then there is a loss.

Loss = Cost price – Selling price

If SP < CP, then there is a loss

Also if SP = CP, then there is no profit no loss

For Example :

If a dealer buys a T.V for ₹ 11, 000 and sells it at ₹ 12,100, then he makes a profit, and,

Profit = ₹ 12100 – ₹11000 = ₹1100

If a dealer sells it at ₹10,000, then he suffers a loss and

Loss = ₹11000 – ₹10000 = ₹1000

PROFIT OR LOSS PERCENTAGE

Often in bussiness, instead of talking about what actual profit or loss is, we have to find out the profit or loss percentage.

Profit or loss percentage is always calculated on the cost price.

$$\text{Profit percentage} = \left[\frac{\text{Profit}}{\text{cost price}} \times 100\right]\%$$

$$\text{Loss percentage} = \left[\frac{\text{Loss}}{\text{cost price}} \times 100\right]\%$$

Example-1 : A dealer buys a watch for ₹ 580 and sells at ₹667. Find his profit and profit percentage.

Sol. C.P of the watch = ₹580

S.P of watch = ₹667

∴ Profit = SP – CP = ₹667 – ₹580 = ₹87

$$\text{Profit percentage} = \left[\frac{\text{Profit}}{\text{cost price}} \times 100\right]\% = \left[\frac{87}{580} \times 100\right]\% = 15\%$$

Example-2 : Sakshi bought a gold ring for ₹5,500 and two years later sold it for ₹4,000. What was her profit or loss ? Also, find her profit or loss percent.

Sol. The selling price of the ring is less than its cost price. Thus, Sakshi suffers a loss on the ring.

Loss = CP – SP = ₹ (5,500 – 4,000)

= ₹1,500

$$\text{Loss percent} = \frac{\text{Loss}}{\text{cost price}} \times 100$$

$$= \frac{1,500}{5,500} \times 100 = 27.27\%$$

Example-3 : A shopkeeper buys an article at ₹150 and sells it at a profit of 12%. Find the selling price

Sol. Cost price = ₹150

$$\text{Profit} = 12\% \text{ of cost price} = \frac{12}{100} \text{ of ₹}150$$

$$= ₹\left[\frac{12}{100} \times 150\right] = ₹\ 18$$

Selling price = Cost price + profit = ₹150 + ₹18

= ₹168

Example-4 : Find the selling price of an article which is purchased for ₹1240 and sold at a loss of 7%

Sol. Cost price = ₹12400

$$\text{Loss} = 7\% \text{ of cost price} = \frac{7}{100} \times ₹12400 = ₹868$$

Selling price = Cost price – Loss

∴ = ₹12400 – ₹868 = ₹11532.

Note : We can also calculate S.P. by using formula

$$\text{S.P.} = \text{C.P.} \times \left[\frac{100 - \text{Loss }\%}{100}\right]$$

Example-5 : By selling an article for ₹ 475 Rahul lost 5%. Find C.P of the article.

Sol.

Let C.P of article = ₹100

Loss = 5% of ₹100

= ₹5

S.P. of article = ₹(100 – 5)

= ₹95

If S.P. of article is ₹95, then C.P = ₹100

If S.P of article is ₹1, then C.P = ₹ $\frac{100}{95}$

If S.P of article is ₹475, then C.P = ₹ $\left[\frac{100}{95} \times 475\right]$

= ₹500

Note → We can also calculate C.P using formula.

$$\text{C.P.} = \text{S.P.} \times \left[\frac{100}{100 - \text{Loss}\%}\right]$$

SIMPLE INTEREST

When you borrow money from a bank or a money lender, you need to pay the money back after a period along with some extra money. This extra money is called the Interest

The amount of interest you pay depends on :

- The money you borrow, called the Principal (P).
- The rate of interest per annum, R (in percent).
- The time for which the money is borrowed, T (in years).

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} \quad \textit{i.e.} \quad I = \frac{P \times R \times T}{100}$$

Note → Given any three quantities out of P, R, T and I;

We can calculate the fourth quantity by using the above formula.

- **Amount :** The total money paid by the borrower to the money lender is called amount.

  Thus, Amount = Principal + Interest

If P denotes the principal, I is interest paid and A the amount, then

$$A = P + I$$

Example-1 : Find the simple interest on ₹1500 at 6% per annum for 3 years. Also find the amount.

Sol. Here, Principal P = ₹1500, rate of interest R = 6% per annum and Time T = 3 years

$$\therefore \quad \text{Simple Interest I} = \frac{P \times R \times T}{100} = ₹\frac{1500 \times 6 \times 3}{100}$$

$$= ₹270$$

$$\text{Amount A} = P + I = ₹1500 + ₹270 = ₹1770$$

Example-2 : On a certain sum the interest paid after 3 years is ₹450 at 5% rate of interest per annum. Find the sum.

Sol. Here I = ₹450, R = 5% p.a. and T = 3 years Let the sum i.e. principal be ₹ P, we want to find P

$$\text{Simple Interest I} = \frac{P \times R \times T}{100}\text{, we get}$$

$$450 = \frac{P \times 5 \times 3}{100} \Rightarrow P = 450 \times \frac{100}{5 \times 3}$$

$$P = 3000$$

Hence, the required sum = ₹ 3000.

Example-3 : Jyoti take a loan of ₹6000 and pays back 7,080 at the end of three years. What is the rate of interest that she paid ?

Sol. Jyoti paid ₹(7,080 – 6,000) = 1,080 as interest, we know that

SI = 1,080, T = 3 years and P = ₹ 6000

We need to find R

The formula for S.I is

$$SI = \frac{P \times R \times T}{100}$$

So,

$$R = \frac{SI \times 100}{P \times T}$$

Substituting the values in the formula

$$R = \frac{1,080 \times 100}{6,000 \times 3} = 6$$

Jyoti paid interest at the rate of 6% per annum

Example-4 : Tanveer lends ₹7,000 to a shopkeeper and charges an interest of 7%. If he gets back ₹8,470 find the time for which tanveer lent the money.

Sol. The interest Tanveer earned = (Amount – Principal)

= ₹(8,470 – 7,000)

= ₹(1,470)

P = ₹7,000, S.I = ₹1,470 and R = 7% we need to find T

The formula for S.I. is

$$S.I = \frac{P \times R \times T}{100}$$

So, $$T = \frac{S.I \times 100}{P \times R}$$

Substituting the values in the formula :

$$T = \frac{1,470 \times 100}{7,000 \times 7} = 3$$

So, Tanveer gave the loan for 3 years.

EXERCISE - 8.3

1. Find what is the profit or loss in the following transactions. Also find profit percent or loss percent in each case.

(*i*) Gardening shears bought for ₹250 and sold for ₹325

(*ii*) A refrigerater bought for ₹12,000 and sold at ₹13,500

(*iii*) A cupboard bought for ₹2,500 and sold at ₹3,000.

(*iv*) A shirt bought for ₹250 and sold at ₹150

2. A shopkeeper buys an article for ₹735 and sold it for ₹850. Find his profit or loss.

3. Kirti bought a saree for ₹ 2500 and sold it for ₹ 2300. Find her loss and loss percent.

4. An article was sold for ₹252 with a profit of 5%. What was its cost price.

5. Amrit buys a book for ₹275 and sells it at a loss of 15%. For how much does she sell it ?

6. Juhi sells a washing machine for ₹13500. She losses 20% in the bargain. What was the price at which she bought it ?

7. Anita takes a loan of ₹ 5000 at 15% per year as rate of interest. Find the interest she has to pay at the end of one year.

8. Find the amount to be paid at the end of 3 years in each case :

(*i*) Principal = ₹1200 at 12% p.a.

(*ii*) Principal = ₹7500 at 5% p.a.

9. Find the time when simple interest on ₹2500 at 6% p.a. is ₹450

10. Find the rate of interest when simple interest on ₹1560 in 3 years is ₹585.

11. If Nakul gives an interest of ₹45 for one year at 9% rate p.a. what is the sum he borrowed?

12. If ₹14,000 is invested at 4% per annum simple interest, how long will it take for the amount to reach ₹16240 ?

13. Multiple Choice Questions :

(*i*) If a man buys an article for ₹ 80 and sells it for ₹100, then gain percentage is

(*a*) 20% (*b*) 25%

(*c*) 40% (*d*) 125%

(*ii*) If a man buys an article for ₹120 and sells it for ₹100, then his loss percentage is

(*a*) 10% (*b*) 20% (*c*) 25%(*d*) $16\frac{2}{3}\%$

(*iii*) The salary of a man is ₹24000 per month. If he gets an increase of 25% in the salary, then the new salary per month is

(*a*) ₹2,500 (*b*) ₹28,000 (*c*) 30,000 (*d*) 36,000

(*iv*) On selling an article for ₹100, Renu gains ₹20 Her gain percintage is

(*a*) 25% (*b*) 20% (*c*) 15% (*d*) 40%

(*v*) The simple interest on ₹6000 at 8% *p.a*. for one year is

(*a*) ₹600 (*b*) ₹480 (*c*) ₹400 (*d*) ₹240

(*vi*) If Rohini borrows ₹4800 at 5% p.a. simple interest, then the amount she has to return at the end of 2 years is.

(*a*) ₹480 (*b*) ₹5040 (*c*) ₹5280 (*d*) ₹5600

WHAT HAVE WE DISCUSSED ?

1. We are often required to compare two quantities in our daily life. They may be heights, weights, salaries, marks, etc.

2. Two ratios can be compared by converting them to like fractions. If two fractions are equal, we say the two given ratios are equivalent.

3. If two ratios are equivalent then the four quantities are said to be in proportion. For example, the ratios 8 : 2 and 16 : 4 are equivalent there fore 8, 2, 16 and 4 are in proportion.

4. A proportion is the equivalence of two ratios.

5. In a proportion, product of middle terms is equal to product of extreme terms.

6. Percent means per hundred or out of hundred. The symbol % stands for percent i.e. $\frac{1}{100}$

7. To convert the percentage into a fraction, replace the % sign with $\frac{1}{100}$ and simplify.

8. To convert a fraction into percentage, multiply the fraction by 100 and put % sign.

9. To convert the percentage into decimal, first convert the percentage into a fraction by replacing the sign % with $\frac{1}{100}$ and then convert fraction to decimal.

10. To convert a decimal into percentage, multiply the decimal by 100 and put the sign %

11. To convert percentage into ratio, first convert the given percentage into a fraction in simplest form and then to a ratio.

12. To convert ratio into percentage, first convert the given ratio into a fraction and then to percentage.

13. To find the percentage of a given quantity, change the percentage into fraction and multiply by the given quantity.

14. Percentage increase / decrease in a quantity. = $\left[\frac{\text{Change in quantity}}{\text{Original quantity}} \times 100\right]\%$

15. The price at which an article is bought by a dealer is called its cost price (C.P.)

16. The price at which the article is sold by a dealer is called its selling price (S.P)

17. If selling price is more than the cost price, then the dealer makes a profit, and Profit = Selling price – Cost price.

18. If the selling price is less than the cost price, then the dealer suffers a Loss and,
Loss = Cost Price – Selling Price.

19. Profit or Loss percentage is calculated on the cost price.

$$\text{Profit percentage} = \left[\frac{\text{profit}}{\text{cost price}} \times 100\right]\%$$

$$\text{Loss percentage} = \left[\frac{\text{loss}}{\text{cost price}} \times 100\right]\%$$

20. $\text{Simple Interest (S.I.)} = \frac{P \times R \times T}{100}$

Where P = Principal
R = Rate of interest per annum
T = Time (in years)

21. Amount = Principal + Interest

22.
- $P = \frac{S.I \times 100}{R \times T}$
- $R = \frac{S.I \times 100}{P \times T}$
- $T = \frac{S.I \times 100}{P \times R}$

LEARNING OUTCOMES

After completion of the chapter, the students are now able to

1. Compare the two quantities in their daily life.
2. Convert two ratios in to like fractions.
3. Understand equivalent ratios.
4. Distinguish quantities that are in proportion.
5. Find the percentage of given problems.
6. Convert percentage into fraction, fraction into percentage, percentage into decimal, decimal into percentage, percentage into ratio and ratio into percentage.
7. Solve the problems related to profit, loss, profit% or loss%.
8. Find simple interest, principal, rate of interest or time in the given problem.
9. Differentiate between simple interest and amount.

ANSWERS

EXERCISE 8.1

1. (*i*) 10 : 1 (*ii*) 500 : 7
(*iii*) 1 : 1 (*iv*) 20 : 1

2. No **3.** 840

4. 180*km* **5.** 12

EXERCISE 8.2

1. (*i*) 12.5% (*ii*) 98%
(*iii*) 125% (*iv*) $137\frac{1}{2}\%$

2. (*i*) $\frac{1}{4}$ (*ii*) $\frac{3}{2}$ (*iii*) $\frac{3}{40}$

3. (*i*) 81% (*ii*) 25% (*iii*) 25%

4. (*i*) $\frac{1}{2}$; 50% (*ii*) $\frac{1}{3}$; $33\frac{1}{3}\%$ (*iii*) $\frac{5}{8}$; 62.5%

5. (*i*) 7 : 50 (*ii*) 7 : 400
(*iii*) 1 : 3

6. (*i*) 125% (*ii*) 100%
(*iii*) $66\frac{2}{3}\%$ (*iv*) $56\frac{1}{4}\%$

7. 12%

8. (*i*) 75%, 25% (*ii*) 20%, 80% (*iii*) $26\frac{2}{3}\%$, $33\frac{1}{3}$, 40%

9. (*i*) 0.28 (*ii*) 0.03 (*iii*) 0.375

10. (*i*) 65% (*ii*) 90% (*iii*) 210%

11. (*i*) 35% (*ii*) 20%

12. 2% **13.** $5\frac{5}{7}\%$

14. (*i*) 37.5 (*ii*) 30 litres
(*iii*) 0.5 (*iv*) ₹300

15. (*i*) (*c*) (*ii*) (*b*) (*iii*) (*d*)
(*iv*) (*c*) (*v*) (*d*)

EXERCISE 8.3

1. (*i*) Profit = ₹75 ; Profit % = 30 (*ii*) Profit = ₹1500 ; profit % = 12.5

(*iii*) Profit = ₹500 ; profit% = 20 (*iv*) Loss = ₹100 ; Loss % = 40

2. Profit = ₹115

3. ₹ 200 ; 8%

4. ₹ 240

5. ₹ 233.75

6. ₹ 16875

7. ₹ 750

8. (*i*) ₹1632 (*ii*) ₹ 8625

9. 3 years

10. 12.5% *p.a.*

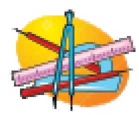
11. ₹ 500

12. 4 years

13. (*i*) (*b*) (*ii*) (*d*)

(*iii*) (*c*) (*iv*) (*a*)

(*v*) (*b*) (*vi*) (*c*)



CHAPTER 9

Rational Numbers

Learning Objectives :-

In this chapter, you will learn :-

1. To define rational numbers and reduce them to their standard form.
2. The concept of equivalent rational numbers.
3. To represent the rational number on a number line.
4. To find more rational numbers between given rational numbers.
5. To compare rational numbers and perform basic mathematical operations on them.
6. To use rational numbers in solving your daily life problems.

3. OUR NATION'S PRIDE

Aryabhata : Aryabhata was a great Indian mathematican born in 476CE at Kusumpura (Patliputra) presently Patna, India and died in 550CE. His research work include place value, number system and many more concepts in the field of mathematics. Great mathematician Laplуce (1749-1829) said that India has offered us a system for expressing all the numbers with the help of only ten symbols. This concept picks up its new height when we recall that the world known mathematicians like Appolonius and Archimedes failed to discover such essential system.

INTRODUCTION

It took a long time to discover the numbers. Earlier, man could not write the numbers but could only express them with the help of fingers or counting objects.

Natural numbers : The numbers which are used for counting are called natural numbers.

For example 1, 2, 3, 4, 5, 6, 7, ...

Whole Numbers : All natural numbers along with zero (0) are called whole numbers.

For example 0, 1, 2, 3, 4, 5, 6, ...

Integers : All the whole numbers and negative of natural numbers are called integers.

For example ....–6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, ...

In the above 0 is an integer which is neither negative nor positive.

Fractions : The numbers of the form $\frac{a}{b}$ are known as fractions, where a is called numerator and b ($\neq 0$) is called the denominator.

Need of Rational number : To convert time, length and distance from one unit into another fractions are used e.g. If we convert 20 minutes into hours it will be $\frac{20}{60} = \frac{1}{3}$ hours. You can repersent a height of 500m above sea level as $\frac{1}{2}$ km. Can this height be repersented below sea level ? Can we denote $\frac{1}{2}$ km below sea level as $\frac{-1}{2}$? We see $\frac{-1}{2}$ is neither an integer nor a fractional number. We need to extend our number system to include such type of numbers.

What are rational numbers

The word 'rational' arises from the term 'ratio' and a ratio of 5 : 6 is written as $\frac{5}{6}$ where 5 is numerator and 6 is denominator.

The numbers of the form $\frac{a}{b}$

Where a and b are Integers and $b \neq 0$ are called rational number. e.g $\frac{5}{6}$, $\frac{-7}{8}$ and $\frac{21}{-9}$ rational numbers.

Equivalent of rational numbers : If we multiply or divide both the numerator and denominator of a rational number by a non zero integer then we get a rational number equivalent to the given rational number.

Example-1 : Write two equivalent rational numbers for the following :

(*i*) $\mathbf{\frac{-3}{5}}$ **(*ii*)** $\mathbf{\frac{-3}{40}}$

Sol. (*i*)

$$\frac{-3}{5} = \frac{-3}{5} \times \frac{2}{2} = \frac{-6}{10}$$

$$\frac{-3}{5} = \frac{-3}{5} \times \frac{3}{3} = \frac{-9}{15}$$

$\therefore$ Equivalent rational numbers of $\frac{-3}{5}$ are $\frac{-6}{10}$ and $\frac{-9}{15}$

(*ii*)

$$\frac{-8}{40} = \frac{-8 \div 4}{40 \div 4} = \frac{-2}{10}$$

$$\frac{-8}{40} = \frac{-8 \div (-8)}{40 \div (-8)} = \frac{1}{-5}$$

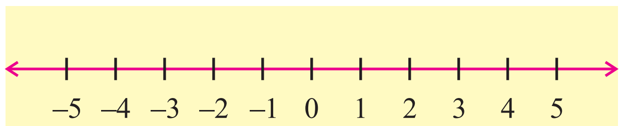
$\therefore$ Thus equivalent rational numbers of $\frac{-8}{40}$ are $\frac{-2}{10}$ and $\frac{1}{-5}$

This way we can write as many equivalent fractions as we want.

Positive rational numbers : A rational number is said to be positive if both the numerator and denominator are either positive or negative. For example : $\frac{3}{7}, \frac{5}{8}, \frac{-15}{-18}, \frac{-25}{-9}$ are positive rational numbers.

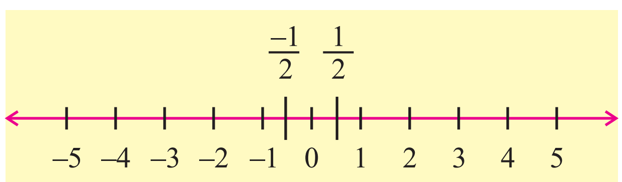
Negative rational numbers : A rational number is said to be negative if either numerator or denominator is negative. For example : $\frac{-6}{8}, \frac{5}{-9}, \frac{-15}{8}, \frac{8}{-17}$ are negative rational numbers.

Rational numbers on a number line : We have already learnt how to repersent integers on a number line.



Now let us repersent rational number $\frac{1}{2}$ and $\frac{-1}{2}$ on a number line. Half the distance between 0 and 1 which will be repersented as $\frac{1}{2}$.

Half the distance between 0 and –1 which will be repersented as $\frac{-1}{2}$.



Rational numbers in the standard form : A rational number is said to be in standard form if its denominator is positive and the highest common factor (HCF) of numerator and denominator is 1.

For example : $\frac{5}{7}, \frac{-4}{9}, \frac{2}{9}$

Example-2 : Find the standard form of

(*i*) $\frac{-21}{48}$ (*ii*) $\frac{42}{-28}$

Sol. (*i*) $\frac{-21}{48}$

$\because$ H.C.F of 21 and 48 is 3

So dividing both the numerator and denominator by 3 we get.

$\therefore$
$$\frac{-21}{48} = \frac{-21 \div 3}{48 \div 3} = \frac{-7}{16}$$

Standard form of $\frac{-21}{48}$ is $\frac{-7}{16}$.

(*ii*) $\frac{42}{-28}$

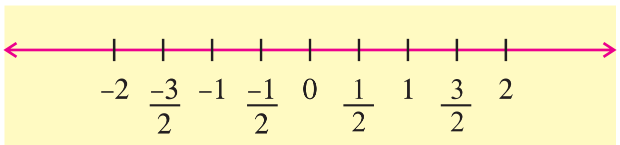
∵ HCF of 42 and 28 is 14.

∴ So dividing both the numerator and denominator by (–14) we get

$$\frac{42}{-28} = \frac{42 \div (-14)}{-28 \div (-14)} = \frac{-3}{2}$$

∴ Standard form of $\frac{42}{-28}$ is $\frac{-3}{2}$.

Comparison of two rational numbers : From a number line it is clear that :-



1. A positive rational number is always greater than zero.

2. A negative rational number is always less than zero.

3. If both the rational number are either positive or negative then they are compared as follows.

(*i*) Express each of the rational number with a positive denominator.

(*ii*) Take L.C.M of the denominators to make the denominators same.

(*iii*) The rational number having the greater numerator is greater.

Example-3 : **Which is greater in each of the following :**

(*i*) $\frac{4}{9}$ **and** $\frac{3}{6}$ **(*ii*)** $\frac{-5}{7}$ **and** $\frac{-4}{9}$

Sol. (*i*) Given rational numbers are $\frac{4}{9}$ and $\frac{3}{6}$.

∵ L.C.M of 9 and 6 is 18.

∴

$$\frac{4}{9} = \frac{4 \times 2}{9 \times 2} = \frac{8}{18}$$

$$\frac{3}{6} = \frac{3 \times 3}{6 \times 3} = \frac{9}{18}$$

∵ Numerator of second rational number is greater than first

i.e., $9 > 8$

so $\frac{3}{6} > \frac{4}{9}$

(*ii*) Given rational numbers are $\frac{-5}{7}$ and $\frac{-4}{9}$.

∵ LCM of 7 and 9 is 63.

$$\therefore \quad \frac{-5}{7} = \frac{-5}{7} \times \frac{9}{9} = \frac{-45}{63}$$

$$\frac{-4}{9} = \frac{-4}{9} \times \frac{7}{7} = \frac{-28}{63}$$

$$\because \quad -28 > -45$$

So, $$\frac{-4}{9} > \frac{-5}{7}$$

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Integers between –4 and 3 are –3, –2, –1, 0, 1, 2. There are exactly six integers between –4 and 3 which are finite. In example 3 part (*ii*) the rational numbers between $\frac{-5}{7}$ and $\frac{-4}{9}$ are

$$\frac{-44}{63} < \frac{-43}{63} < \frac{-42}{63} < \frac{-41}{63} < \text{..........} < \frac{-29}{63}$$

You can insert as many as rational numbers as you want. To insert '*n*' rational numbers, between two rational numbers, we shall multiply both the numerator and the denominator of the given rational number by '*n* + 1'.

So, to find 4 rational numbers between two rational numbers $\frac{2}{5}$ and $\frac{4}{5}$, we shall multiply the numerators and denominators of both the rational numbers by 4 + 1 = 5.

Example-4 : Find three rational numbers between –1 and 0.

Sol. Let's write –1 and 0 as rational numbers with 3 + 1 i.e 4 as denominator

we have $$-1 = -1 \times \frac{4}{4} = \frac{-4}{4}$$

$$0 = 0 \times \frac{4}{4} = \frac{0}{4}$$

$$\frac{-4}{4} < \frac{-3}{4} < \frac{-2}{4} < \frac{-1}{4} < \frac{0}{4}$$

Hence rational number between –1 and 0 are $\frac{-3}{4}, \frac{-2}{4}, \frac{-1}{4}$.

Example-5 : Find five rational number between $\frac{-5}{7}$ and $\frac{-1}{3}$.

Sol. Give rational numbers are $\frac{-5}{7}$ and $\frac{-1}{3}$

Here the denominators are not same

$$\therefore \quad \frac{-5}{7} = \frac{-5}{7} \times \frac{3}{3} = \frac{-15}{21}$$

$$\frac{-1}{3} = \frac{-1}{3} \times \frac{7}{7} = \frac{-7}{21}$$

$$\frac{-15}{21} < \frac{-14}{21} < \frac{-13}{21} < \frac{-12}{21} < \frac{-11}{21} < \frac{-10}{21} < \frac{-7}{21}$$

or $$\frac{-5}{7} < \frac{-2}{3} < \frac{-13}{21} < \frac{-12}{21} < \frac{-11}{21} < \frac{-10}{21} < \frac{-1}{3}$$

Hence five rational number between $\frac{-5}{7}$ and $\frac{-1}{3}$ are :

$$\frac{-2}{3}, \frac{-13}{21}, \frac{-4}{7}, \frac{-11}{21}, \frac{-10}{21}$$

EXERCISE - 9.1

1. Write two equivalent rational numbers of the following :-

(*i*) $\frac{4}{5}$ (*ii*) $\frac{-5}{9}$

(*iii*) $\frac{3}{-11}$

2. Find the standard form of the following rational numbers :-

(*i*) $\frac{35}{49}$ (*ii*) $\frac{-42}{56}$

(*iii*) $\frac{19}{-57}$ (*iv*) $\frac{-12}{-36}$

3. Which of the following pairs represent same rational number ?

(*i*) $\frac{-15}{25}$ and $\frac{18}{-30}$ (*ii*) $\frac{2}{3}$ and $\frac{-4}{6}$

(*iii*) $\frac{-3}{4}$ and $\frac{-12}{16}$ (*iv*) $\frac{-3}{-7}$ and $\frac{3}{7}$

4. Which is greater in each of the following ?

(*i*) $\frac{3}{7}, \frac{4}{5}$ (*ii*) $\frac{-4}{12}, \frac{-8}{12}$

(*iii*) $\frac{-3}{9}, \frac{4}{-18}$ (*iv*) $-2\frac{3}{5}, -3\frac{5}{8}$

5. Write the following rational numbers in ascending order.

(*i*) $\frac{-5}{7}, \frac{-3}{7}, \frac{-1}{7}$ (*ii*) $\frac{-1}{5}, \frac{-2}{15}, \frac{-4}{5}$

(*iii*) $\frac{-3}{8}, \frac{-2}{4}, \frac{-3}{2}$

6. Write five rational numbers between following rational numbers.

(*i*) –2 and –1 (*ii*) $\frac{-4}{5}$ and $\frac{-2}{3}$ (*iii*) $\frac{1}{3}$ and $\frac{5}{7}$

7. Write four more rational numbers in each of the following.

(*i*) $\frac{-1}{5}, \frac{-2}{10}, \frac{-3}{15}, \frac{-4}{20}, \ldots\ldots\ldots$ (*ii*) $\frac{-1}{7}, \frac{2}{-14}, \frac{3}{-21}, \frac{4}{-28}, \ldots\ldots\ldots\ldots$

8. Draw a number line and represent the following rational number on it.

(*i*) $\frac{2}{4}$ (*ii*) $\frac{-3}{4}$ (*iii*) $\frac{5}{8}$ (*iv*) $\frac{-6}{4}$

9. **Multiple choice questions :-**

(*i*) $\frac{3}{4} = \frac{?}{12}$, then ? =

(*a*) 3 (*b*) 6 (*c*) 9 (*d*) 12

(*ii*) $\frac{-4}{7} = \frac{?}{14}$, then ? =

(*a*) –4 (*b*) –8 (*c*) 4 (*d*) 8

(*iii*) The standard form of rational number $\frac{-21}{28}$ is

(*a*) $\frac{-3}{4}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{-3}{7}$

(*iv*) Which of the following rational number is not equal to $\frac{7}{-4}$?

(*a*) $\frac{14}{-8}$ (*b*) $\frac{21}{-12}$ (*c*) $\frac{28}{-16}$ (*d*) $\frac{7}{-8}$

(*v*) Which of the following is correct ?

(*a*) $0 > \frac{-4}{9}$ (*b*) $0 < \frac{-4}{9}$ (*c*) $0 = \frac{4}{9}$ (*d*) None

(*vi*) Which of the following is correct ?

(*a*) $\frac{-4}{5} < \frac{-3}{10}$ (*b*) $\frac{-4}{5} > \frac{3}{-10}$ (*c*) $\frac{-4}{5} = \frac{3}{-10}$ (*d*) None

OPERATIONS ON RATIONAL NUMBERS

Addition of Rational Numbers : To add two or more rational numbers, their denominators have to be positive and same. In case, the denominators are not same, we will make them same by taking their L.C.M., as we do for comparing the rational numbers and then we shall simply add their numerators.

Example-1 : Add $\frac{5}{9}$ and $\frac{-8}{9}$.

Sol. We have $\frac{5}{9} + \frac{-8}{9}$

$$= \frac{5+(-8)}{9}$$

$$= \frac{5-8}{9}$$

$$= \frac{-3}{9}$$

$$= \frac{-1}{3}$$

Example-2 : Add $\frac{9}{-17}$ and $\frac{-5}{17}$.

Sol. We have $\frac{9}{-17} + \frac{-5}{17}$

$$\frac{9}{-17} = \frac{9}{-17} \times \frac{-1}{-1} = \frac{-9}{17}$$

Now, $$\frac{9}{-17} + \frac{-5}{17} = \frac{-9}{17} + \frac{-5}{17}$$

$$= \frac{-14}{17}$$

Example-3 : Find the sum of $\frac{-4}{6}$ and $\frac{5}{9}$.

Sol. The rational number are $\frac{-4}{6}$ and $\frac{5}{9}$.

Here denominators are not same
so L.C.M of 6 and 9 = 2 × 3 × 3 = 18

2	6, 9
3	3, 9
3	1, 3
	1, 1

Now $$\frac{-4}{6} = \frac{-4}{6} \times \frac{3}{3} = \frac{-12}{18}$$

$$\frac{5}{9} = \frac{5}{9} \times \frac{2}{2} = \frac{10}{18}$$

Thus $$\frac{-4}{6} + \frac{5}{9} = \frac{-12}{18} + \frac{10}{18}$$

$$= \frac{-12+10}{18}$$

$$= \frac{-2}{18} = \frac{-1}{9}$$

Example-4 : Add $\frac{5}{-27}$ and $\frac{13}{36}$.

Sol. LCM of denominators = (3 × 3 × 3 × 4)
= 108

3	27, 36
3	9, 12
	3, 4

Now $\frac{5}{-27} = \frac{5 \times -4}{-27 \times -4} = \frac{-20}{108}$

$\frac{13}{36} = \frac{13 \times 3}{36 \times 3} = \frac{39}{108}$

Thus $\frac{5}{-27} + \frac{13}{36} = \frac{-20}{108} + \frac{39}{108}$

$= \frac{-20+39}{108}$

$= \frac{19}{108}$

Additive Inverse : The additive Inverse of a rational number $\frac{a}{b}$ is $\frac{-a}{b}$ which is again a rational number.

- Sum of a rational number and its additive inverse is zero $\frac{a}{b} + \left(\frac{-a}{b}\right) = 0$
- 0 is only rational number which is additive inverse of itself.

Subtraction of a rational number : If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then

$$\left(\frac{a}{b} - \frac{c}{d}\right) = \frac{a}{b} + \left(\frac{-c}{d}\right)$$

$$= \frac{a}{b} + \text{additive inverse of } \left(\frac{c}{d}\right)$$

From above we conclude that while subtracting two rational numbers we add the additive inverse of the rational number that is being subtracted.

Example-5 : Find

(*i*) $\frac{3}{9} - \left(\frac{-4}{9}\right)$ **(*ii*)** $\frac{5}{12} - \frac{7}{24}$

Sol. (*i*) $\frac{3}{9} - \left(\frac{-4}{9}\right) = \frac{3}{9} + (\text{additive inverse of } -\frac{4}{9})$

$= \frac{3}{9} + \frac{4}{9}$

$= \frac{3+4}{9}$

$= \frac{7}{9}$

(*ii*) $\frac{5}{12} - \frac{7}{24} = \frac{5}{12} + (\text{additive inverse of } \frac{7}{24})$

$= \frac{5}{12} + \left(\frac{-7}{24}\right)$

L.C.M of 12 and 24 $= 24$

Now $\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24}$

$\therefore$ $\frac{5}{12} - \frac{-7}{24} = \frac{10}{24} + \left(\frac{-7}{24}\right)$

$= \frac{10-7}{24}$

$= \frac{3}{24}$

$= \frac{1}{8}$

Multiplication of Rational numbers : Product of two rational numbers is defined as follows

$$\text{Product of two rational numbers} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

For any two rational number $\frac{a}{b}$ and $\frac{c}{d}$

$$\frac{a}{b} \times \frac{c}{d} = \frac{(a \times c)}{(b \times d)}$$

Example-6 : Find the product of

(*i*) $\frac{9}{5} \times \frac{3}{7}$ (*ii*) $\frac{3}{7} \times \frac{-7}{3}$

Sol (*i*) $\frac{9}{5} \times \frac{3}{7} = \frac{9 \times 3}{5 \times 7}$

$= \frac{27}{35}$

(*ii*) $\frac{3}{-7} \times \frac{-7}{3} = \frac{3 \times -7}{-7 \times 3}$

$= 1$

Reciprocal of a rational number : Reciprocal of a rational number $\frac{a}{b}$ is $\frac{b}{a}$.

- Product of a rational number and its reciprocal is always one. $\left(\frac{a}{b} \times \frac{b}{a} = 1\right)$
- Reciprocal of 1 is 1.
- Reciprocal of 0 does not exist.

Division of Rational numbers : If $\frac{a}{b}$ and $\frac{c}{d}$ be two rational number such that $\frac{c}{d} \neq 0$ then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times (\text{Reciprocal of } \frac{c}{d})$$
$$= \frac{a}{b} \times \frac{d}{c}$$

Example-7 : Divide

(*i*) $\frac{9}{21}$ **by** $\frac{3}{7}$ **(*ii*)** $\frac{-5}{9}$ **by** $\frac{7}{27}$

Sol. (*i*) Given numbers are $\frac{9}{21}$ and $\frac{3}{7}$

Now $\frac{9}{21} \div \frac{3}{7} = \frac{9}{21} \times (\text{Reciprocal of } \frac{3}{7})$

$$= \frac{9}{21} \times \frac{7}{3}$$
$$= 1$$

(*ii*) Given numbers are $\frac{-5}{9}$ and $\frac{7}{27}$

Now $\frac{-5}{9} \div \frac{7}{27} = \frac{-5}{9} \times (\text{Reciprocal of } \frac{7}{27})$

$$= \frac{-5}{9} \times \frac{27}{7}$$
$$= \frac{-15}{7}$$

Example-8 : What number should be added to $\frac{-7}{12}$ to get $\frac{5}{9}$?

Sol. Let the required number to be added be x,

then $\frac{-7}{12} + x = \frac{5}{9}$

$\Rightarrow$ $x = \frac{5}{9} - \left(\frac{-7}{12}\right)$

$\Rightarrow$ $x = \frac{5}{9} + \frac{7}{12} = \frac{5 \times 4 + 7 \times 3}{36}$

$$= \frac{20+21}{36} = \frac{41}{36} = 1\frac{5}{36}$$

Hence the required number to be added is $1\frac{5}{36}$.

Example-9 : What number should be subtracted from $\frac{-3}{4}$ to get $\frac{-11}{4}$?

Sol. Let the required number to be subtracted be x, then

$$\frac{-3}{4} - x = \frac{-11}{4}$$

$$\Rightarrow \quad \frac{-3}{4} - \left(\frac{-11}{4}\right) = x$$

$$\Rightarrow \quad x = \frac{-3}{4} - \left(\frac{-11}{4}\right) = \frac{-3}{4} + \frac{11}{4} = \frac{-3+11}{4} = \frac{8}{4}$$

$$x = 2$$

Hence the required number to be subtracted is 2.

Example-10 : The product of two rational numbers is $\frac{-9}{16}$. If one of the number is $\frac{3}{14}$, find the other number.

Sol. Let the required number be x, then

$$\frac{3}{14} \times x = \frac{-9}{16}$$

$$\Rightarrow \quad x = \frac{-9}{16} \div \frac{3}{14}$$

$$x = \frac{-9}{16} \times \frac{14}{3} = \frac{(-9)\times 14}{16\times 3} = \frac{-126}{48} = \frac{-21}{8}$$

$$x = -2\frac{5}{8}$$

EXERCISE - 9.2

1. Find the sum

(i) $\frac{6}{9} + \frac{2}{9}$ (ii) $\frac{-15}{7} + \frac{9}{7}$ (iii) $\frac{17}{11} + \left(\frac{-9}{11}\right)$

(iv) $\frac{-5}{6} + \frac{3}{18}$ (v) $\frac{-7}{19} + \frac{-3}{38}$ (vi) $-3\frac{4}{7} + 2\frac{3}{7}$

(vii) $\frac{-5}{14} + \frac{8}{21}$ (viii) $-4\frac{1}{15} + 3\frac{2}{20}$

2. Find

(*i*) $\frac{7}{12} - \frac{11}{36}$ (*ii*) $\frac{-5}{9} - \frac{3}{5}$ (*iii*) $\frac{-7}{13} - \left(\frac{-5}{91}\right)$

(*iv*) $\frac{6}{11} - \frac{-3}{4}$ (*v*) $3\frac{4}{9} - \frac{28}{63}$

3. Find the product of

(*i*) $\frac{5}{9} \times \frac{-3}{8}$ (*ii*) $\frac{-3}{7} \times \frac{7}{-3}$ (*iii*) $\frac{3}{13} \times \frac{5}{8}$

(*iv*) $\frac{3}{10} \times (-18)$

4. Find the value of

(*i*) $-9 \div \frac{3}{5}$ (*ii*) $\frac{-4}{7} \div 4$ (*iii*) $\frac{7}{18} \div \frac{5}{6}$

(*iv*) $\frac{-8}{35} \div \left(\frac{-2}{7}\right)$ (*v*) $\frac{-9}{15} \div -18$

5. What rational number should be added to $\frac{-5}{12}$ to get $\frac{-7}{8}$?

6. What number should be subtracted from $\frac{-2}{3}$ to get $\frac{-5}{6}$?

7. The product of two rational numbers is $\frac{-11}{2}$. If one of them is $\frac{33}{8}$, find the other number.

8. **Multiple choice questions**

(*i*) The sum of $\frac{5}{4} + \left(\frac{25}{-4}\right) =$

(*a*) –5 (*b*) 5 (*c*) 4 (*d*) –4

(*ii*) $\frac{17}{11} - \frac{6}{11} =$

(*a*) 1 (*b*) –1 (*c*) 6 (*d*) 3

(*iii*) $\frac{2}{-5} \times \frac{-5}{2} =$

(*a*) 1 (*b*) –1 (*c*) 2 (*d*) –5

(*iv*) $\frac{7}{12} \div \left(\frac{-7}{12}\right) =$

(*a*) 1 (*b*) –1 (*c*) 7 (*d*) –7

(*v*) Which of the following is value of $(-4) \times [(-5) + (-3)]$

(*a*) –32 (*b*) 120 (*c*) 32 (*d*) –23

WHAT WE HAVE DISCUSSED ?

1. The number of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called rational numbers.
2. If we multiply or divide both the numerator and the denominator of a rational number by a non-zero integer, we get an equivalent ralional number.
3. A rational number is said to be positive if both the numerator and the denominator are either positive or negative.
4. A rational number is said to be negative if either numerator or denominator is negative.
5. The number zero (0) is neither positive nor negative rational number.
6. A rational number $\frac{a}{b}$ is said to be in standard form, if b is positive, a and b have no common divisor, other than 1.
7. Additive inverse of a rational number $\frac{a}{b}$ is $\frac{-a}{b}$.
8. Multiplicative inverse (Reciprocal) of a non-zero rational number $\frac{a}{b}$ is $\frac{b}{a}$.

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Define rational numbers and reduce the given rational numbers to the standard form.
2. Write equivalent rational numbers.
3. Represent the given rational numbers on a number line.
4. Find more rational numbers between given rational numbers.
5. Add, subtract, multiply and divide two or more rational numbers.
6. Solve problems related to daily life situations involving rational numbers.

ANSWERS

EXERCISE 9.1

1. (*i*) $\frac{8}{10}, \frac{12}{15}$ (*ii*) $\frac{-10}{18}, \frac{15}{27}$

(*iii*) $\frac{6}{-22}, \frac{9}{-33}$

2. (*i*) $\frac{5}{9}$ (*ii*) $\frac{-3}{4}$

(*iii*) $\frac{-1}{3}$ (*iv*) $\frac{1}{3}$

3. (*i*), (*iii*), (*iv*)

4. (*i*) $\frac{4}{5} > \frac{3}{7}$ (*ii*) $\frac{-4}{12} > \frac{-8}{12}$

(*iii*) $\frac{4}{-18} > \frac{-3}{9}$ (*iv*) $-2\frac{3}{5} > -3\frac{5}{8}$

5. (*i*) $\frac{-5}{7}, \frac{-3}{7}, \frac{-1}{7}$ (*ii*) $\frac{-4}{5}, \frac{-1}{5}, \frac{-2}{15}$

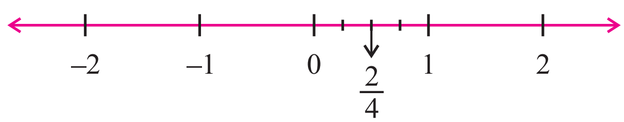
(*iii*) $\frac{-3}{2}, \frac{-2}{4}, \frac{-3}{8}$

6. (*i*) $\frac{-11}{6}, \frac{-5}{3}, \frac{-3}{2}, \frac{-4}{3}$ and $\frac{-7}{6}$ (*ii*) $\frac{-7}{9}, \frac{-34}{45}, \frac{-11}{15}, \frac{-32}{45}$ and $\frac{-31}{45}$

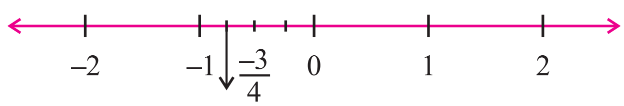
(*iii*) $\frac{8}{21}, \frac{3}{7}, \frac{10}{21}, \frac{11}{21}$ and $\frac{4}{7}$

7. (*i*) $\frac{-5}{25}, \frac{-6}{30}, \frac{-7}{35}, \frac{-8}{40}$ (*ii*) $\frac{5}{-35}, \frac{6}{-42}, \frac{7}{-49}, \frac{8}{-56}$

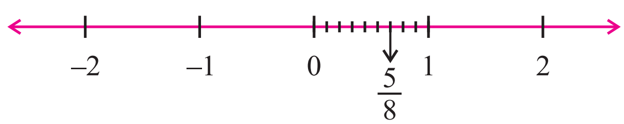
8. (*i*)



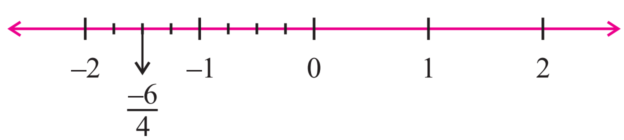
(*ii*)



(*iii*)



(*iv*)



9. (*i*) *c* (*ii*) *b*

(*iii*) *a* (*iv*) *d*

(*v*) *a* (*vi*) *a*

EXERCISE 9.2

1. (*i*) $\frac{8}{9}$ (*ii*) $\frac{-6}{7}$

(*iii*) $\frac{8}{11}$ (*iv*) $\frac{-2}{3}$

(*v*) $\frac{-17}{38}$ (*vi*) $\frac{-8}{7}$

(*vii*) $\frac{1}{42}$ (*viii*) $\frac{-29}{30}$

2. (*i*) $\frac{5}{18}$ (*ii*) $\frac{-52}{45}$

(*iii*) $\frac{-44}{91}$ (*iv*) $\frac{57}{44}$

(*v*) 3

3. (*i*) $\frac{-5}{24}$ (*ii*) 1

(*iii*) $\frac{15}{104}$ (*iv*) $\frac{-27}{5}$

4. (*i*) –15 (*ii*) $\frac{-1}{7}$

(*iii*) $\frac{7}{15}$ (*iv*) $\frac{4}{5}$

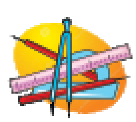
(*v*) $\frac{1}{30}$

5. $\frac{-11}{24}$ **6.** $\frac{1}{6}$ **7.** $\frac{-4}{3}$

8. (*i*) *a* (*ii*) *a*

(*iii*) *a* (*iv*) *b*

(*v*) *c*



CHAPTER 10

M2T1Y7

Practical Geometry

Learning Objectives :-

In this chapter you will learn :-

1. To draw a line parallel to a given line.
2. To construct triangles using different construction criterias.
3. To determine whether the construction of a triangle with given measurements is possible or not.

INTRODUCTION

We have already learnt how to draw a line segment of given length, a line prependicular to a given line segment, an angle, an angle bisector and a circle etc previously. Now in this chapter. We will learn, how to

- Construct a line parallel to a given line.
- Construct triangles.

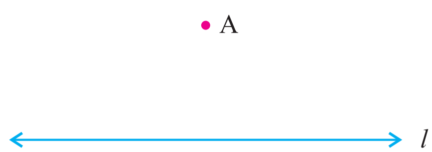
CONSTRUCTION OF PARALLEL LINE TO A GIVEN LINE

Construction of a line parallel to a given line '*l*' through a point 'A', That does not lie on the given line. We shall draw it either.

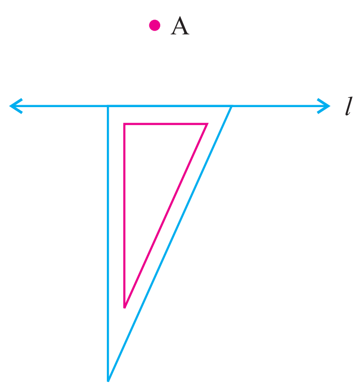
(*i*) By using ruler and set square Or (*ii*) By using ruler and compasses

(*i*) Construction of a parallel line using ruler and set square

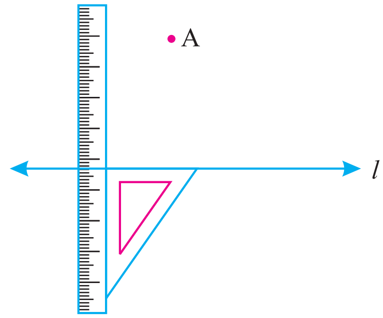
Step 1 : Draw a line '*l*' and take a point 'A' that does not lie on the given line.



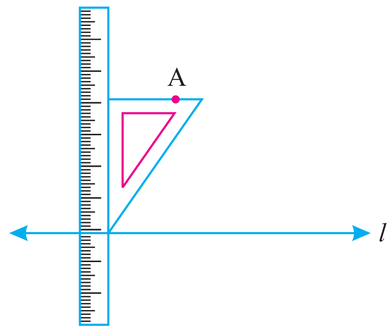
Step 2 : Place a set square such that one of its shorter edge lies along line '*l*' as shown.



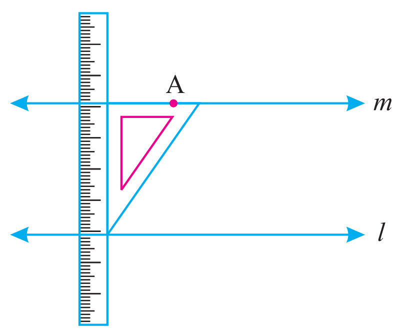
Step 3 : Now place a ruler along the other side of the set square so that it touches the standing edge of the set square.



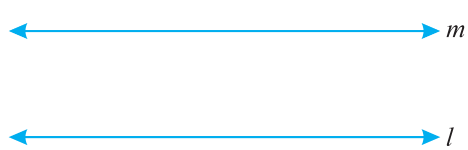
Step 4 : Hold the ruler firmly and slide the set square along the ruler unitl the edge of the set square passes through 'A'.



Step 5 : Draw a straight line *m* along the horizontal edge of the set square passing through point 'A'.

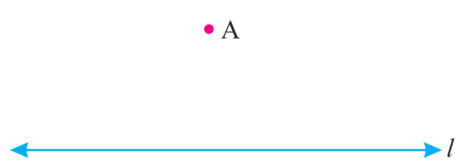


Step 6 : Line '*m*' is the required line parallel to '*l*'

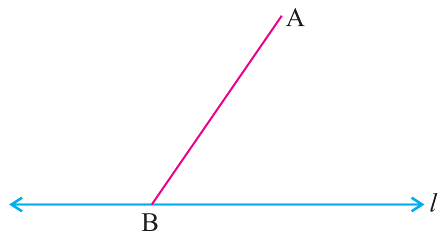


(*ii*) Construction of a parallel line by using ruler and compass.

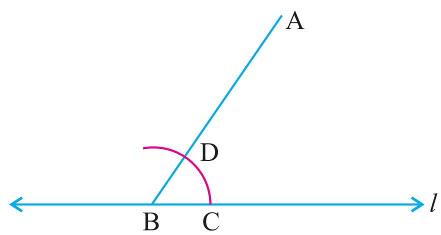
Step 1 : Draw a line '*l*' and a point 'A' not lying on '*l*'



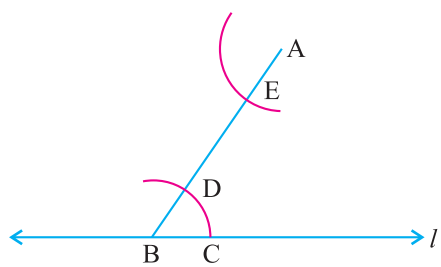
Step 2 : Take any point B on l and join B to A.



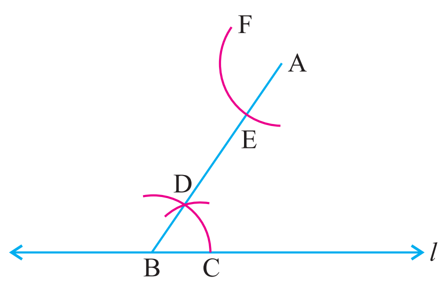
Step 3 : With B as centre draw an arc of any radius intersecting l at C and AB at D.



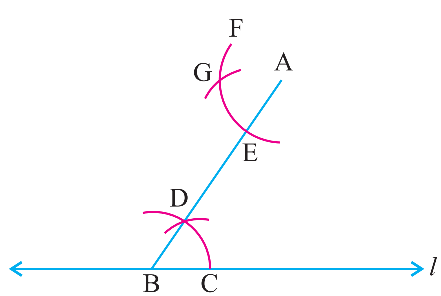
Step 4 : Now with A as centre and the same radius, draw another arc intersecting AB at a point E.



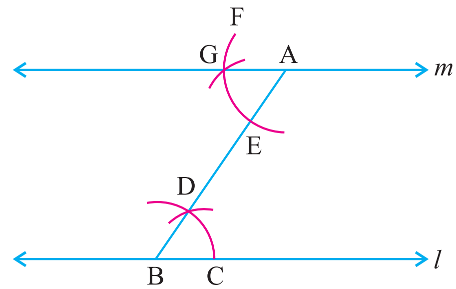
Step 5 : Measure the arc CD with compass.



Setp 6 : With E as centre cut an arc EG = arc CD



Step 7 : Join A to G and extend the line segment to both sides.



Note that $\angle EAG = \angle DBC$ are the alternate angles therefore $m \parallel l$.

EXERCISE - 10.1

1. Draw a line l, take a point p outside it, Through p draw a line parallel to l using ruler and compass only.
2. Draw a line parallel to a line l at a distance of 3.5cm from it.
3. Let l be a line and P be a point not on l. Through P draw a line m paralel to l. Now, join P to any point Q on l. Choose any other point R on m. Through R draw a line parallel to PQ. Let this meets l at S. What shape do the two sets of parallel lines enclose.
4. (*i*) How many parallel lines can be drawn, passing through a point not lying on the given line ?
   (*a*) 0 (*b*) 2
   (*c*) 1 (*d*) 3
   (*ii*) Which of the following is used to draw a line parallel to a given line ?
   (*a*) A protractor (*b*) A ruler
   (*c*) A compasses (*d*) A ruler and compasses.

CONSTRUCTION OF TRIANGLES

Let us recall some important propoerties of triangle.

1. Sum of lengths of any two sides of a triangle is greater than the length of the third side.
2. Sum of measures of the three angles of a triangle is 180°.
3. Exterior angle of a triangle is equal to the sum of opposite interior angles.
4. Pythagoras property i.e. In right angled triangle
   $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

A triangle can be drawn, if any one of the following set of measurements are given in the questions.

1. Three sides (SSS)
2. Two sides and angle between them (SAS)
3. Two angles and the side between them (ASA)
4. The length of Hypotenuse and a side in case of a right angled triangle.

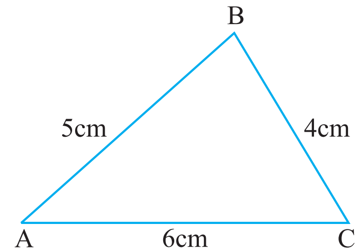
- Note : To construct a triangle with a given measure we should first draw a rough sketch to indicate the given measure.

CONSTRUCTION OF A TRIANGLE USING SSS CRITERION

SSS stands for side-side-side. In this section we would construct triangle when all its sides are known. To understand the criteria see the following example.

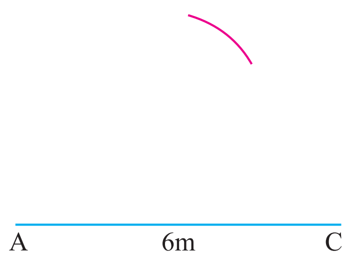
Example-1 : Construct a triangle ABC, given that AB = 5*cm*, BC = 4*cm*, AC = 6*cm*

Step 1 : Draw a rough sketch of ΔABC with given measures.

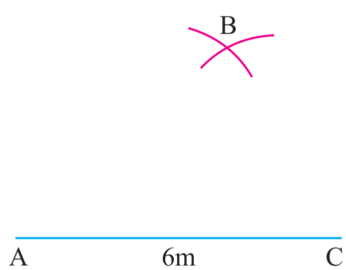


Step 2 : Draw a line segment AC of length 6cm (Note : Take longest side as a base it is optional but not compulsory)

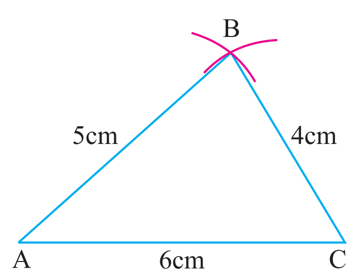
Step 3 : With A as centre and radius 5*cm* (∵ AB = 5*cm*) draw on arc.



Step 4 : With C as centre and radius 4cm (∵ BC = 4*cm*) draw another arc intersecting the previous arc at B.



Step 5 : Join AB and CB. ΔABCis the required triangle.



EXERCISE - 10.2

1. Construct a ΔABC in which AB = 3.5*cm*, BC = 5*cm* and CA = 7*cm*.

2. Construct a triangle ABC in which AB = BC = 6.5*cm* and CA = 4*cm*. Also name the kind of triangle drawn.

3. Construct a triangle XYZ such that length of each side is 5*cm.* Also name the kind of triangle drawn.

4. Construct a triangle PQR such that PQ = 2.5, QR = 6cm and RP = 6.5cm. Measure ∠PQR and also name the kind of triangle drawn.

5. Construct a triangle ABC, in which AB = 6*cm*, BC = 2*cm*, CA = 3*cm*. (If possible). If not possible give the reason.

6. (*i*) Which of the following can be used to construct a triangle ?

(*a*) The lengths of the three sides

(*b*) The perimeter of the triangle

(*c*) The measures of three angles

(*d*) The name of three vertices

(*ii*) A triangle can be constructed by taking its sides as

(*a*) 1.8*cm*, 2.6*cm*, 4.4*cm* (*b*) 3*cm*, 4*cm*, 8*cm*

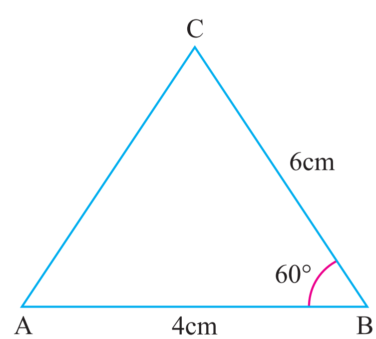
(*c*) 4*cm*, 7*cm*, 2*cm*. (*d*) 5*cm*, 4*cm*, 4*cm*

CONSTRUCTION OF A TRIANGLE USING SAS CRITERION

SAS stands for side-angle-side. Here, we have two given sides and the one angle between them. We first draw a rough sketch. Follow example (1) to understand. the concept of construction of triangle using SAS.

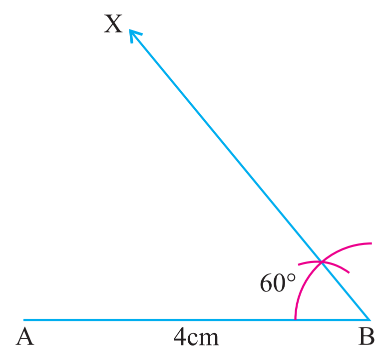
Example-1 : **Construct a triangle ABC, Such that AB = 4*cm*, BC = 6*cm* and ∠ABC = 60°**

Step 1 : Draw a rough sketch of ΔABC with given measures.

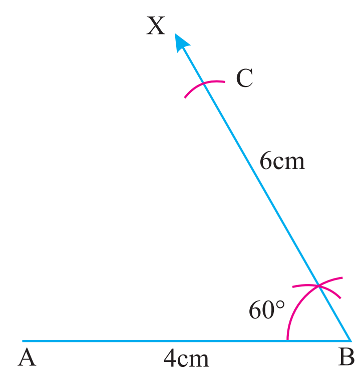


Step 2 : Draw a line segment AB of length 4*cm*.

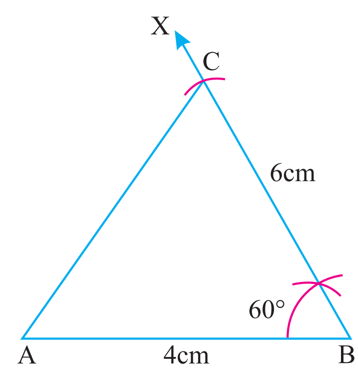
Step 3 : With the help of compass, at B, draw a ray BX making an angle 60° with AB.



Step 4 : With B as centre and radius 6cm. draw an arc intersecting the ray BX at point C.



Step 5 : Join AC. ΔABC is required triangle.



EXERCISE - 10.3

1. Construct ΔPQR such that AB = 4*cm*, ∠B = 30°, BC = 4*cm*. Also name the type of this triangle on the basis of sides.
2. Construct ΔABC with AB = 7.5*cm*, BC = 5*cm* and ∠B = 30°.
3. Construct a triangle XYZ such that XY = 6*cm*, YZ = 6*cm* and ∠Y = 60°. Also name the type of this triangle.

4. (*i*) Which of the following triangle can be constructed using SAS criterion.

(*a*) AB = 5*cm*, BC = 5*cm*, CA = 6*cm*

(*b*) AB = 5*cm*, BC = 5*cm*, ∠B = 40°

(*c*) ∠A = 60°, ∠B = 60°, ∠C = 60°

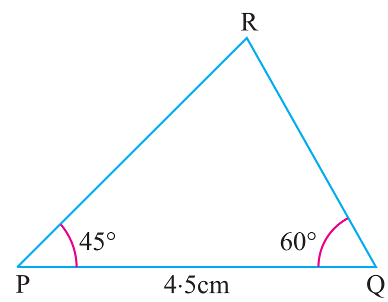
(*d*) BC = 5*cm*, ∠B = ∠C = 45°

CONSTRUCTION OF A TRIANGLE USING ASA CRITERION

ASA stands for Angle-Side-Angle. First draw rough sketch of given measures, then draw the given line segment. Make angles on the both ends of line of given measures as shown in the following example.

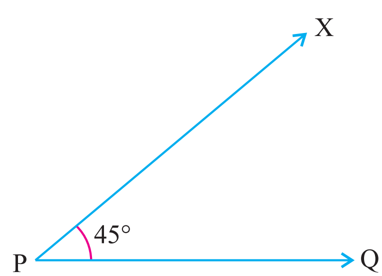
Example-1 : Construct a triangle PQR such that PQ = 4.5*cm*, ∠P = 45°, ∠Q = 60°

Step 1 : First we draw rough sketch of triangle PQR with given measures.

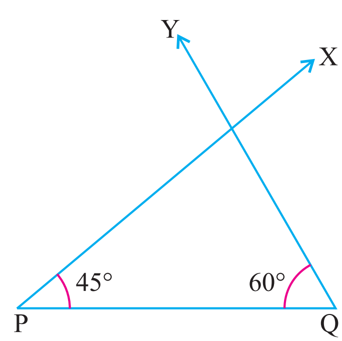


Step 2 : Draw a line segment PQ = 4.5*cm*.

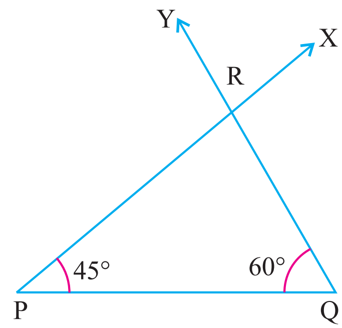
Step 3 : At P, Draw a ray PX making an angle 45° with PQ (with the help of compass as discussed in earlier classes)



Step 4 : With the help of compass, At Q draw a ray QY making an angle 60° with the line segment PQ.

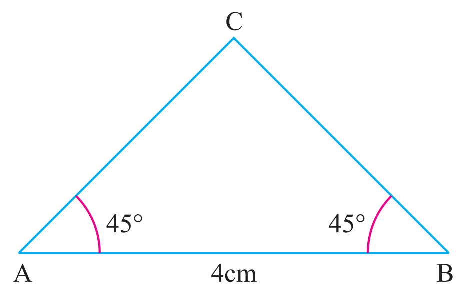


Step 5 : Rays PX and QY intersect at a point say R. then ΔPQR is the required triangle.



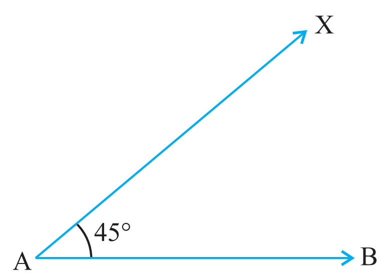
Example-2 : Construct an isosceles ΔABC with Base AB = 4*cm* and each base angle measuring 45°.

Step 1 : Draw a rough sketch of ΔABC with given measures.

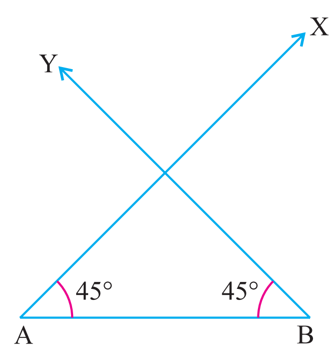


Step 2 : Draw a line segment AB = 4*cm*.

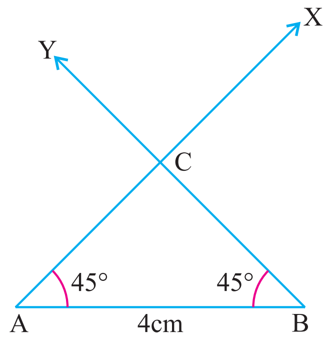
Step 3 : Taking A as centre with the help of compass, Draw a ray AX making an angle 45° with AB.



Step 4 : With the help of compass at taking B as a centre. Draw a ray BY making an angle 45° with the line segment AB.



Step 5 : Rays AX and BY intersect, at a point, say C, then ΔABC is the required triangle.



EXERCISE - 10.4

1. Construct ΔABC, given AB = 6*cm*, ∠A = 30° and ∠B = 75°
2. Construct an isosceles ΔABC such that base AB = 5.3*cm* and each base angle = 45°
3. Construct ΔXYZ if XY = 4*cm*, ∠X = 45° and ∠Z = 60°

   (Hint : ∠Y = 180° – 45° – 60° = 75°)
4. Examine whether you can construct ΔPQR such that ∠P = 100°, ∠Q = 90° and PQ = 4.3 cm. If not possible give reason.
5. (*i*) In which of the following cases a unique triangle can be drawn ?
   (*a*) BC = 5*cm*, ∠B = 90° and ∠C = 100°
   (*b*) AB = 4*cm*, BC = 7*cm* and CA = 2*cm*
   (*c*) XY = 5*cm*, ∠X = 45°, ∠Y = 60°
   (*d*) An isosceles triangle with length of each equal side equal to 5 cm.

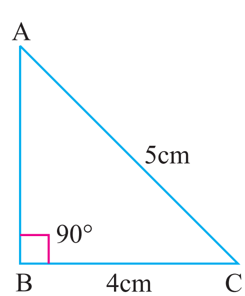
   (*ii*) A triangle can be constructed by taking two of its angles as.
   (*a*) 110°, 40° (*b*) 70°, 115°
   (*c*) 135°, 45° (*d*) 90°, 90°

CONSTRUCTION OF A TRIANGLE USING RHS CRITERION

RHS stands for right angle, hypotenuse and side of a right angled triangle. First draw a rough sketch of given measure. Draw a line segment of given measure. Construct a right angle. Now mark length of side of hypotenuse of triangle. See example 1.

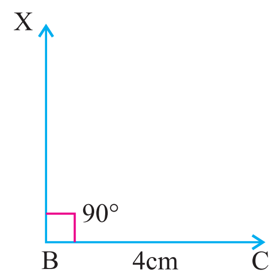
Example-1 : Construct a ΔABC such that ∠B = 90° BC = 4*cm* and AC = 5*cm*

Step 1 : Draw a rough sketch of given measures.

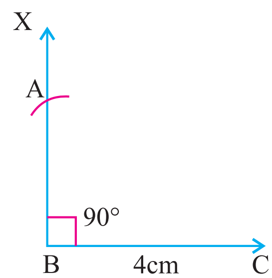


Step 2 : Draw a line segment BC = 4*cm*.

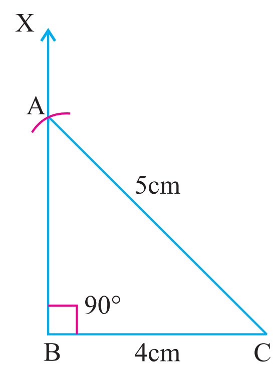
Step 3 : With the help of a compass taking B as centre, draw a ray BX making an angle 90° with BC.



Step 4 : With C as centre and radius 5cm (= AC) draw an arc interseting ray BX at a point A.



Step 5 : Join A and C therefore ΔABC is required triangle.



EXERCISE - 10.5

1. Construct a right angled triangle ABC with ∠C = 90°, AB = 5*cm* and BC = 3*cm*.
2. Construct an isosceles right angled triangle DEP where ∠E = 90° and EF = 6*cm*.
3. Construct a right angled triangle PQR in which ∠Q = 90°, PQ = 3.6*cm* and PR = 8.5*cm*.
4. (*i*) Which of the following is a pythagorian triplet ?

   (*a*) 1, 2, 3 (*b*) 2, 3, 4

   (*c*) 4, 5, 6 (*d*) 12, 13, 5

(*ii*) Construction of unique triangle is not possible when

(*a*) Three sides are given.

(*b*) Two sides and an included angle are given.

(*c*) Three angles are given.

(*d*) Two angles & included side are given.

WHAT HAVE WE DISCUSSED ?

1. A line can be drawn parallel to a given line through a point not lying on it. With compass and ruler by alternate angle or by corresponding angle method.
2. A triangle can uniquely be constructed by using indirectly the concept of congurence of triangles.
3. **ASA :** Measure of two angles and length of the side included between them is given.
4. **SSS :** Length of three sides of a triangle are given.
5. **SAS :** Measure of two sides and angle between the two sides are given.
6. **RHS :** Length of hypotenuse and one of the other two sides of a triangles are given.

LEARNING OUTCOMES

After completion of the chapter, students are now able to :

1. Handle geometrical instruments like scale, compass, protractor etc.
2. Construct a line parallel to the given line from a point outside it.
3. Construct a triangle with given measurements.
4. Check whether a triangle is possible with given measurements.

ANSWERS

EXERCISE 10.1

4. (*i*) *c* (*ii*) *d*

EXERCISE 10.2

6. (*i*) *a* (*ii*) *d*

EXERCISE 10.3

4. (*i*) *b*

EXERCISE 10.4

5. (*i*) *c* (*ii*) *a*

EXERCISE 10.5

4. (*i*) *d* (*ii*) *c*

CHAPTER 11

Perimeter And Area

Learning Objectives :-

In this chapter you will learn :-

1. About measurements.
2. To convert the units of length and area.
3. The difference between perimeter and area of different plane figures.
4. To compute perimeter and area of square, rectangle, triangle and parallelogram using formulae.
5. To compute the circumference and area of a circle.
6. To apply your knowledge of perimeter and area in real life situations.

OUR NATIONS' PRIDE

History : Indian mathematicians played a crucial role in finding the area of plane figures. Aryabhata (476–550 AD) gave the formula for area of a triangle. He worked on the approximation for *pi* (π). In the second part of Aryabhatiya, he wrote that the ratio of the circumference of a circle to its diameter is 3.1416. Another mathematician Brahmagupt (598 – 668 AD) gave the formula for the area of a cyclic quadrilateral.

INTRODUCTION

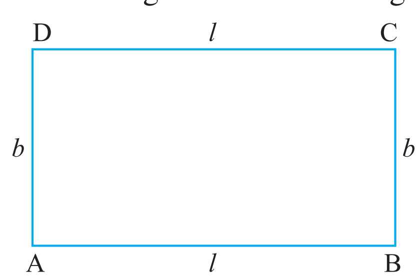
We have already learnt about the perimeter of plane figures and the area of a square and rectangle in class VI.

Perimeter : Perimeter of a simple closed figure is the length of its boundary. Its units are same as the units of length *i.e*, *cm* and *m* etc.

Area : Area of a simple closed figure is the measure of the surface enclosed in it. Units of area are cm^2 and m^2 etc.

PERIMETER AND AREA OF A RECTANGLE AND A SQUARE

Rectangle : Let us consider a rectangle ABCD with length = l units and breadth = b units



Then, Perimeter of rectangle = AB + BC + CD + DA

= $l + b + l + b$

= $2l + 2b$

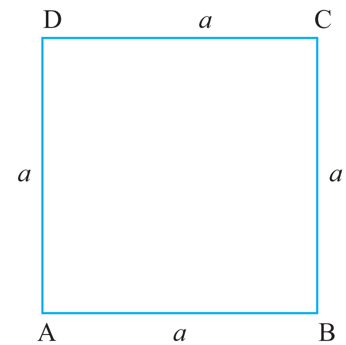
= $2(l + b)$ units

Area of rectangle = (Length × Breadth) sq. units

We can find length and breadth by using the formula

$$\text{length} = \frac{\text{Area}}{\text{Breadth}} \text{ units and Breadth} = \frac{\text{Area}}{\text{Length}} \text{ units}$$

Square : Let us consider a square ABCD having each side equal to a units then,



Perimeter of square = AB + BC + CD + DA

= $a + a + a + a$

= 4a units

= (4 × side) units

Area of square = Side × side

A = $a \times a$

A = a^2 sq units

Example-1 : Find the perimeter and area of a rectangle whose length is 18cm and breadth 9*cm*

Sol. Given length of rectangle = 18*cm*

Breadth of rectangle = 9*cm*

Perimeter of rectangle = 2 (Length + Breadth)

= 2 (18 + 9)

= 2 (27)

= 54*cm*

Area of rectangle = Length × Breadth

= 18 × 9

= 162cm^2

Example-2 : Find the perimeter and area of a square with side 3.5*cm*

Sol. Side of square = 3.5 *cm*

Perimeter of square = 4 × side

= 4 × 3.5

= 14.0 *cm*

Area of square = $(\text{Side})^2$

= $(3.5)^2$

= 3.5 × 3.5

= 12.25 cm^2

Example-3 : Area of Rectangular park is 1386 m^2. If length of park is 42 *m*, find the breadth and the perimeter of the park.

Sol. Area of rectangular park = 1386m^2

Length = 42*m*

$$\text{Area} = \text{Length} \times \text{Breadth}$$

$$\therefore \quad \text{Breadth} = \frac{\text{Area}}{\text{Length}} = \frac{1386}{42}$$

$$= 33m$$

$$\text{Perimeter of rectangular park} = 2\,(\text{Length} + \text{Breadth})$$

$$\text{Perimeter of rectangular park} = 2\,(42 + 33)$$

$$= 2\,(75)$$

$$= 150m$$

Example-4 : The area of square park is same as that of a rectangular park. If side of a square park is 36*m* and length of rectangular park is 54*m*. Find the breadth of rectangular park.

Sol. Side of a square park = 36*m*

$$\text{Area of the square park} = (\text{side})^2$$

$$= 36 \times 36$$

$$= 1296m^2$$

$$\text{Length of a rectangular park} = 54m$$

$$\text{Let breadth of the rectangular park} = b$$

According to question,

$$\text{Area of rectangular park} = \text{Area of square park}$$

$$54 \times b = 1296$$

$$b = \frac{1296}{54}$$

$$b = 24m$$

Hence breadth of rectangular park = 24m

Example-5 : A wire is in the shape of a square of side 15 cm. If the wire is rebent into a rectangle of length 16cm. Find its breadth. Which encloses more area, square or rectangle ?

Sol. Side of square = 15 *cm*

$$\text{Perimeter of square} = 4 \times \text{side}$$

$$= 4 \times 15$$

$$= 60\ cm$$

$$\text{Length of rectangle} = 16\ cm$$

$$\text{Let breadth of rectangle} = b\ cm$$

$$\text{Perimeter of rectangle} = 2\,(l + b)$$

$$= 2\,(16 + b)\ cm$$

According to question,

$$\text{Perimeter of square} = \text{Perimeter of rectangle}$$

$$60 = 2\,(16 + b)$$

$$\frac{60}{2} = 16 + b$$

$$16 + b = 30$$

$$b = 30 - 16$$

$$b = 14cm$$

$\therefore$ Breadth of rectangle = $14cm$

Area of square = $(\text{side})^2$

= 15×15

= $225cm^2$

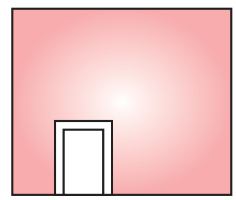
Area of Rectangle = Length × Breadth

= 16×14

= $224\ cm^2$

Square encloses more area

Example-6 : A door of dimensions $3m \times 2m$ is fitted in a wall. The length of the wall is $8m$ and the breadth is $5m$. Find the cost of painting the wall, if the rate of painting is ₹25 per m^2.



Sol. Length of wall = $8m$

Breadth of wall = $5m$

Area of wall = Length × Breadth

= 8×5

= $40m^2$

Area of door = $3m \times 2m$

= $6m^2$

Now area of wall for painting = Area of wall including door – Area of door

= $40 - 6$

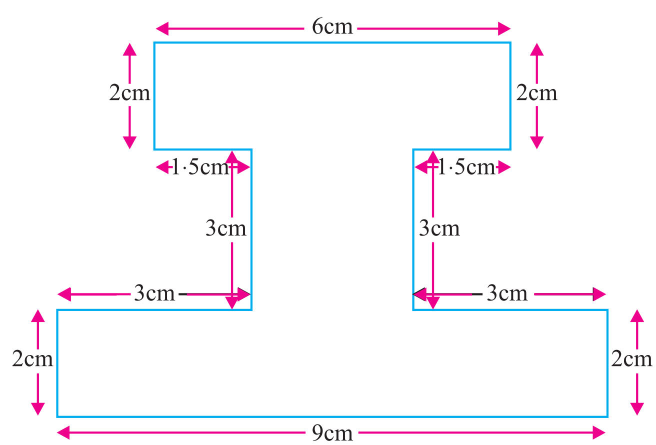
= $34m^2$

Cost of painting $1m^2$ of wall = ₹ 25

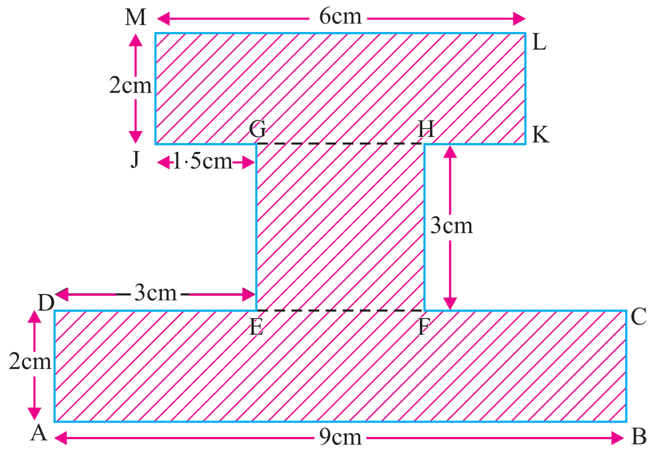
Cost of painting $34m^2$ of wall = 34×25

= ₹ 850

Example-7 : Find the perimeter and area of the given figure.



Sol.



$$AB = DC$$
$$AB = DE + EF + FC$$
$$9 = 3 + EF + 3$$
$$9 = 6 + EF$$
$$EF = 3cm$$

We obtain the rectangles ABCD, JKLM and a square EFHE

Area of the figure = Area of rectangle ABCD + Area of rectangle JKLM + Area of square EFHG

$= (9 \times 2)cm^2 + (6 \times 2)cm^2 + (3 \times 3)cm^2$

$= (18 + 12 + 9)cm^2$

$= 39cm^2$

Perimeter of the given figure = MJ + JG + GE + DE + DA + AB + BC + CF + FH + HK + KL + ML

$= 2 + 1.5 + 3 + 3 + 2 + 9 + 2 + 3 + 3 + 1.5 + 2 + 6$

$= 38cm$

EXERCISE - 11.1

1. Find the perimeter and the area of a rectangle having

(*i*) Length = 28*cm*, Breadth = 15*cm*

(*ii*) Length = 9.4*cm* Breadth = 2.5*cm*

2. Find the perimeter and the area of a square whose side measures

(*i*) 29*cm*

(*ii*) 8.3*cm*

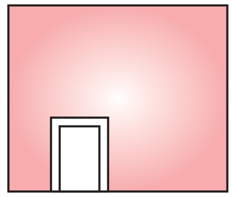
3. The perimeter of a square park is 148*m*. Find its area.

4. The area of a rectangle is 580cm^2. Its length is 29*cm*. Find its breadth and also, the perimeter.

5. A wire is in the shape of a rectangle. Its length is 48cm and breadth is 32*cm*. If the same wire is rebent into the shape of a square, what will be the measure of each side. Also, find which shape encloses more area and by how much ?

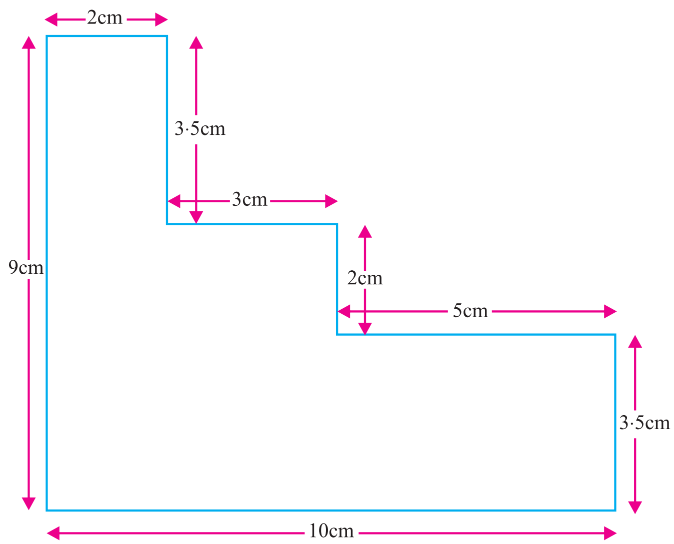
6. The area of a square park is the same as that of a rectangular park. If the side of the square park is 75*m* and the length of the rectangular park is 125*m*, find the breadth of the rectangular park. Also, find the perimeter of rectangular park.

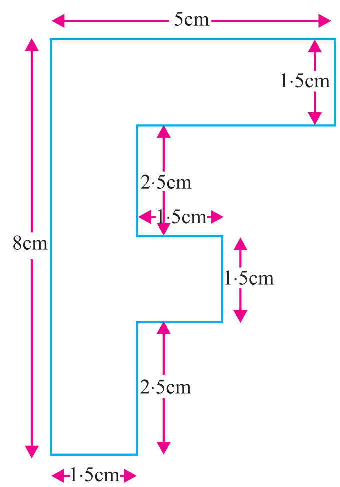
7. A door of length 2.5*m* and breadth 1.5*m* is fitted in a wall. The length of wall is 9*m* and breadth is 6*m*. Find the cost of painting the wall, if the rate of painting the wall is ₹ 30 per m^2.



8. A door of dimensions 3*m* × 2*m* and a window of dimensions 2.5*m* × 1.5*m* is fitted in a wall. The length of the wall is 7.8*m* and breadth is 3.9m. Find the cost of painting the wall, if the rate of painting the wall is ₹ 25 per m^2.

9. Find the area and the perimeter of the following figures.





10. Multiple choice questions

(*i*) What is the area of a rectangle of dimensions 12*cm* × 10*cm* ?

(*a*)	$44cm^2$	(*b*)	$120cm^2$
(*c*)	$1200cm^2$	(*d*)	$1440cm^2$

(*ii*) Find the breadth of a rectangle whose length is 12*cm* and perimeter is 36*cm*.

(*a*)	6*cm*	(*b*)	3*cm*
(*c*)	9*cm*	(*d*)	12*cm*

(*iii*) If each side of a square is 1*m* then its area is ?

(*a*)	$10cm^2$	(*b*)	$100cm^2$
(*c*)	$1000cm^2$	(*d*)	$10000cm^2$

(*iv*) Find the area of a square whose perimeter is 96*cm*.

(*a*)	$576cm^2$	(*b*)	$626cm^2$
(*c*)	$726cm^2$	(*d*)	$748cm^2$

(*v*) The area of a rectangular sheet is $500 cm^2$. If the length of the sheet is $25 cm$, what is its breadth ?

(*a*) $30 cm$ (*b*) $40 cm$

(*c*) $20 cm$ (*d*) $25 cm$

(*vi*) What happens to the area of a square, if its side is doubled ?

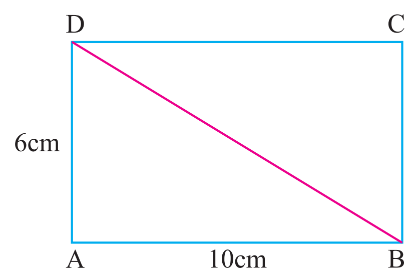
(*a*) The area becomes 4 times, the area of original square.

(*b*) The area becomes $\frac{1}{4}$ times, the area of original square.

(*c*) The are becomes 16 times, the area of orignal square.

(*d*) The area becomes $\frac{1}{6}$ times, the area of original square.

TRIANGLES AS PARTS OF A RECTANGLE



Consider a rectangle ABCD of length $10 cm$ and breadth $6 cm$. Now, if we draw a diagonal of the rectangle ; It divides the rectangle into two triangles and sum of the area of two triangles is equal to the area of rectangle.

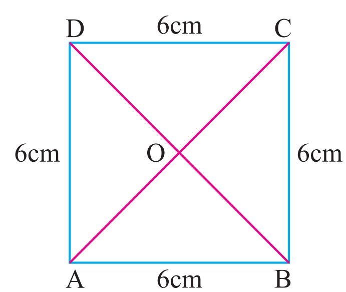
i.e; Area of ΔABD + Area of ΔBCD = Area of Rectangle ABCD

$$\therefore \quad \text{Area of each triangle} = \frac{1}{2} \times \text{area of rectangle}$$

$$= \frac{1}{2} \times \text{Length} \times \text{Breadth}$$

$$= \frac{1}{2} \times 10 \times 6$$

$$= 30 cm^2$$



On the other hand if we divide the rectangle into 4 triangles, then also the result is same i.e. sum of area of triangles is equal to the area of rectangle.

Let a Square ABCD of side $6 cm$ is divided into four triangles as shown in the figure. Then

Area of ΔAOB + Area of ΔBOC + Area of ΔCOD + area of ΔDOA = Area of square ABCD

$$\text{Area of each triangle} = \frac{1}{4} \times \text{Area of square}$$

$$= \frac{1}{4} \times (\text{Side})^2$$

$$= \frac{1}{4} \times 6 \times 6 = 9 cm^2$$

Example-1 : Estimate the area of the following figures by counting unit squares.

(*i*)

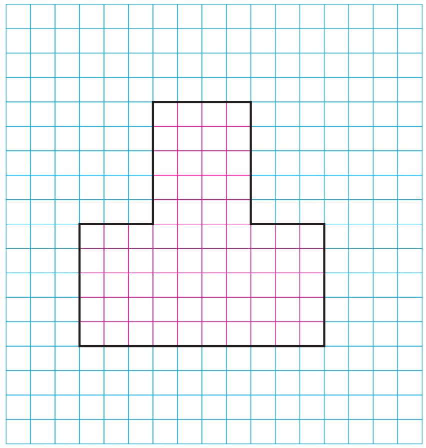


Figure 1

(*ii*)

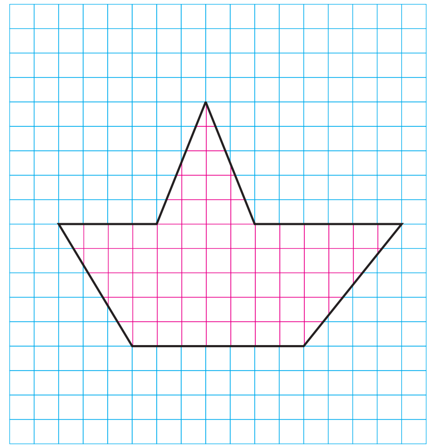


Figure 2

Sol. (*i*) In figure 1 no. of squares covered completely = 70

Area of 1 square = 1 sq. unit

Area of the figure = 70 sq. units.

(*ii*) No. of squares covered completely = 51

No. of squares covered half = 6

No. of squares to be considered in lieu of half squares = $\frac{1}{2} \times 6 = 3$

No. of squares covered more than half = 8

No. of squares to be considered in lieu of more than half = 8

No. of squares covered less than half = 9

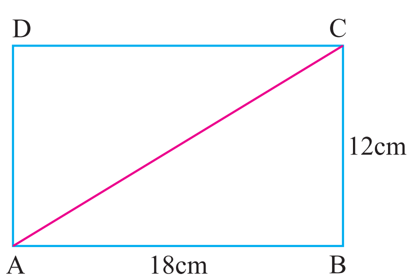
No. of squares to be considered in lieu of less than half = 0

Total no. of squares to be considered = 51 + 3 + 8 + 0 = 62

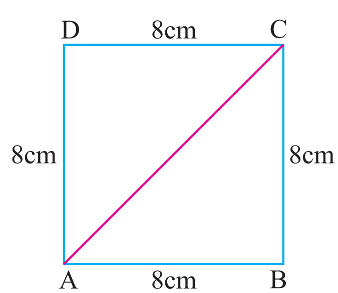
$\therefore$ Area of figure = 62 square units approx.

Example-2 : In- the following figures find the area of ΔABC

(*i*)



(*ii*)



Sol. (*i*) Given, Length of rectangle = $18cm$

Breadth of rectangle = $12cm$

AC the diagonal of rectangle ABCD, divides it into two equal triangles, ΔABC and ΔCDA

So, Area of $\Delta ABC = \frac{1}{2} \times$ Area of Rectangle ABCD

$= \frac{1}{2} \times \text{Length} \times \text{Breadth}$

$= \frac{1}{2} \times 18 \times 12$

$= 108 cm^2$

(*ii*) Given, side of a square = 8*cm*.

AC, the diagonal of square ABCD, divides it into two equal triangles, ΔABC and ΔCDA

So, Area of $\Delta ABC = \frac{1}{2} \times$ Area of square ABCD

$= \frac{1}{2} \times (\text{Side})^2$

$= \frac{1}{2} \times 8 \times 8$

$= 32\ cm^2$

GENERALISING FOR OTHER CONGRUENT PARTS OF RECTANGLES

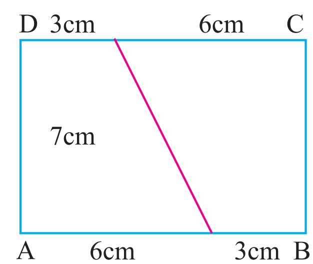
A rectangle of length 9*cm* and breadth 7*cm* is divided into two parts, congruent to each other.

$\therefore$ Area of each congruent part $= \frac{1}{2} \times$ Area of Rectangle ABCD

$= \frac{1}{2} \times \text{Length} \times \text{Breadth}$

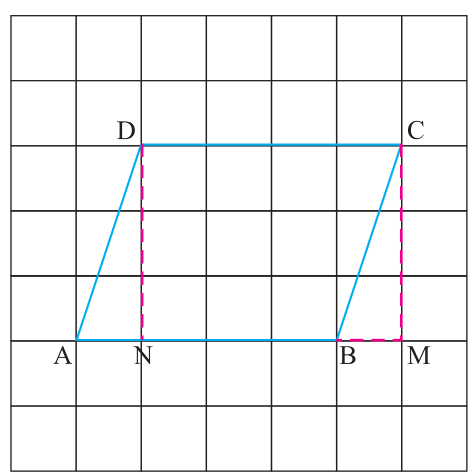
$= \frac{1}{2} \times 9 \times 7$

$= 31.5 cm^2$



AREA OF PARALLELOGRAM

A quadrilateral in which each pair of opposite sides is parallel and equal is called a parallelogram. In the figure, we see that the length of the rectangle DNMC is equal to the base of parallelogram ABCD and the breadth of the rectangle is equal to the height of parallelogram.



Area of parallelogram = Area of Rectangle

Area of parallelogram = Length × Breadth

Area of parallelogram = Base × height

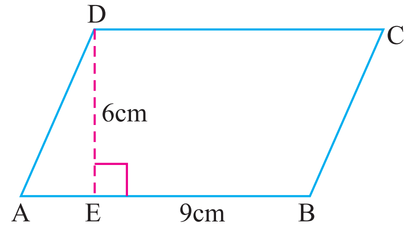
From above, we also have the following relatioship

$$\text{Base} = \frac{\text{Area of parallelogram}}{\text{Height}}$$

or $$\text{Height} = \frac{\text{Area of parallelogram}}{\text{Base}}$$

[Length of Rectangle = Base of Parallelogram
Breadth of Rectangle = height of Parallelogram]

Example-3 : Find the area of the following parallelogram.



Sol. Given,

Base of parallelogram $= 9cm$

Height of parallelogram $= 6cm$

$\therefore$ Area of parallelogram $=$ Base $\times$ height

$= 9 \times 6$

$= 54cm^2$

Example-4 : Find the height of parallelogram, if the area of parallelogram is $42cm^2$ and base is $6cm$.

Sol. Given,

Base of parallelogram $= 6cm$

Area of Parallelogram $= 42cm^2$

Base $\times$ height $= 42$

$6 \times$ height $= 42$

$$\text{height} = \frac{42}{6}$$

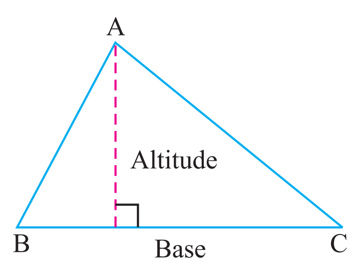
height $= 7cm$

$\therefore$ Height of parallelogram $= 7cm$

AREA OF A TRIANGLE

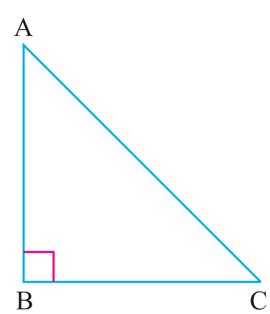
A closed plane figure made by joining three line segments is called a triangle.

(*i*) In a scalene triangle all the three sides have different lengths. If base and corresponding altitude are given.



Then area of $\Delta = \frac{1}{2} \times$ (base $\times$ height) sq. units

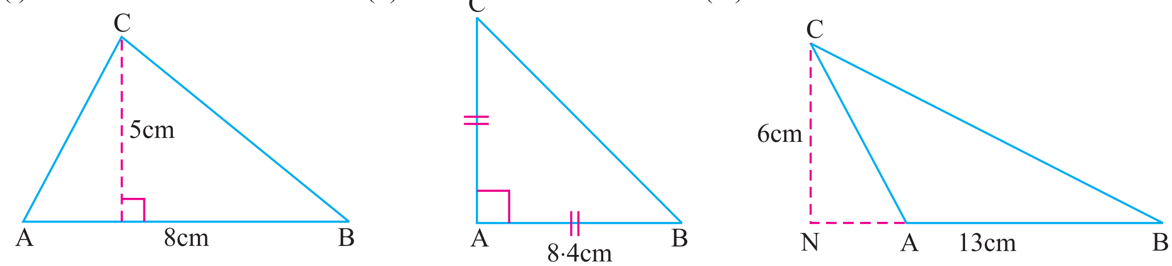
(*ii*) In a right angled triangle, the side opposite to the right angle is called the hypotenuse, the adjacent sides to right angle are called legs.



The area of right angled triangle $= \frac{1}{2} \times$ (base $\times$ height) sq unit.

Example-5 : Find the area of following triangles.

(*i*) (*ii*) (*iii*)



Sol. (*i*) Given,

$$\text{Base of triangle} = 8cm$$
$$\text{Height of triangle} = 5\text{cm}$$
$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 8 \times 5$$
$$= 20cm^2$$

(*ii*) In ΔCAB,

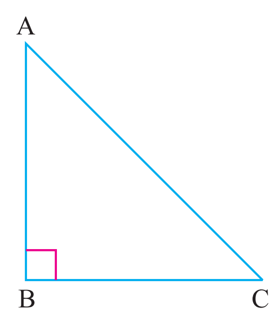
$$AB = AC$$
$$AB = 8.4cm$$
$$\therefore \quad AC = 8.4cm$$
$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 8.4 \times 8.4$$
$$= 35.28cm^2.$$

(*iii*)

$$\text{Base of triangle} = 13 \text{ cm}$$
$$\text{Height of triangle} = 6 \text{ cm}$$
$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 13 \times 6$$
$$= 39 \ cm^2.$$

Example-6 : Area of a right angled triangle is 108cm^2, if length of one leg is 9cm. Find the length of other leg.

Sol. Let ABC be the triangle right angled at B



$$\text{Let BC} = 9cm$$
$$AB = ?$$
$$\text{Area of Triangle} = 108cm^2$$
$$\frac{1}{2} \times \text{Base} \times \text{height} = 108$$
$$\frac{1}{2} \times 9 \times \text{height} = 108$$

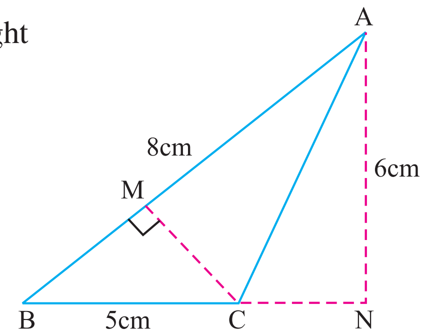
$$\text{height} = \frac{108 \times 2}{9}$$

$$\text{height} = 24cm$$

Example-7 : In ΔABC, BC = 5*cm* AN = 6*cm* and AB = 8*cm* find

(*i*) Area of ΔABC **(*ii*) Length of CM**

Sol. In ΔABC, BC = 5*cm* AN = 6*cm*



$$\text{Area of triangle ABC} = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times AN$$

$$= \frac{1}{2} \times 5 \times 6$$

$$= 15cm^2$$

(*ii*) In ΔABC, AB = 8*cm*

$$CM = ?$$

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$15 = \frac{1}{2} \times AB \times CM$$

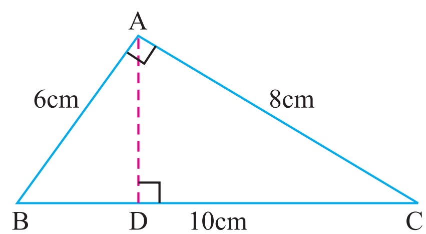
$$15 = \frac{1}{2} \times 8 \times CM$$

$$15 = 4 \times CM$$

$$CM = \frac{15}{4}$$

$$CM = 3.75cm$$

Example-8 : ΔABC is right angled at A as shown in figure. AD is prependicular to BC. If AB = 6*cm*, BC = 10*cm* and AC = 8*cm*. Find area of ΔABC. Also, find the length of AD.



Sol. Given ΔABC is right angled at A, AB = 6*cm*, BC = 10*cm* and AC = 8*cm*.
On taking AC as a base and AB as height we get

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times AC \times AB = \frac{1}{2} \times 8 \times 6$$

$$= 24cm^2$$

Also, AD is prependicular to BC

Now, on taking BC as base and AD as height, We get

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AD$$

$$24 = \frac{1}{2} \times 10 \times AD$$

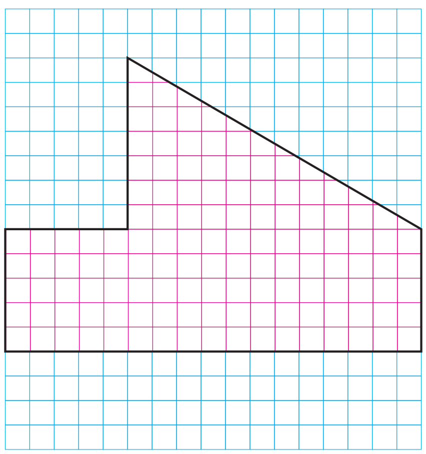
$$24 = 5 \times AD$$

$$AD = \frac{24}{5} = 4.8\,cm.$$

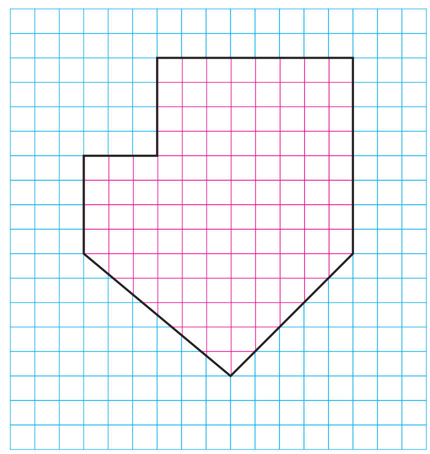
EXERCISE - 11.2

1. Estimate the area of the following figures by counting unit squares.

(*i*)

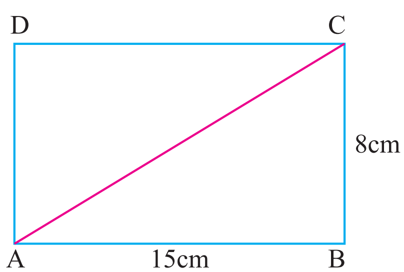


(*ii*)

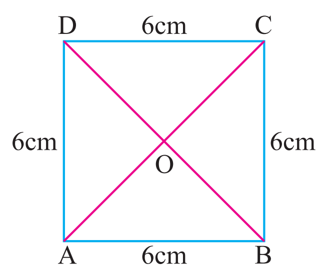


2. In the following figures find the area of

(*i*) ΔABC

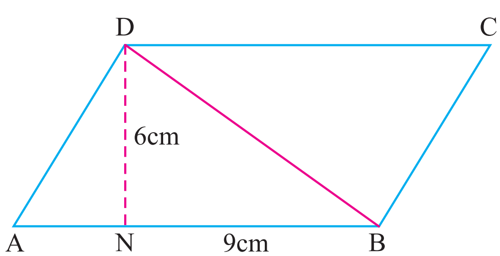


(*ii*) ΔCOD

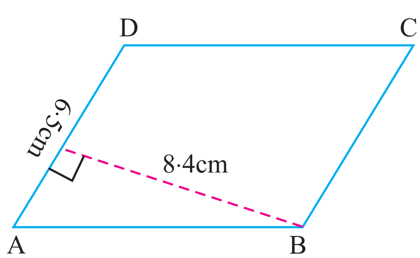


3. Find the area of following parallelograms.

(*i*)

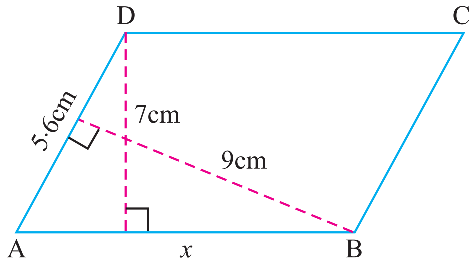


(*ii*)

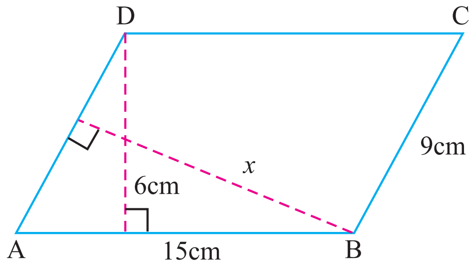


4. Find the value of x in the following parallelograms

(*i*)

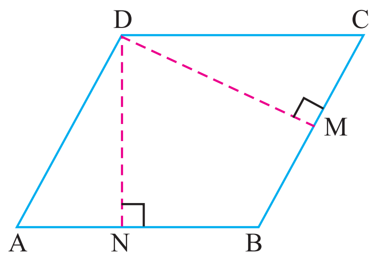


(*ii*)



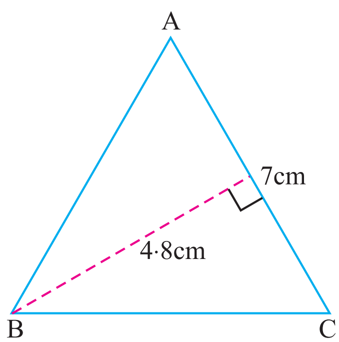
5. The adjacent sides of a parallelogram are 28*cm* and 45*cm* and the altitude on longer side is 18*cm*. Find the area of parallelogram.

6. ABCD is a parallelogram given in figure. DN and DM are the altitudes on side AB and CB respectively. If area of the the parallelogram is 1225cm^2, AB = 35*cm* and CB = 25*cm,* find DN and DM.

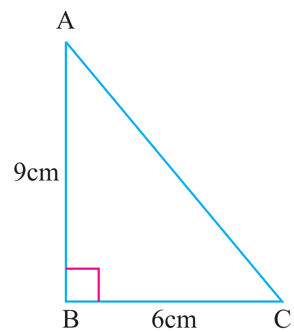


7. Find the area of the following triangles

(*i*)

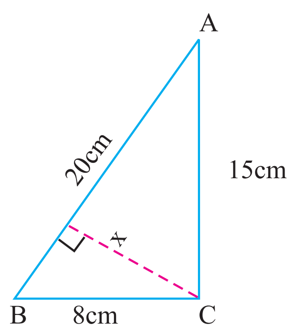


(*ii*)

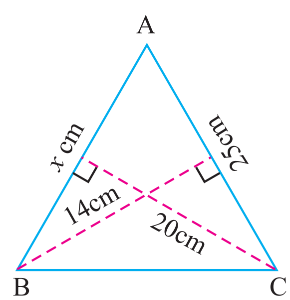


8. Find the value of x in the following triangles.

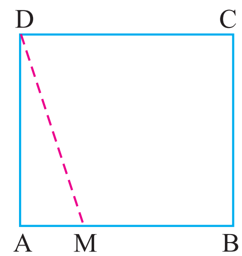
(*i*)



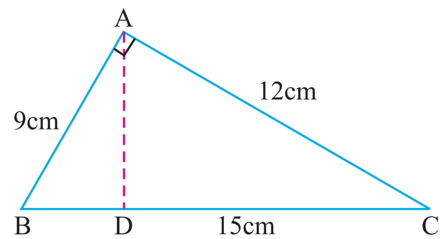
(*ii*)



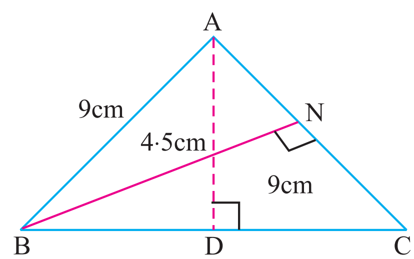
9. ABCD is a square, M is a point on AB such that AM = 9*cm* and area of ΔDAM is 171cm^2. What is the area of the square ?



10. ΔABC is right angled at A as shown in figure. AD is prependicular to BC, if AB = 9*cm*, BC = 15*cm* and AC = 12*cm*. Find the area of ΔABC, also find the length of AD.



11. ΔABC is isosceles with AB = AC = 9*cm*, BC = 12*cm* and the height AD from A to BC is 4.5*cm*. Find the area of ΔABC. What will be the height from B to AC i.e BN ?



12. Multiple choice questions :-

(*i*) Find the height of a parallelogram whose area is 246cm^2 and base is 20*cm*.

(*a*) 1.23cm^2 (*b*) 13.2cm^2

(*c*) 12.3cm^2 (*d*) 1.32cm^2

(*ii*) One of the side and the corresponding height of *a* parallelogram are 7*cm* and 3.5*cm* respectively. Find the area of the parallelogram.

(*a*) 21cm^2 (*b*) 24.5cm^2

(*c*) 21.5cm^2 (*d*) 24cm^2

(*iii*) The height of a triangle whose base is 13*cm* and area is 65cm^2 is.

(*a*) 12*cm* (*b*) 15*cm*

(*c*) 10*cm* (*d*) 20*cm*

(*iv*) Find the area of an isosceles right angled triangle, whose equal sides are of length 40*cm* each.

(*a*) 400cm^2 (*b*) 200cm^2

(*c*) 600cm^2 (*d*) 800cm^2

(*v*) If the sides of a parallelogram are increased to twice of its original length, how much will be the perimeter of the new parallelogram.

(*a*) 1.5 times (*b*) 2 times

(*c*) 3 times (*d*) 4 times

(*vi*) In a right angled triangle one leg is double the other and area is $64cm^2$ find the smaller leg.

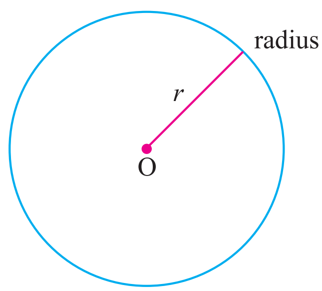
(*a*) 8*cm* (*b*) 16*cm*

(*c*) 24*cm* (*d*) 32*cm*

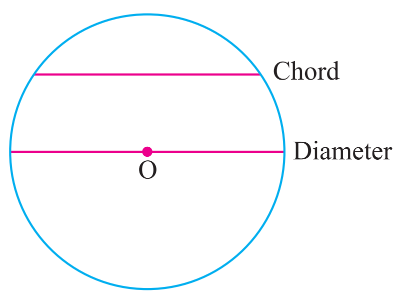
CIRCLE

A circle is a simple closed curve consisting of all those points in a plane each of which is at a constant distance from a fixed point O inside it. Let this constant distance be *r*.

- The fixed point O is called the **centre of the circle.**
- Line segment joining any point on the circle to its centre is called the **radius of the Circle.** It is denoted by *r*.



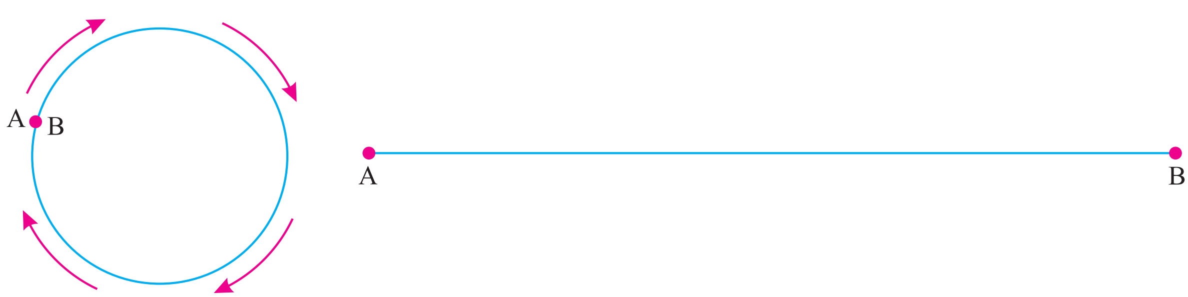
- A line segment joining any two points of a circle is called the **chord** of the circle.
- A chord passing through the centre of the circle is known as the **diameter of circle.** It is denoted by *d*.



Circumference of circle : The perimeter of a circle is called its circumference.

Circumference of a circle = Measure of the boundary of a circle.

To measure the boundary of a circle, we can put a piece of thread along its boundary and then straighten it to measure its length as shown in figure.



LAB ACTIVITY TO FIND THE VALUE OF Pi (π) AND FORMULA OF CIRCUMFERENCE

Aim : To find the value of Pi (π)

Material required : (*i*) Paper (*ii*) Thread (*iii*) Scissors (*iv*) Geometry box

Method : Draw six circles of different radii say 1cm, 2cm, 3cm, 4cm, 5cm and 6cm. Measure their circumference using thread. Now straighten the thread and measure its length.

Sr. No	Radius (in cm)	Diameter (in cm)	Circumference in cm (i.e., length of thread)	Value of $\pi = \frac{\text{Circumference}}{\text{Diameter}}$
1	1	2	6.3	3.15
2	2	4	12.5	3.125
3	3	6	18.8	3.133
4	4	8	25.1	3.14
5	5	10	31.4	3.14
6	6	12	37.6	3.133

From the above table we observe that the ratio of circumference to its diameter is constant and denoted by *pi* (π). The approximated value π is 3.14

From the above activity, we conclude that

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

$$\pi = \frac{C}{d}$$

$$C = \pi d$$

$$C = \pi(2r) \quad [\because \text{ Diameter is double the radius of circle}]$$

$$C = 2\pi r$$

$$\therefore \quad \text{circumference of circle} = 2\pi r$$

$$\text{circumference of semi circle} = \frac{1}{2} \times 2\pi r = \pi r$$

Example-1 : Find the circumference of circle whose diameter is 12cm (Take $\pi = 3.14$)

Sol.

$$\text{Diameter of circle } d = 12cm$$

$$\text{circumference of circle} = \pi d$$

$$= 3.14 \times 12$$

$$= 37.68cm$$

Example-2 : Circumference of circle is 88 *cm* find the radius of circle (Take $\pi = \frac{22}{7}$)

Sol.

$$\text{Circumference of circle} = 88 \ cm$$

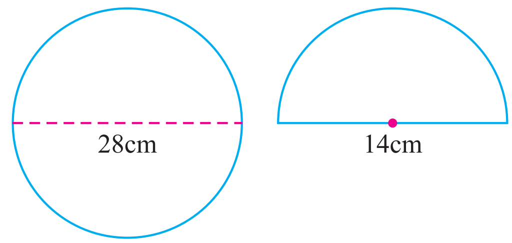
$$2\pi r = 88$$

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{2 \times 22}$$

$$r = 14cm$$

Example-3 : A circular disc of diameter 28cm is divided into two parts. What is the perimeter of each semi circular disc ?



Sol. Diameter of circular disc $= 28cm$

Radius of circular disc $= 14cm$

Circumference of semicircle $= \pi r$

$= \frac{22}{7} \times 14$

$= 44cm$

Perimeter of semi circular disc = Circumference of semi circle + length of diameter

$= 44 + 28$

$= 72cm$

Example-4 : A gardener wants to fence a circular garden of diameter 28*m*. Find the length of barbed wire if he makes 2 rounds of fence.

Sol. Diameter of circular garden $= 28m$

Circumference of garden $= \pi d$

$= \frac{22}{7} \times 28$

$= 88m$

Length of barbed wire needed to make one round of fence = 88m

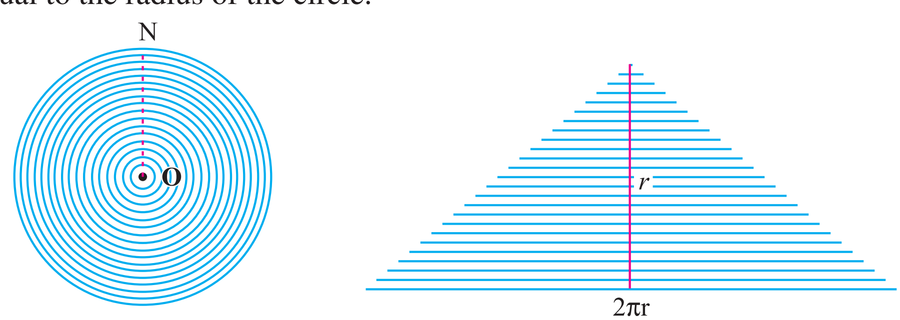
Length of barbed wire needed to make two round of fence = $88 \times 2 = 176m$

LAB ACTIVITY TO FIND AREA OF CIRCLE

Aim : To find the area of circle.

Material required : (*i*) wool of different colours (*ii*) scissor (*iii*) Fevicol (*iv*) coloured pens (*v*) geometery box

Method : Draw a circle of any radius. Fill up this circle with concentric circles with the help of wool. Without leaving any gap from the centre O cut off all the pieces of wool along ON and now arrange them starting from the outermost circular piece (as shown in figure). It will take the shape of a triangle whose base is equal to the circumference of the outermost circle and height is equal to the radius of the circle.



$$\text{Base of Triangle} = 2\pi r$$
$$\text{height of triangle} = r$$
$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 2\pi r \times r$$
$$\text{Area of triangle} = \pi r^2 \text{ sq. unit}$$
$$\text{Hence area of circle} = \pi r^2 \; sq. \text{ unit}$$

Example-5 : Find the area of circle whose radius is 21cm (Take $\pi = \frac{22}{7}$)

Sol.

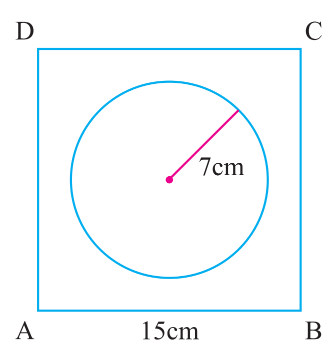
$$\text{Radius of circle} = 21cm$$
$$\text{Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 21 \times 21$$
$$= 1386\ cm^2$$

Example-6 : Find the area of circle whose circumference is 88*cm*.

Sol.

$$\text{Circumference of circle} = 88cm$$
$$2\pi r = 88$$
$$\pi r = \frac{88}{2}$$
$$\frac{22}{7} \times r = 44$$
$$r = \frac{44 \times 7}{22}$$
$$r = 14cm$$
$$\text{Then Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 14 \times 14$$
$$= 616 \text{ cm}^2$$

Example-7 : From a square sheet of side 15 cm *a* circular sheet of radius 7*cm* is removed. Find the area of remaining sheet (Take $\pi = \frac{22}{7}$)



Sol.

$$\text{Side of square sheet} = 15cm$$
$$\text{Area of square sheet} = (\text{side})^2$$
$$= 15 \times 15$$
$$= 225 \text{ cm}^2$$
$$\text{Radius of removed circle} = 7cm$$
$$\text{Area of removed circle} = \pi r^2$$
$$= \frac{22}{7} \times 7 \times 7$$
$$= 154 cm^2$$

Area of Remaining sheet = Area of square sheet – Area of removed circular sheet

$= 225 - 154$

$= 71 cm^2$

EXERCISE - 11.3

1. Find the circumference of circle whose

(*i*) Radius (r) = 21*cm* (*ii*) Radius (r) = 3.5*cm*

(*iii*) Diameter = 84*cm*

2. If the circumference of a circular sheet is 176*m*, find its radius.

3. A circular disc of diameter 8.4*cm* is divided into two parts what is the perimeter of each semicircular part ?

4. Find the area of the circle having

(*i*) Radius r = 49*cm* (*ii*) Radius r = 2.8*cm*

(*iii*) Diameter = 4.2*cm*

5. A gardener wants to fence a circular garden of radius 15*m*. Find the length of wire, if he makes three rounds of fence. Also, find the cost of wire if it costs ₹ 5 per meter (Take $\pi = 3.14$)

6. Which of the following has larger area and by how much ?

(*a*) Rectangle with length 15*cm* and breadth 5.4*cm*

(*b*) Circle of diameter 5.6*cm*.

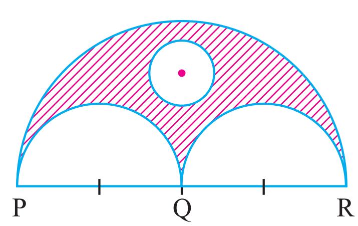
7. From a rectangular sheet of length 15*cm* and breadth 12*cm* a circle of radius 3.5*cm* is removed. Find the area of remaining sheet.

8. From a circular sheet of radius 7*cm*, a circle of radius 2.1*cm* is removed, find the area of remaining sheet.

9. Smeep took a wire of length 88*cm* and bent it into the shape of a circle, find the radius and area of the circle. If the same wire is bent into a square, what will be the side of the square? Which figure encloses more area ?

10. A garden is 120*m* long and 85*m* broad. Inside the garden, there is a circular pit of diameter 14*m*. Find the cost of planting the remaining part of the garden at the rate of ₹5.50 per square meter.

11. In the figure PQ = QR and PR = 56*cm*. The radius of inscribed circle is 7*cm*. Q is centre of semi circle. What is the area of shaded region ?



12. The minute hand of a circular clock is 18*cm* long. How far does the tip of minute hand move in one hour ?

13. Multiple choice questions :-

(*i*) The circumference of a circle of diameter 10*cm* is

(*a*) 31.4*cm* (*b*) 3.14*cm*

(*c*) 314*cm* (*d*) 35.4*cm*

(*ii*) The circumference of a circle with radius 14*cm* is

(*a*) 88*cm* (*b*) 44*cm*

(*c*) 22*cm* (*d*) 85*cm*

(*iii*) What is the area of the circle of radius 7*cm* ?

(*a*) 49*cm* (*b*) $22cm^2$

(*c*) $154cm^2$ (*d*) $308cm^2$

(*iv*) Find the diameter of a circle whose area is $154cm^2$.

(*a*) 4*cm* (*b*) 6*cm*

(*c*) 14*cm* (*d*) 12*cm*

(*v*) A circle has area 100 times the area of another circle. What is the ratio of their cirumferences ?

(*a*) 10 : 1 (*b*) 1 : 10

(*c*) 1 : 1 (*d*) 2 : 1

(*vi*) Diameter of a circular garden is 9.8*cm*. Which of the following is its area ?

(*a*) $75.46cm^2$ (*b*) $76.46cm^2$

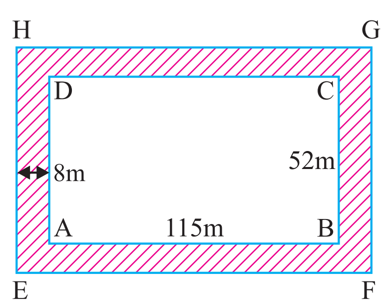
(*c*) $74.4cm^2$ (*d*) $76.4cm^2$

Conversion of units :-

Lenght units	Area Units
1*cm* = 10*mm*	1 cm^2 = (10 × 10) mm^2 = 100 mm^2
1dm = 10cm	1 dm^2 = (10 × 10) cm^2 = 100 cm^2
1m = 10dm	1 m^2 = (10 × 10) dm^2 = 100 dm^2
1m = 100cm	1 m^2 = (100 × 100) cm^2 = 10000 cm^2
1 dam = 10m	1 dam^2 = (10 × 10) m^2 = 100 m^2
1 hm = 100m	1 hm^2 = (100 × 100) m^2 = 10000 m^2
1km = 1000m	1 km^2 = (1000 × 1000) m^2 = 1000000 m^2
	1 are = 100 m^2
	1 hectare = 10000 m^2

AREA OF PATHS, CROSS ROADS AND BORDERS

Example-1 : **A retangular garden is 115*m* long and 52*m* broad. A path of uniform width of 8*m* has to be constructed around it, on its outside. Find the cost of gravelling the path at 10.50 per m^2.**



Sol. Let ABCD represents the rectangular garden and the shaded region represents the path of width 8*m* around the garden.

Length of rectangular garden $l = 115m$

Breadth of rectangular garden $b = 52m$

Area of rectangular garden ABCD = $(115 \times 52)m^2$

$= 5980m^2$

Length of rectangular garden including path = $115m + (8m + 8m) = 131m$

Breadth of rectangular garden including path = $52m + (8m + 8m) = 68m$

Area of garden including path $= (131 \times 68)\ m^2 = 8908\ m^2$

Area of path = Area of garden including path – Area of garden

Area of path $= (8908 - 5980)m^2$

$= 2928m^2$

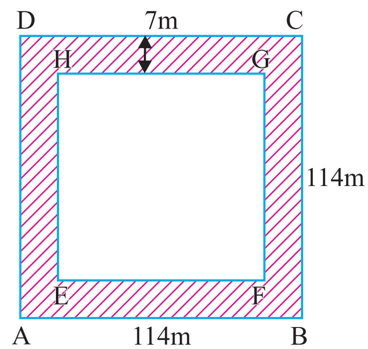
Cost of gravelling $1m^2$ of path = ₹ 10.50

Cost of gravelling $2928m^2$ of path = 2928×10.50

= ₹ 30744

Example-2 : A path of 7*m* wide runs along inside a square park of side 114*m*. Find the area of the path. Also, find the cost of cementing it at the rate of ₹225 per 15m^2

Sol. Let ABCD be the square park of side 114m and the shaded region represents the path 7m wide



EF $= 114m - (7 + 7)m$

$= 100m$

Area of square park ABCD = $(\text{Side})^2$

$= 114 \times 114$

$= 12996m^2$

Area of EFGH = $(\text{Side})^2$

$= 100 \times 100$

$= 10000m^2$

Area of path = Area of square park ABCD – Area of EFGH

$= (12996 - 10000)\ m^2$

$= 2996m^2$

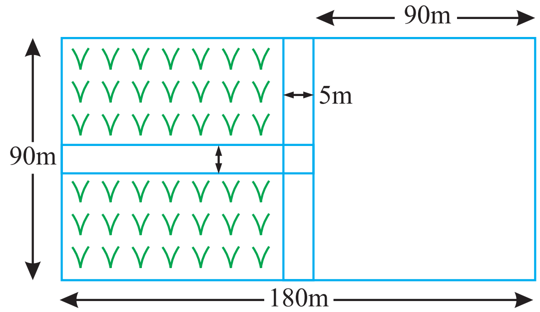
Cost of cementing $15m^2$ = ₹225

Cost of cementing $1m^2 = \dfrac{225}{15}$

Cost of cementing $2996m^2 = \dfrac{225}{15} \times 2996$

= ₹44940

Example-3 : A School ground is 180*m* long and 90*m* wide. An area of 90*m* × 90*m* is kept for morning assembly. In the remaining portion there is 5*m* wide path parallel to its width and parallel to its remaining length (as shown in figure). The remaining area is covered by grass. Find the area covered by grass.



Sol. Area of school ground = $180m \times 90m$

= $16200\ m^2$

Area kept for morning assembly = 90×90

= $8100\ m^2$

Area of path parallel to width of ground = 90×5

= $450\ m^2$

Area of path parallel to remaining length of ground = $90 \times 5 = 450\ m^2$

Area common to both parths = 5×5

= $25m^2$

Total area covered by path = $(450 + 450 - 25)$

= $875m^2$

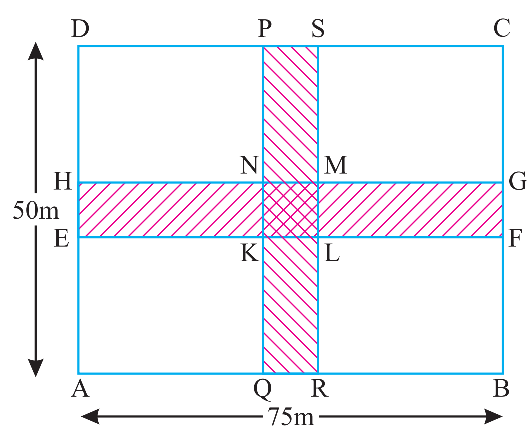
Area covered by grass = Area of ground – (Area kept for morning assembly + area covered by paths)

= $16200 - (8100 + 875)$

= $16200 - 8975$

= $7225m^2$

Example-4 : Two crossroads each of width 6m, runs at right angle through the centre of rectangular park of length 75m and breadth 50*m* and parallel to its sides. Find the area of roads. Also find the cost of constructing the roads at the rate of ₹120 per m^2.



Sol. ABCD represent the rectangular park of length AB = 75m and breadth BC = 50m. Area of shaded portion i.e, area of rectangle EFGH and PQRS represent the area of cross roads, but the area of square KLMN is taken twice, So it will be subtracted

Now EF = $75m$, FG = $6m$, PQ = $50m$, QR = $6m$, KL = 6m. Area covered by road = Area of rectangle EFGH + area of rectangle PQRS – Area of square KLMN

= $(EF \times FG) + (PQ \times QR) - (KL)^2$

= $(75 \times 6) + (50 \times 6) - (6 \times 6)$

$= 450 + 300 - 36$

$= 714m^2$

Cost of constructing $1m^2$ of road = ₹ 120

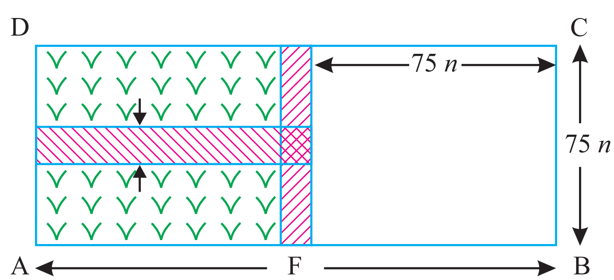
Cost of constructing $714m^2$ of road = 714×120

= ₹ 85680

Hence, the cost of constructing the road is ₹ 85680

EXERCISE - 11.4

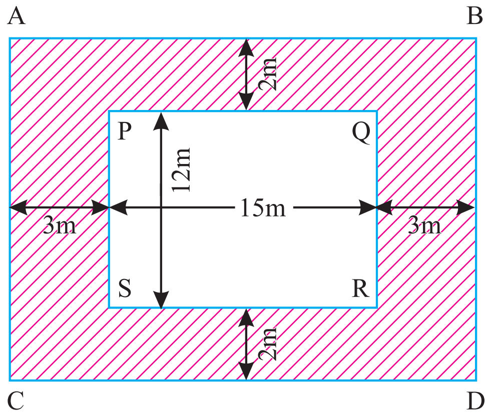
1. A rectangular park is 80*m* long and 65*m* wide. A path of 5*m* width is constructed outside the park. Find the area of path.
2. A rectangular garden is 110*m* long and 72*m* broad. A path of uniform width 8*m* has to be constructed around it. Find the cost of gravelling the path at ₹11.50 per m^2.
3. A room is 12*m* long and 8*m* broad. It is surrounded by a verandah, which is 3*m* wide all around it. Find the cost of flooring the verandah with marble at ₹275 per m^2.
4. A sheet of paper measures 30*cm* × 24*cm*. A strip of 4*cm* width is cut from it, all around. Find the area of remaining sheet and also the area of cut out strip.
5. A path of 2*m* wide is built along the border inside a square garden of side 40*m*. Find
   (*i*) The Area of path.
   (*ii*) The cost of planting grass in the remaining portion of the garden at the rate of ₹50 per m^2.
6. A nursery school play ground is 150*m* long and 75*m* wide. A portion of 75*m* × 75*m* is kept for see-saw slides and other park equipments. In the remaining portion 3*m* wide path parallel to its width and parallel to remaing length (as shown in fig). The remaining area is covered by grass. Find the area covered by grass.



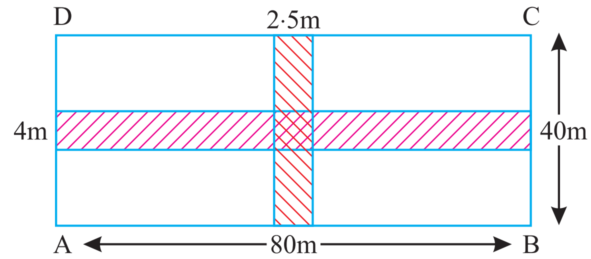
7. Two cross roads each of width 8*m* cut at right angle through the centre of a rectangular park of length 480m and breadth 250*m* and parallel to its sides. Find the area of roads. Also, find the area of park excluding cross roads.
8. In a rectangular field of length 92*m* and breadth 70*m*, two roads are constructed which are parallel to the sides and cut each other at right angles through the centre of field. If the width of each road is 4*m*, find.
   (*i*) The area covered by roads.
   (*ii*) The cost of constructing the roads at the rate of ₹150 per m^2.

9. Find the area of shaded region in each of the following figures.

(*i*)



(*ii*)



WHAT HAVE WE DISCUSSED ?

1. For rectangle and square

(*i*) Perimeter of a rectangle = 2 (Length + Breadth) units

(*ii*) Area of a rectangle = (Length × Breadth) sq. units

(*iii*) Length = $\frac{\text{Area}}{\text{Breadth}}$ and Breadth = $\frac{\text{Area}}{\text{Length}}$

(*iv*) Perimeter of a square = (4 × side) units

(*v*) Area of a square = $(\text{side})^2$ sq units

(*vi*) Side of a square = $\sqrt{\text{Area}}$ units

2. For a parallelogram

(*i*) Area of a parallelogram = (Base × Height) sq. units

(*ii*) Base = $\frac{\text{Area of parallelogram}}{\text{Height}}$ and Height = $\frac{\text{Area of parallelogram}}{\text{Base}}$

3. For a triangle

(*i*) Area of a triangle = $\frac{1}{2}$ × (base × height) sq. units

4. For a circle

(*i*) Circumference of a circle = πd unit or $2\pi r$ units

(*ii*) Area of a circle = πr^2 sq. units.

LEARNING OUTCOMES

After completion of the chapter the students are now be able to :

1. Differentiate between perimeter and area of plane figures.
2. Find out approximate area of closed shapes by using unit square grid/graph sheet.
3. Calculate the perimeter and area of plane figures viz. square, rectangle, triangle, parallelogram.
4. Find the circumference and area of circle using formulae.
5. Convert various units of area and perimeter, wherever necessary.
6. Apply the formulae learnt to solve the problems related to their day to day life.

ANSWERS

EXERCISE 11.1

1. (*i*) $86cm$; $420cm^2$ (*ii*) $23.8cm$; $23.5cm^2$

2. (*i*) $116cm$; $841cm^2$ (*ii*) $33.2cm$; $68.89cm^2$

3. $1369m^2$ **4.** $20cm$; $98cm$

5. $40cm$; Square encloses more area ; $64cm^2$

6. $45m$; $340m$ **7.** ₹1507.50 **8.** ₹516.75

9. (*i*) $38cm$; $52cm^2$ (*ii*) $29cm$; $19.5cm^2$

10. (*i*) *b* (*ii*) *a* (*iii*) *d*
(*iv*) *a* (*v*) *c* (*vi*) *a*

EXERCISE 11.2

2. (*i*) 135 sq. units approx. (*ii*) 114 sq. units approx.

2. (*i*) $60m^2$ (*ii*) $9cm^2$

3. (*i*) $54cm^2$ (*ii*) $54.6cm^2$

4. (*i*) $7.2cm$ (*ii*) $10cm$

5. $810cm^2$ **6.** $35cm$; $49cm$

7. (*i*) $16.8cm^2$ (*iii*) $27cm^2$

8. (*i*) $6cm$ (*ii*) $17.5cm$

9. $324cm^2$ **10.** $54cm^2$; $7.2cm$

11. $27cm^2$; $6cm$

12. (*i*) *c* (*ii*) *b* (*iii*) *c*
(*iv*) *d* (*v*) *b* (*vi*) *a*

EXERCISE 11.3

1. (*i*) $132cm$ (*ii*) $22cm$ (*iii*) $264cm$

2. $28m$ **3.** $21.6cm$

4. (*i*) $7546cm^2$ (*ii*) $24.64cm^2$ (*iii*) $13.86cm^2$

5. $282.6\ m$; ₹1413 **6.** Rectangle has more area ; $56.36cm^2$

7. $141.5cm^2$ **8.** $140.14cm^2$

9. $14cm$; $616cm^2$; $22cm$; circle encloses more area **10.** ₹55243

11. $462cm^2$ **12.** $113.04cm$

13. (*i*) *a* (*ii*) *a* (*iii*) *c*
(*iv*) C (*v*) *a* (*vi*) *a*

EXERCISE 11.4

1.	$1550m^2$	**2.**	₹36432	**3.**	₹42900
4.	$352cm^2$; $368cm^2$	**5.**	(*i*) $304m^2$	(*ii*)	64800
6.	$5184m^2$	**7.**	$5776m^2$; $114224m^2$		
8.	(*i*) $632m^2$	(*ii*)	₹94800		
9.	(*i*) $156m^2$	(*ii*)	$410m^2$		

CHAPTER 12

T1W4G3

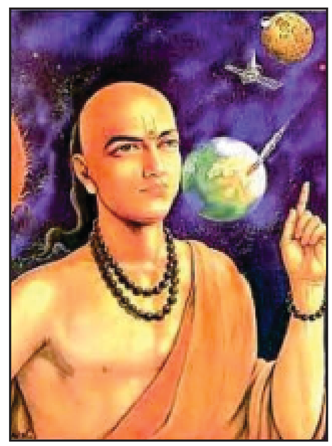
Algebraic Expressions

Learning Objectives :-

In this chapter, you will learn :-

1. To identify terms related to algebra like constant, variable, terms, coefficient of terms.
2. To generate algebraic expression with one or two variables.
3. About addition and subtraction of algebraic expressions.
4. To find the value of an expression for a given value of variable.
5. To apply your knowledge of algebraic expressions in your daily life.

OUR NATIONS' PRIDE



Bhaskaracharya was an Indian mathematician and astronomer. He was born in Bijapur in Karnatka. His works represent a significant contribution to mathematical and astronomical knowledge in the 12th century. He has been called the great mathematican of medieval India. His main work is divided into four parts called Lilavati, Bijaganita, Grahganita and Goladhyaya. These four sections deal with Arithmetic, Algebra, mathematics of the planets and sphere respectively. On 20 Nov. 1981, the Indian space research organisation launched the Bhaskara II satellite honouring the great mathematican and astronomer.

INTRODUCTION

In class VI, you have already learnt about simple algebraic expressions like $x + 5$, $2x - y$, $3x + y$, $2y - 7$ etc and we have seen the use of such expressions in forming words problems and simple equations.

In this chapter, we shall learn more about algebraic expressions. We shall study "How are Algebraic expressions formed" Factors of a term, coefficient of a term, like and unlike terms, types of polynomials. We shall learn to find the value of an expression for a particular value of the variable.

Algebraic Expression : Before proceeding further let us revise some definition related to algebraic expressions.

1. **Constant :** Constant is a term that has a fixed value. Some examples of a constant are $3, 5, 0, -7, \frac{-2}{3}, \sqrt{3}$ etc.
   - Infact every number is a constant.

2. Variable : Variable is a term that does not have a fixed value. We use letters of english alphabets for variables. For example x, y, z, s, t etc.

Let us assume any number less than 3. It may be –10, –7, –6, –3, –1, 0, 1, 2 and many more. Therefore, when we think any number less than 3. We observe that we do not have single fixed number less than 3 we will write $x < 3$.

Where x may have varying value. Which is less than 3. $\therefore$ x is variable.

3. Term : A term is a number (constant), a variable or a combination (Product or quotient) of numbers and variables. For example

$$7, y, 5b, xy, \frac{-3x}{2y}, \frac{7m}{8}, \frac{5}{t} \text{ etc.}$$

Algebraic expressions : A combination of one or more terms, which are seperated by addition, subtraction is called on Algebraic expression. For example. $4 + 10x$, $5x - 7y$, $3a + 7b$, $ax + by - cz$ etc.

- Only (–) minus and (+) plus signs seperate the terms. Where as the division and product do not seperate the terms.

Factors : The terms are made of the product of factors. For example the term $2xy$ of expression $2xy + 7z$ has three factors 2, x and y and the term 7z has 7 and 'z' two factors and expression $2xy + 7z$ has two terms.

Coefficient : Any of the factors of a term is called the coefficient of the product of all the remaining factors. In particular, the constant part is called the "numerical coefficient" and the remaining part is called the 'Literal coefficient" of term.

For example : Consider the expression.

$3x^2y + 7xy - 8$

In the term $3x^2y$ Numerical coefficient = 3

Coefficient of y = $3x^2$

Coefficient of x^2 = $3y$

Coefficient of x = $3xy$

Simlarly : In the term $7xy$

Numerical coefficient = 7

Coefficient of x = $7y$

Coefficient of y = $7x$

Example-1 : Write the terms, factors and numerical coefficient for the following expressions.

(a) $\boldsymbol{xy - x}$ **(b)** $\boldsymbol{17xy + 3}$

(c) $\boldsymbol{30\,x^2yz + 70x}$ **(d)** $\boldsymbol{10m^2n + 3pq + 17z}$

Sol.

Algebraic Expressiosn	Terms	Factors	Numerical Coefficient
(a) $xy - x$	xy $-x$	x, y x	1 –1
(b) $17xy + 3$	$17xy$ 3	17, x, y 3	17 3

(*c*) $30\,x^2yz + 70x$	$30\,x^2yz$ $70\,x$	$30, x, x, y, z$ $70, x$	30 70
(*d*) $10\,m^2n + 3pq + 17z$	$10m^2n$ $3pq$ $17z$	$10, m, m, n$ $3, p, q$ $17, z$	10 3 17

Like Terms : The terms having same variable factors are called like terms. For example $3x^2y$ and $-7x^2y$, $2xyz$ and $7xyz$, $-3x^2yz^2$ and $2x^2yz^2$ etc.

Note : Like term may have different numerical coefficient, but same literal coefficient.

Unlike terms : The terms having different variable factors are called unlike terms. For example xy^2 and xyz, x^2y^2z and xyz^2, $3x^2$ and $3y^2$ etc.

Example-2 : Group the like terms.

(*a*) $2xy, 3x^2, -7x^2, 3xyz$ and $7xy$

(*b*) $7x^2yz, 3x^2y^2, 2xy^2, -8x^2y^2$

Sol. (*a*) Terms having xy as variable factor $2xy$, $7xy$.

∴ $2xy$, $7xy$ are like terms.

Terms having x^2 as varibale factor are

$3x^2, -7x^2$

∴ $3x^2, -7x^2$ are like terms

(*b*) Terms having x^2y^2 as variable factor are

$3x^2y^2, -8x^2y^2$

∴ $3x^2y^2$ and $-8x^2y^2$ are like terms.

Example-3 : State whether the given pair of terms is of like or unlike term.

(*a*) $10\,p^2q$ and $10\,pq^2$

(*b*) $7xy^2$ and $-3xy^2$

Sol. (*a*) $10p^2q$ and $10pq^2$

Variable coefficient in $10\,p^2q = p^2q$

Variable coefficient in $10\,pq^2 = pq^2$

∴ $10p^2q$ and $10pq^2$ are unlike terms.

(*b*) $7xy^2$ and $-3xy^2$

Variable coefficient in $7xy^2 = xy^2$

Variable coefficient in $-3xy^2 = xy^2$

∴ $7xy^2$ and $-3xy^2$ are like terms.

TYPES OF ALGEBRAIC EXPRESSIONS

Number of Terms	Name of Expression	Examples
One	Monomial	$x, 2y, \frac{5z}{3}, -\frac{7x^2}{9}$ etc
Two	Binomial	$x + 9$, $3x - 2y$ $3x^2 - z^2$
Three	Trinomial	$x + y + z$, $p^2 + q^2 + r^2$, $pq + r + t^2$
Two or more than two terms	Polynomial	x, $3x + 2y$, $p + q + z$, $x^2 + y^2 + zx$

- Every binomial, every trinomial, is a polynomial.

Example-4 : Classify the following algebraic expression as monomial, binomial or trinomial

(*a*) $xy + x - z$ (*b*) $16pqr$ (*c*) $m^2 + n^2$ (*d*) $\frac{7x}{2} + \frac{3y}{5}$

Sol. (*a*) Algebraic expression is $xy + x - z$

Number of terms = 3

$\therefore$ It is a trinomial

(*b*) Algebraic expression is $16\,pqr$

Number of terms = 1

$\therefore$ It is a monomial

(*c*) Algebraic expression is $m^2 + n^2$

Number of terms = 2

$\therefore$ It is a binomial

(*d*) Algebraic expresions is $\frac{7x}{2} + \frac{3y}{5}$

Number of terms = 2

$\therefore$ It is a binomial

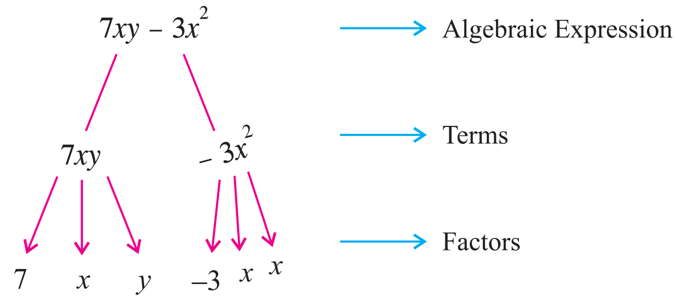
Tree diagram : It is a diagrammatic way of representing terms and factors of an algebraic expression. The terms and the factors of each terms of an algebraic expression can be shown by a tree diagram as shown in the following examples.

Example-5 : Identify the terms and their factors in the following expressions by tree diagrams.

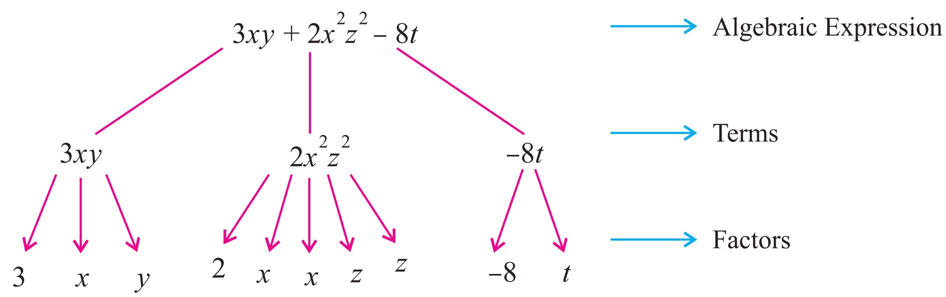
(*a*) $7xy - 3x^2$

(*b*) $3xy + 2x^2z^2 - 8t$

Sol. (*a*) Given expression is $7xy - 3x^2$
Tree diagram.



(*b*) Given expression is $3xy + 2x^2 z^2 - 8t$
Tree diagram



EXERCISE - 12.1

1. **Generate algebraic expressions for the following :**

(*i*) The sum of *a* and *b*

(*ii*) The number *z* multiplied by itself.

(*iii*) The product of *x* and *y* added to the product of *m* and *n*.

(*iv*) The quotient of *p* by 5 is multiplied by *q*.

(*v*) One half of *z* added to twice the number *t*.

(*vi*) Sum of squares of the numbers *x* and *z*.

(*vii*) Sum of the numbers *x* and *z* is subtracted from their product.

2. Separate constant terms and variable terms from the following.

$7, xy, \frac{3x^2}{2}, \frac{72z}{3}, \frac{-8z}{3x^2}$

3. Write the terms and factors for each of the following algebraic expression.

(*a*) $2x^2 + 3yz$ (b) $15x^2y + 3xy^2$

(*c*) $-7xyz^2$ (*d*) $100pq + 10\,p^2q^2$

(*e*) $xy + 3x^2\,y^2$ (*f*) $-7x^2yz + 3xy^2\,z + 2xyz^2$

4. Classify the following algebraic expression into monomial, binomial and trinomial.

(*a*) $7x + 3y$ (*b*) $5 + 2x^2y^2z$

(*c*) $ax + by^2 + cz^2$ (*d*) $3x^2 y^2$

(*e*) $1 + x$ (*f*) 10

(g) $\frac{3}{2}p + \frac{7}{6}q$

5. Write numerical coefficient of each of the following algebraic expression.

(*a*) 2x (*b*) $\frac{-3}{2}xyz$

(*c*) $\frac{7}{2}x^2 p$ (*d*) $-p^2q^2$

(*e*) $-5mn^2$

6. State whether the given pairs of terms is of like or unlike terms.

(*a*) $-3y, \frac{7}{8}y$ (*b*) $-32, -32x$

(*c*) $3x^2y$, $3xy^2$ (*d*) $14mn^2$, $14mn^2q$

(*e*) $8pq$, $32pq^2$ (*f*) 10, 15

7. In the following algebraic expressions write the coefficient of

(*a*) x in x^2y (*b*) xyz in $15x^2yz$

(*c*) $3pq^2$ in $3p^2q^2 r^2$ (*d*) m^2 in $m^2 + n^2$

(*e*) xy in $x^2 y^2 + 2x + 3$

8. Identify the terms and their factors in the following algebraic expressions by tree diagrams

(*a*) $12xy + 7x^2$ (*b*) $p^2q^2 + 3mn^2 - pqr$

(*c*) $2x^2 y^2 + xyz^2 + zy$ (*d*) $\frac{3}{2}x^3 + 2x^2y^2 - 7y^3$

9. Multiple Choice Questions :

(*i*) An expression with only one term is called a

(*a*) Monomial (*b*) Binomial

(*c*) Trinomial (*d*) None of these

(*ii*) The coefficient of x in $8 - x + y$ is

(*a*) –1 (*b*) 1

(*c*) 8 (*d*) 0

(*iii*) Which of the following are like terms ?

(*a*) $7x$, $12y$ (*b*) $15x$, $12x$

(*c*) $3xy$, $3x$ (*d*) $2y$, $-2yx$

(*iv*) Terms are added to form

(*a*) Expressions (*b*) Variables

(*c*) Constants (*d*) Factors

ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

Suppose, you have 15 apples and your brother has 12 apples, then how many Apples do you both have together ? The answer is simple 15 + 12 = 27 Apples.

If we denote an apple by x then you have $15x$ and your brother has $12x$ which can be added as $15x + 12x = 27x$.

Now again suppose you have 12 pens and your brother has 8 pencils can you add 12 pens and 8 pencils. The answer is no. We can only say that we have 12 pens and 8 pencils.

Addition of like terms : The sum of two or more like terms is again the like terms whose numerical coefficient is the sum of the numerical coefficient of the given terms.

For example :

$$2y + 3y = (2 + 3)\,y = 5y$$
$$3x + 7x + 8x = (3 + 7 + 8)x = 18\,x$$
$$2ab + 5ab + 7ab = (2 + 5 + 7)ab = 14ab$$

Example-1 : Add $3xy^2$, $7xy^2$, $-2xy^2$

Sol. Given terms are like terms.

Their coefficients are 3, 7 and –2 respectively.

$$3xy^2 + 7xy^2 + (-2xy^2)$$
$$= (3 + 7 - 2)xy^2$$
$$= (10 - 2)xy^2$$
$$= 8xy^2$$

Example-2 : Add $9xy$, $-3xy$, $-8xy$, $5xy$

Sol. Required sum $= 9xy + (-3xy) + (-8xy) + (5xy)$

$$= (9 - 3 - 8 + 5)xy$$
$$= (9 + 5 - 3 - 8)xy$$
$$= (14 - 11)xy$$
$$= 3xy$$

Addition of Algebraic expressions : To add algebraic expressions, we have to group like terms and then carry out addition on them. This can be done by two methods.

(*i*) **Horizontal Method :** In this method, All expressions are written in a horizontal line and then the terms are arranged in the group of like terms and then like terms are added.

(*ii*) **Column method :** In this method each expression is written in a seperate row such that there like terms are arranged one below the other in a column. Then the addition of terms is done column wise.

Example-3 : (*i*) Add the algebraic expressions $2x + 3y - 7z$ and $3x + 4y + 8z$

Sol. Horizontal Method

$$(2x + 3y - 7z) + (3x + 4y + 8z)$$
$$= 2x + 3y - 7z + 3x + 4y + 8z$$
$$= 2x + 3x + 3y + 4y - 7z + 8z$$
$$= (2 + 3)x + (3 + 4)y + (-7 + 8)z$$
$$= 5x + 7y + z$$

Column Method :

$$\begin{array}{r} 2x + 3y - 7z \\ 3x + 4y + 8z \\ \hline 5x + 7y + z \\ \hline \end{array}$$

(*ii*) Add the algebraic expressions $5x + 7y - 2z$, $3x + 3y + 8z$ and $7x + 2y - 3z$

Sol. Horizontal method

$(5x + 7y - 2z) + (3x + 3y + 8z) + (7x + 2y - 3z)$

$= 5x + 3x + 7x + 7y + 3y + 2y - 2z + 8z - 3z$

$= (5 + 3 + 7)x\ (7 + 3 + 2)y + (-2 + 8 - 3)z$

$= 15x + 12y + 3z$

Column Method :

$$\begin{array}{r} 5x + 7y - 2z \\ 3x + 3y + 8z \\ 7x + 2y - 3z \\ \hline 15x + 12y + 3z \\ \hline \end{array}$$

(*iii*) Add the algebraic expressions $3x^3 + 7xy - 8z^2x$, $2x^3 - 3xy + 3z^2x$, $x^3 - 2xy + 5z^2x$

Sol. Horizontal method

$(3x^3 + 7xy - 8z^2x) + (2x^3 - 3xy + 3z^2x) + (x^3 - 2xy + 5z^2x)$

$= 3x^3 + 2x^3 + x^3 + 7xy - 3xy - 2xy - 8z^2\,x + 3z^2\,x + 5z^2x$

$= (3 + 2 + 1)x^3 + (7 - 3 - 2)xy + (-8 + 3 + 5)\,z^2\,x$

$= 6x^3 + 2xy + 0z^2\,x$

$= 6x^3 + 2xy$

Column Method

$$\begin{array}{r} 3x^3 + 7xy - 8z^2x \\ 2x^3 - 3xy + 3z^2x \\ +\ \ x^3 - 2xy + 5z^2x \\ \hline 6x^3 + 2xy + 0z^2x \\ \hline \end{array}$$

Subtraction of like terms : Subtraction of like terms can be done in a manner exactly similar to that of integers. In other words change the sign of each term to be subtracted and then add.

Example-4 : Subtract (*a*) $3x^2$ from $7x^2$ (*b*) $-3xy^2$ from $2xy^2$

Sol. (*a*) $7x^2 - 3x^2 = (7 - 3)x^2 = 4x^2$

(*b*) $2xy^2 - (-3xy^2) = 2xy^2 + 3xy^2$

$= (2 + 3)xy^2 = 5xy^2.$

Subtraction of algebraic expression : Subtraction of algebraic expressions can be done by two methods.

(*i*) Horizontal method : Change the sign of each term of the expression to be subtracted and then add.

(*ii*) Column Method : Write both expressions one below the other such that the expression to be subtracted comes in the second row and the like terms come one below the other. Change the sign of every term of the expression in the second row and then add.

Example-5 : Subtract $15x^2 + 3xy$ from $20x^2 - 2xy$

Sol. Horizontal method

$20\,x^2 - 2xy - (15\,x^2 + 3xy)$

$= 20x^2 - 2xy - 15x^2 - 3xy$

$= 20x^2 - 15x^2 - 2xy - 3xy$

$= (20 - 15)x^2 + (-2 -3)xy$

$= 5x^2 - xy$

Column Method

$$\begin{array}{r} 20x^2 - 2xy \\ 15x^2 + 3xy \\ - \quad\;\; - \quad \\ \hline 5x^2 - 5xy \\ \hline \end{array}$$

Example-6 : Subtract $3a^2 - b^3 + 5c - 1$ from $2a^2 + 3b^3 - 7c + 2$

Sol. Horizontal method

$2a^2 + 3b^3 - 7c + 2 - (3a^2 - b^3 + 5c - 1)$

$= 2a^2 + 3b^3 - 7c + 2 - 3a^2 + b^3 - 5c + 1$

$= 2a^2 - 3a^2 + 3b^3 + b^3 - 7c - 5c + 2 + 1$

$= (2 - 3)a^2 + (3 + 1)b^3 + (-7 - 5)c + (\,2 + 1)$

$= -a^2 + 4b^3 - 1\,2c + 3$

Column Method

$$\begin{array}{r} 2a^2 + 3b^3 - 7c + 2 \\ 3a^2 - b^3 + 5c - 1 \\ - \quad + \quad - \quad + \quad \\ \hline -a^2 + 4b^3 - 12\,c + 3 \\ \hline \end{array}$$

Example-7 :From the sum of $2x^2 + 7x - 2$ and $3x^2 - 8x + 7$ Subtract $2x^2 + x -1$

Sol. We first add $2x^2 + 7x - 2$ and $3x^2 - 8x + 7$

$$\begin{array}{r} 2x^2 + 7x - 2 \\ 3x^2 - 8x + 7 \\ \hline 5x^2 - \;\; x + 5 \\ \hline \end{array} \qquad (1)$$

Now we subtract $2x^2 + x - 1$ from sum (1)

$$\begin{array}{r} 5x^2 - x + 5 \\ 2x^2 + x - 1 \\ - \quad - \quad + \\ \hline 3x^2 - 2\,x + 6 \\ \hline \end{array}$$

EXERCISE - 12.2

1. Fill in the blanks :-

(*i*) $5y + 7y =$ .... (*ii*) $3xy + 2xy =$ ....

(*iii*) $12a^2 - 7a^2 =$ .... (*iv*) $8mn^2 - 3mn^2 =$ ....

2. Add the following algebraic expressions

(*a*) $3x^2 y^2, 7x y^2$ (*b*) $7x, -3x, 2x$

(*c*) $12p^2q, 3p^2q, -5p^2q$ (*d*) $3x^2, -8x^2, -5x^2, 13x^2$

3. Add the following algebraic expressions.

(*a*) $x + y$ and $2x - 3y$ (*b*) $5a + 7b$ and $3a - 2b$

(*c*) $3m + 2n, 7m - 8n, 2m - n$ (*d*) $3x^2 + 2x - 7$ and $5x^2 - 7x + 8$

(*e*) $m^2 + 2n^2 - p^2, -3m^2 + n^2 + 2p^2$ and $4m^2 - 3n^2 + 5p^2$

(*f*) $3xy + 7x^2 - 2y^2, 2xy + y^2$ and $2x^2 + y^2$

4. Simplify the following algebraic expressions by combining like terms.

(*a*) $-5ax + 3xy + 2xy - 8ax$

(*b*) $3m - 2n + 5m - 3m + 8n$

(*c*) $3pq - 15r^2 - 3\,l^2m^2 + 2r^2 + 2l^2m^2 - 5pq.$

(*d*) $4x^3 + 7x^2 - 3x + 2 - 2x^3 - 2x^2 + 7x - 3$

5. Subtract the algebraic expressions.

(*a*) $-3x^2$ from $7x^2$ (*b*) $-3ab$ from $10ab$

(*c*) $a + b$ from $a - b$ (*d*) $15m + 10n$ from $2m - 16n$

(*e*) $2x + 8y - 3z$ from $-3x + 2y + z$

(*f*) $18m^2 + 3n^2 - 2mn - 7$ from $3m^2 - 2n^2 + 8mn - 8m + 4$

6. What should be subtracted from $l - 2m + 5n$ to get $2l - 3m + 4n$

7. What should be added to $3x^2 + 2xy - y^2$ to obtain $x^2 - 7xy + 3y^2$.

8. Subtract $3a^2 + 2b^2 - 8ab + 8$ from the sum of $a^2 - b^2 + 7ab + 3$ and $2a^2 + 4b^2 - 18ab + 7$

9. How much $x^2 + 3xy + y^2$ is less than $2x^2 + 5xy - y^2$.

10. Multiple Choice Questions :

(*i*) The algebraic expression for "Number 5 added to three times the product of numbers *m* and *n*" is.

(*a*) $5 + 3mn$ (*b*) $3 + 5mn$

(*c*) $(5 + 3)mn$

(*ii*) The sum of algebraic expressions $3x + 11$ and $2x - 7$ is

(*a*) $5x + 4$ (*b*) $x + 4$

(*c*) $5x - 18$

(*iii*) Subtraction of $a + b$ from $2a + 3b$

(*a*) $a + 2b$ (*b*) $-a - 2b$

(*c*) $3a + 4b$ (*d*) $a + b$

VALUE OF AN ALGEBRAIC EXPRESSION

The value of an Algebriac expression varies with the change in the value of variable forming the expression. There are number of situation in which we need to find the value of an algebraic expression such as when we want to check whether a particular value of variable satisfies a given equation or not. The process of finding the value of an algebraic expression by replacing the variable by their particular value is called substitution.

Example-1 : Find the value of the following expressions for $x = 1$

(a) $x + 5$ (b) $3x - 7$ (c) $7x^2 - 2x$

Sol. (a) $x + 5$

Putting $x = 1$ in $x + 5$ we get

$= 1 + 5$

$= 6$

(b) $3x - 7$

Putting $x = 1$ in $3x - 7$ we get

$= 3\,(1) - 7$

$= 3 - 7$

$= -4$

(c) $7x^2 - 2x$

Putting $x = 1$ in $7x^2 - 2x$ we get

$= 7\,(1)^2 - 2\,(1)$

$= 7(1) - 2 = 7 - 2$

$= 5$

Example-2 : Find the value of the following expressions when $p = -3$

(a) $p^2 - 7$ (b) $3p^2 + p - 2$ (c) $10p^3 - 100p^2$

Sol. (a) $p^2 - 7$

Putting $p = -3$ in $p^2 - 7$ we get

$= (-3)^2 - 7 = 9 - 7 = 2$

(b) $3p^2 + p - 2$

Putting $p = -3$ in $3p^2 + p - 2$ we get

$= 3(-3)^2 + (-3) - 2 = 3(9) - 3 - 2$

$= 27 - 3 - 2$

$= 27 - 5 = 22$

(c) $10p^3 - 100p^2$

Putting $p = -3$ in $10p^3 - 100p^2$ we get

$= 10(-3)^3 - (100)\,(-3)^2$

$= 100\,(-27) - 100\,(9)$

$= -2700 - 900$

$= -3600$

Example-3 : Find the value of the following expressions when $a = -2, b = 3$

(i) $a + b$ (ii) $a^2 + b^2$ (iii) $10a - 8b$ (iv) $a^2 + 2ab + b^2$

Sol. (i) $a + b$

Putting $a = -2, b = 3$ in $a + b$ we get

$= (-2) + 3 = +1$

(ii) $a^2 + b^2$

Putting $a = -2, b = 3$ in $a^2 + b^2$ we get

$= (-2)^2 + (3)^2 = 4 + 9 = 13$

(*iii*) $10a - 8b$

Putting $a = -2$, $b = 3$ in $10a - 8b$ we get

$= 10(-2) - 8(3) = -20 - 24 = -44$

(*iv*) $a^2 + 2ab + b^2$

Putting $a = -2$, $b = 3$ in $a^2 + 2ab + b^2$ we get

$= (-2)^2 + 2(-2)(3) + (3)^2$

$= 4 - 12 + 9$

$= 13 - 12$

$= 1$

USING ALGEBRAIC EXPRESSIONS – FORMULA AND RULE

We have already learnt some rules and formulas in mathematics. Here we shall see that these rules and formula can be written in a concise and general form using algebraic expressions.

Perimeter formula :

(*i*) To find the perimeter of an equilateral triangle if we denote the length of the side of the equilateral triangle by l. Then the perimeter of the equilateral triangle = $3l$.

(*ii*) To find the perimeter of rectangle we use the algebraic expression $2(l + b)$ where l and b are length and breadth of a rectangle.

(*iii*) To find the perimeter of a square we use the algebraic expression 4s. Where 's' is the side of square.

Area Formula :

(*i*) If length of side of a square is 's'. Then area of square = s^2.

(*ii*) If l is the length and b is the breadth of a rectangle. Then area of rectangle = $l \times b$

Once a formula i.e the algebraic expression for a given quantity is known. Then we can compute the value of the quantity according to requirement.

For example : If the length of a rectangle is 4 unit and breadth is 6 unit. Then perimeter of rectangle is.

$= 2(4 + 6) = 2(10) = 20$ unit

Area of rectangle $= 4 \times 6 = 24$ sq. unit

Rules for numbers :

1. If n is any natural number then its successor is $n + 1$. We can check this for any natural number. For example. If $n = 15$, Then $n + 1 = 15 + 1 = 16$ which is succesor of 15.
2. If n is any natural number, then $2n$ is always an even number and $2n-1$ is an odd number for example if $n = 3$.

   $2n = 2(3) = 6$ is an even number

   $2n-1 = 2(3) - 1 = 6 - 1 = 5$ is an odd number
3. If n is an odd number n^3 is also an odd number and if n is an even number n^3 is also an even number for example.

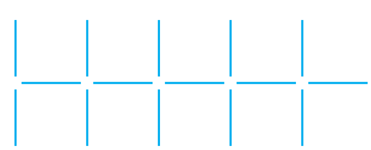
   If $n = 5$

   $n^3 = 5^3 = 5 \times 5 \times 5 = 125$ is an odd number

   If $n = 4$

   $n^3 = 4^3 = 4 \times 4 \times 4 = 64$ is an even number

Take some matchsticks, Tooth picks or pieces of straw cut into smaller pieces of equal length. Join them in patterns as show in given figure.



It consist of the repetition of the letter H made from 5 line segments. As we note that the number of letters (H) formed and the number of the line segment required to form these letters. We get the following table.

Number of letters formed	Number of line segments required
1	5
2	8
3	11
4	14
....	....

i.e. If n is the number of letter, then the algebraic expression to represent the number of required line segments is $3n + 2$.

You may verify this by taking different values for n. such that $n = 1, 2, 3,$ .... so on.

For example : If the number of letter formed is 5. then the number of line segment required is $3n + 2 = 3(5) + 2 = 15 + 2 = 17$ as seen from figure.

Pattern of shapes of Rectangles :

In consist of the repetition of the shapes  made from 6 line segment. We note that number of shapes of rectangle formed and number of line segment required to form the shapes we get the following table.

Number of shapes formed	Number of line segments required
1	6
2	11
3	16
4	21
...	...

i.e. If n is the number of shapes, then the algebraic expression to represent the number of required line segment $5n + 1$.

For example, if the number of shapes formed is 3 than the number of line segment required is $5(3) + 1 = 15 + 1 = 16$ as seen from figure

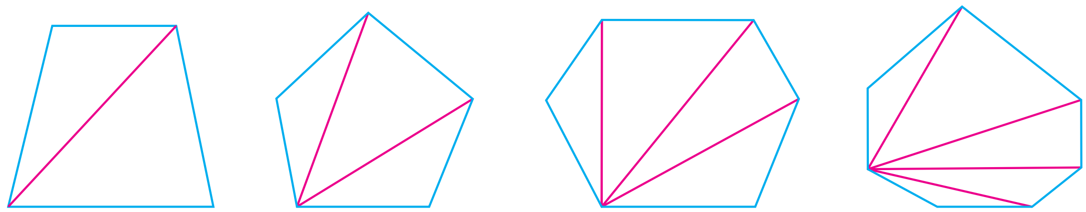
Pattern in Geometry : Here we are going to check the number of diagonals from one vertex in a simple closed polygon. Take four polygons, a quadrilateral, a pentagen, a hexagon and a heptagon

Number of diagonals from one vertex of a quadrelateral = 1

Number of diagonals from one vertex of a pentagon = 2

Number of diagonals from one vertex of a hexagon = 3

Number of diagonals from one vertex of a heptagon = 4



We observe that number of diagonals, we can draw from one vertex of a polygon of 'n' sides is $(n-3)$ check it for octagon. By drawing figure check what is the number of diagonals for a triangle.

For example, If a polygon has 12 sides i.e $n = 12$ then the number of diagonals, we can draw from one vertex $= n - 3 = 12 - 3 = 9$

EXERCISE - 12.3

1. Fill in the Table by substituting the values in the given expressions.

Expression	Value of the expression for			
	$x = 1$	$x = -2$	$x = 3$	$x = 10$
(*i*) $3x + 7$				
(*ii*) $x^2 - 2x + 3$				
(*iii*) $8x^3 - 3x^2$				
(*iv*) $-10x^2 + 20x$				

2. If a = 1, $b = -2$ find the value of given expressions

(*i*) $a^2 - b^2$

(*ii*) $a + 2\,ab - b^2$

(*iii*) $a^2b + 2ab^2 + 5$

3. Simplify the following expressions and find their values for m = 1, $n = 2$, $p = -1$.

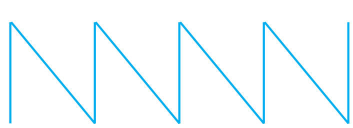
(*i*) $2m + 3n - p + 7m - 2n$ (*ii*) $3p + n - m + 2n$

(*iii*) $m + p - 2p + 3m$ (*iv*) $3n + 2m - 5p - 3m - 2n + p$

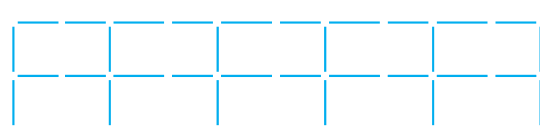
4. What should be the value of a if the value of $2a^2 + b^2 = 10$ when $b = 2$?

5. Find the vlaue of x if $-3x + 7y^2 = 1$ when $y = 1$?

6. Observe the pattern of shapes of letters formed from line segment of equal lengths.



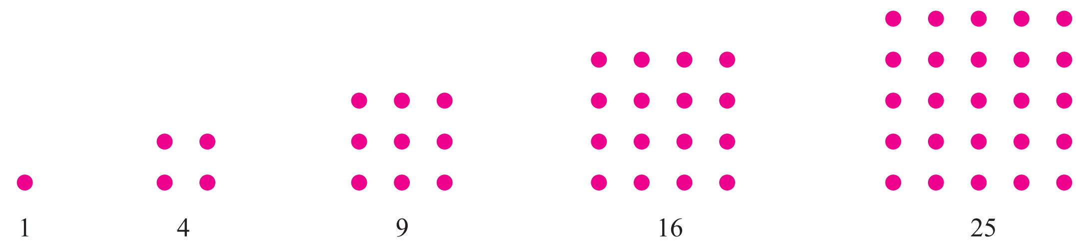
(*i*)



(*iii*)

If n shapes of letters are formed, then write the algebraic expression for the number of line segment requred for making these n shapes in each case.

7. Observe the following pattern of squares made using dots.

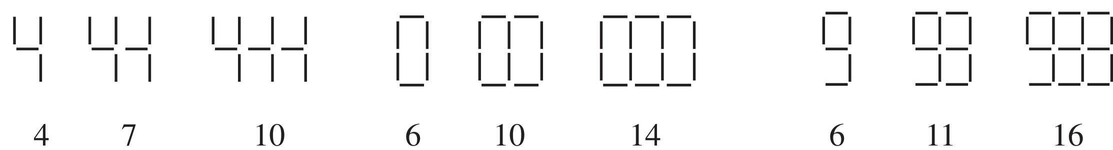


If n is taken as the number of dots in each row then find the algebraic expression for number of dots in nth figure. Also find number of dots if.

(*i*) $n = 3$ (*ii*) $n = 7$

(*iii*) $n = 10$

8. Observe the pattern of shapes of digits formed from line segment of equal lengths.



If n shapes of digits are formed then write the algebraic expression for the numbers of line segment required to make n shapes.

9. Multiple Choice Questions :

(*i*) If l is the length of the side of the regular pentagon, perimeter of a regular Pentagon is.

(*a*) $3\,l$ (*b*) $4\,l$ (*c*) $5\,l$ (*d*) $8\,l$

(*ii*) The value of the expression $5n–2$ when $n = 2$ is.

(*a*) 12 (*b*) –12 (*c*) 8 (*d*) 3

(*iii*) The value of $3x^2 – 5x + 6$ when $x = 1$

(*a*) 3 (*b*) 4 (*c*) –8 (*d*) 14

WHAT HAVE WE DISCUSSED ?

1. A symbol having a fixed numerical value is called a constant.
2. A symbol which takes on various numerical value is called a variable.
3. An algebraic expression is formed by using mathematical operations (addition, subtraction, multiplication and division) on variables and constant.
4. Expressions are formed by addition of terms. Terms can be negative as well as positive.
5. The numerical factor of the term is called the numerical coefficient of the term.
6. The terms having same algebraic factors are called like terms and the terms having different algebraic factors are called unlike terms.
7. Monomial, Binomial and trinomials have one, two and three (unlike) terms respectively.
8. Two or more algebraic expressions can be added by arranging their terms and combining the like terms.
9. To find the value of an algebraic expression, we replace the variable of the expression with their respective values.

10. Algebraic expressions are used in finding perimeters and areas of various geometrical shapes and also in forming patterns etc.

LEARNING OUTCOMES

After completions of the chapter the students are now able to :

1. Differentiate between constants and variables.
2. Generate algebraic expressions.
3. Add and subtract algebraic expressions.
4. Find the value of an expression for a particular value of a variable.
5. Find an algebraic expression for a given pattern.
6. Apply their knowledge about algebraic expressions in their daily life.

ANSWERS

EXERCISE 12.1

1. (*i*) $a + b$ (*ii*) z^2 (*iii*) $xy + mn$

(*iv*) $\frac{p}{5}q$ (*v*) $2t + \frac{z}{2}$ (*vi*) $x^2 + z^2$

(*vii*) $xy - (x + y)$

2. Constant Terms 7, $\frac{72}{3}$

Variable Terms xy, $\frac{3x^2}{2}$, $\frac{72}{3}z$, $\frac{-8z}{3x^2}$

3.

Expreission	Terms	Factors
(*a*) $2x^2 + 3xy$	$2x^2$ $3xy$	$2, x, x$ $3, x, y$
(*b*) $15x^2 y + 3xy^2$	$15x^2y$ $3xy^2$	$15, x, x\ y$ $3, x, y, y$
(*c*) $-7x\ y\ z^2$	$-7xy\ z^2$	$-7, x, y, z, z$
(*d*) $100pq + 10p^2q^2$	$100pq$ $10p^2q^2$	$100, p, q$ $10, p, p, q, q$
(*e*) $xy + 3x^2y^2$	xy $3x^2y^2$	x,y $3, x, x, y, y$
(*f*) $-7x^2yz + 3xy^2z$ $+2xy\ z^2$	$-7x^2\ yz$ $3xy^2z$ $2xyz^2$	$-7, x, x, y, z$ $3, x, y, y, z$ $2, x, y, z, z$

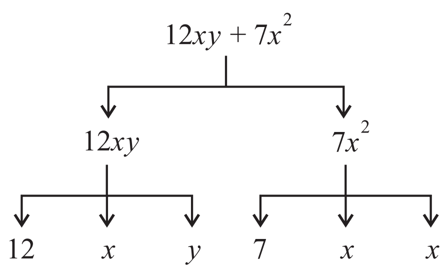
4. (*a*) Binomial (*b*) Binomial (*c*) Trinomial
(*d*) Monomial (*e*) Binomial (*f*) Monomial
(*g*) Binomial

5. (*a*) 2 (*b*) $\frac{-3}{2}$ (*c*) $\frac{7}{2}$
(*d*) –1 (*e*) –5

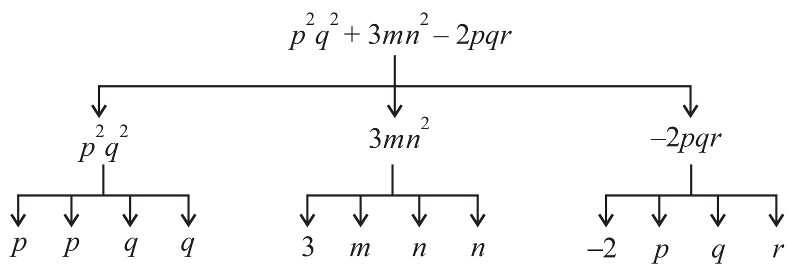
6. (a) Like (*b*) Unlike (*c*) unlike
(*d*) unlike (*e*) unlike (*f*) like

7. (*a*) xy (*b*) $15x$ (*c*) pr^2
(*d*) 1 (*e*) xy

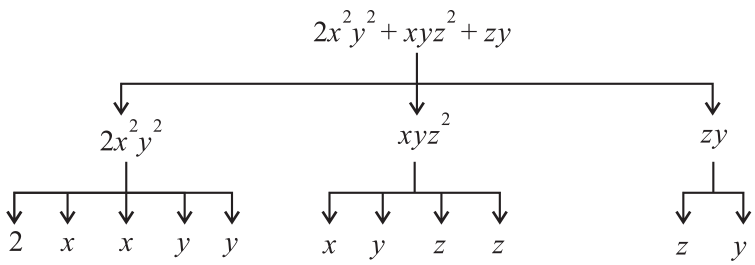
8. (*a*)



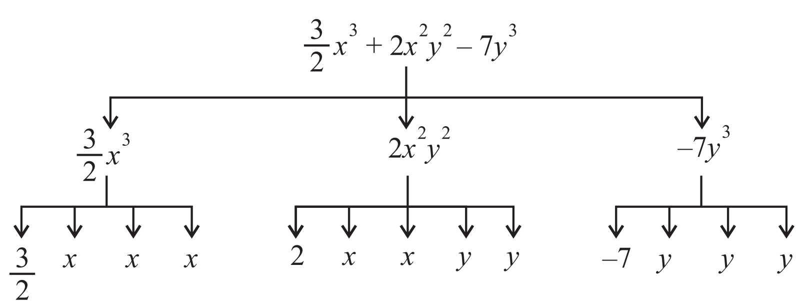
(*b*)



(*c*)



(*d*)



9. (*i*) *a* (*ii*) *a*
(*iii*) *b* (*iv*) *a*

EXERCISE 12.2

1. (*i*) $12y$ (*ii*) $5xy$
(*iii*) $5a^2$ (*iv*) $5mn^2$

2. (*a*) $10xy^2$ (*b*) $6x$
(*c*) $10p^2q$ (*d*) $3x^2$

3. (*a*) $3x - 2y$ (*b*) $8a + 5b$
(*c*) $12m - 7n$ (*d*) $8x^2 - 5x + 1$
(*e*) $2m^2 + 0n^2 + 6p^2$ (*f*) $5xy + 9x^2$

4. (*a*) $-13ax + 5xy$ (*b*) $5m + 6n$
(*c*) $-2pq - 13r^2 - 2\,l^2m^2$ (*d*) $2x^3 + 5x^2 + 4x - 1$

5. (*a*) $4x^2$ (*b*) $13ab$
(*c*) $-2b$ (*d*) $-13m - 26n$
(*e*) $-5x - 6y + 4z$ (*f*) $-15m^2 - 5n^2 + 10m\,n - 8m + 11$

6. $-1 + m + n$ **7.** $-2x^2 - 9xy + 4y^2$

8. $2b^2 - 3ab + 2$ **9.** $x^2 + 2xy - 2y^2$

10. (*i*) a (*ii*) a
(*iii*) a

EXERCISE 12.3

1. (*i*) 10, 1, 16, 37 (*ii*) 2, 11, 6, 83
(*iii*) 5, –76, 189, 7700 (*iv*) 10, –80, –30, –800

2. (*i*) –3 (*ii*) –7
(*iii*) 9

3. (*i*) 12 (*ii*) 2
(*iii*) 5 (*iv*) 5

4. $a = 3$ **5.** $x = 2$

6. (*i*) $2n + 1$ (*ii*) $4n + 2$

7. (*i*) n^2 (*ii*) 9
(*iii*) 49 (*iv*) 100

8. (*i*) $3n + 1$ (*ii*) $4n + 2$
(*iii*) $5n + 1$

9. (*i*) c (*ii*) c
(*iii*) b

CHAPTER 13

Exponents And Powers

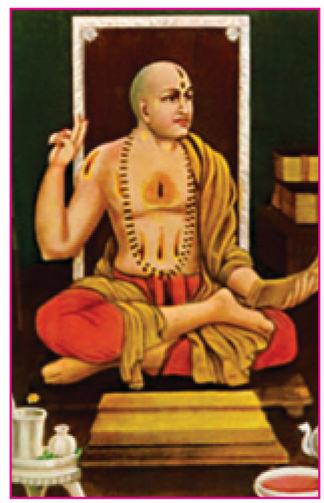
T3Z6U9

Learning Objectives :-

In this chapter, you will learn :-

1. To identify the base and exponent.
2. About the exponential notation.
3. To write the number in exponential form.
4. To apply the laws of exponents.
5. About the standard form of numbers.

OUR NATION'S PRIDE



Apastamba has been known as India's most complicated Mathematician, who is known to have lived around 600 BC. According to the Hindu tradition, he was the student of Baudhayana. Sulabha Sutras given by Apastamba, are known to be one of the oldest known Mathematics text in existence. His major contribution in the field of mathematics include the numeric solution of Pythagoras Theorem. Apastamba's rules for altar construction led to the discovery of irrational numbers though he has never been given the due credit for the same.

INTRODUCTION

In many situations, we come across numbers that are very large or very small. For example, the age of the universe in years, the mass of the earth in tons, the distance of the sun from the earth (in km), the size of bacteria etc. are numbers that are either very large or very small. Such numbers are generally approximate (due to complexity of their measurement). Therefore these are represented by some numbers followed by certain number of zeros. For example, the age of the universe is approximated to the 8,000,000,000 years and the mass of the Earth is approximately 5980000000000000000 metric tons. Such numbers are usually written by using exponents. For example, the number 8000000000 can be written as 8×10^9 or 80×10^8 or 800×10^7. Similarly, $5980000000000000000000 = 598\times10^{19}$. This is known as exponential notation. The exponential notation helps us to write very large and very small numbers easily.

EXPONENTIAL FORM

Consider the number 125 we write $125 = 5 \times 5 \times 5$. Therefore $125 = 5^3$, 5^3 is the exponential form of 125. Here '5' is base and '3' is exponent. The number 5^3 is read as 5 raised to power of 3 or simply 5 cubed.

Consider another number $\frac{16}{81}$.

$$\frac{16}{81} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \left(\frac{2}{3}\right)^4$$

Therefore, the exponential form of $\frac{16}{81}$ is $\left(\frac{2}{3}\right)^4$.

Here $\frac{2}{3}$ is base and 4 is exponent.

This leads to, if a is any rational number and n is a natural number then $a^n = a \times a \times a$ .... multiplied n times, where a is called the base and n is called the exponent or index and a^n is the exponential form, a^n is read as a raised to the power n. In particular, $a^1 = a$.

For example : $10^4 = 10 \times 10 \times 10 \times 10$ *i.e.* $10^4 = 10000$ here base = 10, exponent (or index) = 4 and 10^4 is the exponential form of the number 10000.

Example-1 : Expand the following :

(i) $(-3)^4$ **(ii)** 2^6 **(iii)** $(-1)^5$ **(iv)** $\left(-\frac{1}{2}\right)^2$

Sol. (*i*) $(-3)^4$ means that -3 is multiplied to itself 4 times.

Therefore, $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$

$= (+9) \times (+9) = 81$

(*ii*) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(*iii*) $(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

(*iv*) $\left(-\frac{1}{2}\right)^2 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = +\frac{1}{4}$

Example-2 : Express the following in the exponential form :

(i) **343** **(ii)** **3125**

Sol. (*i*) 343

$343 = 7 \times 7 \times 7 = 7^3$

7	343
7	49
7	7
	1

(*ii*) 3125

$3125 = 5 \times 5 \times 5 \times 5 \times 5$

$= 5^5$

5	3125
5	625
5	125
5	25
5	5
	1

Example-3 : Which is greater 5^3 or 3^5

Sol.
$$5^3 = 5 \times 5 \times 5 = 125$$
$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$
$$243 > 125$$
$$\therefore \quad 3^5 > 5^3$$

Example-4 : Express 540 as product of powers of their prime factors.

Sol. 540

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 \times 3^3 \times 5$

2	540
2	270
3	135
3	45
3	15
5	5
	1

Example-5 : Simplify (*i*) $5^2 \times 3^3$ (*ii*) 0×10^2

Sol. (*i*) $5^2 \times 3^3 = 5 \times 5 \times 3 \times 3 \times 3$

$= 25 \times 27 = 675$

(*ii*) $0 \times 10^2 = 0 \times 10 \times 10$

$= 0 \times 100 = 0$

Example-6 : Find the value of x if $3^x = 729$

Sol. $3^x = 729$

$3^x = 3^6$

$\therefore \quad x = 6$

3	729
3	243
3	81
3	27
3	9
3	3
	1

Example-7 : Check : $(1)^5, (-1)^3, (-1)^4, (-10)^3, (-5)^4$

Sol. (*i*) We have $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1$

In fact, you will realise that 1 raised to any power is 1

(*ii*) $(-1)^3 = (-1) \times (-1) \times (-1) = 1 \times (-1) = -1$ $\quad [(-1)^{\text{odd number}} = -1]$

(*iii*) $(-1)^4 = (-1)\times(-1)\times(-1)\times(-1) = 1\times 1 = 1$ $[(-1)^{\text{even number}} = +1]$

> You may check that (– 1) raised to any odd power is (– 1) and (– 1) raised to any even power is (+ 1).

(*iv*) $(-10)^3 = (-10)\times(-10)\times(-10) = 100\times(-10) = -1000$

(*v*) $(-5)^4 = (-5)\times(-5)\times(-5)\times(-5) = 25\times 25 = 625$

$$(-1)^{\text{odd number}} = -1$$
$$(-1)^{\text{even number}} = 1$$

EXERCISE - 13.1

1. Fill in the blanks :

(*i*) In the expression 3^7, base = _________ and exponent = _________.

(*ii*) In the expression $\left(\frac{2}{5}\right)^{11}$, base = _______ and exponent = _______.

2. Find the value of the following :

(*i*) 2^6 (*ii*) 9^3 (*iii*) 5^5

(*iv*) $(-6)^4$ (*v*) $\left(-\frac{2}{3}\right)^5$

3. Express the following in the exponential form :

(*i*) $6\times 6\times 6\times 6$ (*ii*) $b\times b\times b\times b$ (*iii*) $5\times 5\times 7\times 7\times 7$

4. Simplify the following :

(*i*) 2×10^3 (*ii*) $5^2\times 3^2$ (*iii*) $3^2\times 10^4$

5. Simplify :

(*i*) $(-3)\times(-2)^3$ (*ii*) $(-4)^3\times 5^2$ (*iii*) $(-1)^{99}$

(*iv*) $(-3)^2\times(-5)^2$ (*v*) $(-1)^{132}$

6. Identify the greater number in each of the following :

(*i*) 4^3 or 3^4 (*ii*) 5^3 or 3^2 (*iii*) 2^3 or 8^2

(*iv*) 4^5 or 5^4 (*v*) 2^{10} or 10^2

7. Write the following numbers as power of 2 :

(*i*) 8 (*ii*) 128 (*iii*) 1024

8. Write the following numbers as power of 3 :

(*i*) 27 (*ii*) 2187

9. Find the value of x in each of the following :

(*i*) $7^x = 343$ (*ii*) $9^x = 729$ (*iii*) $(-8)^x = -512$

10. To what power (– 2) should be raised to get 16 ?

11. Write the prime factorization of the following numbers in the exponential form :

(*i*) 72 (*ii*) 360 (*iii*) 405

(*iv*) 648 (*v*) 3600

LAWS OF EXPONENTS

We can multiply and divide **rational numbers** expressed in exponential form.

Multiplication of Identical bases with different powers

Let us find $2^4 \times 2^3$

$= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

Let us find $(-3)^2 \times (-3)^3$

$[(-3) \times (-3)] \times [(-3) \times (-3) \times (-3)]$

$= (-3) \times (-3) \times (-3) \times (-3) \times (-3) = (-3)^5$

Note that $(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5$

In fact, this is true in general we have :

> **Law 1 :** If a is any **rational number** and m, n are integers, then $a^m \times a^n = a^{m+n}$

Division of Identical bases with different Powers

$$\text{Let us find } 5^7 \div 5^4 = \frac{5^7}{5^4} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}$$

$$= 5 \times 5 \times 5 = 5^3$$

$$\text{Note that } 5^7 \div 5^4 = \frac{5^7}{5^4} = 5^{7-4} = 5^3$$

In fact, this is true in general we have :

> **Law 2 :** If a is any (non zero) **rational number** and m, n are integers such that $\boldsymbol{m > n}$, then
>
> $$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}.$$

Zero Exponent

$$\text{Let us find } \frac{3^3}{3^3} = \frac{3 \times 3 \times 3}{3 \times 3 \times 3} = \frac{27}{27} = 1$$

$$\text{Also } \frac{3^3}{3^3} = 3^{3-3} = 3^0$$

Though we have calculated $\frac{3^3}{3^3}$ in two different ways, the answer remains the same. It follows, that $3^0 = 1$.

In fact, this is true in general. So, we have :

> **Law 3 :** If a is any (non-zero) rational number, then $a^0 = 1$

Taking power of a power

$$(2^3)^2 = 2^3 \times 2^3 = 2^{3+3}$$

$$= 2^6 = 2^{3 \times 2}$$

$$\text{Thus } (2^3)^2 = 2^{3 \times 2}$$

In fact, this is true in general we have.

Law 4 : If a is any **rational number** and m, n are integers then $(a^m)^n = a^{m \times n}$

Multiply different bases with same exponent

Let us find $3^4 \times 5^4$

$$= (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5)$$
$$= (3 \times 5) \times (3 \times 5) \times (3 \times 5) \times (3 \times 5)$$
$$= (3 \times 5)^4$$

In fact, this is true in general. We have :

Law 5 : If a, b are any rational numbers and n is an integer then $a^n \times b^n = (ab)^n$

Division of different bases with same exponent

$$\text{Let us find } \frac{2^4}{7^4} = \frac{2 \times 2 \times 2 \times 2}{7 \times 7 \times 7 \times 7} = \left(\frac{2}{7}\right)^4$$

In fact, this is true in general. We have :

Law 6 : If a, $(b \neq 0)$ are any numbers and n is an integer, then $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ or

$$a^n \div b^n = \left(\frac{a}{b}\right)^n$$

NEGATIVE EXPONENT

For any (non – zero) **rational number** a and natural number n, we have

$$\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}, \text{ thus } a^{-n} = \frac{1}{a^n}$$

Law 7 : If a is any (non – zero) **rational number** and n is an integer, then $a^{-n} = \frac{1}{a^n}$ In partcular, $a^{-1} = \frac{1}{a}$.

Example-1 : Simplify and write in exponential form :-

(a) $2^3 \times 2^2$ **(b)** $4^2 \times 4^3$ **(c)** $3^2 \times 3^3 \times 3^4$ **(d)** $(-4)^3 \times (-4)^2$

Sol. (a) $2^3 \times 2^2 = 2^{3+2} = 2^5$

(b) $4^2 \times 4^3 = 4^{2+3} = 4^5$

(c) $3^2 \times 3^3 \times 3^4 = 3^{2+3+4} = 3^9$

(d) $(-4)^3 \times (-4)^2 = (-4)^{3+2} = (-4)^5$

Example-2 : Simplify and write in exponential form

(a) $13^6 \div 13^4$ **(b)** $10^4 \div 10$ **(c)** $18^{16} \div 18^{10}$ **(d)** $(-5)^6 \div (-5)^2$

Sol. (a) $13^6 \div 13^4 = 13^{6-4} = 13^2$

(b) $10^4 \div 10 = 10^{4-1} = 10^3$

(c) $18^{16} \div 18^{10} = 18^{16-10} = 18^6$

(d) $(-5)^6 \div (-5)^2 = (-5)^{6-2} = (-5)^4$

Example-3 : Simplify and express in exponential form

(a) $(3^2)^3$ **(b)** $(4^3)^2$ **(c)** $[(10)^2]^3$ **(d)** $(2^{100})^2$

Sol. (a) $(3^2)^3 = 3^{2\times3} = 3^6$

(b) $(4^3)^2 = 4^{3\times2} = 4^6$

(c) $[(10)^2]^3 = (10)^{2\times3} = (10)^6$

(d) $(2^{100})^2 = 2^{100\times2} = 2^{200}$

Example-4 : Simplify (a) $\left(\frac{2}{5}\right)^4$ **(b)** $\left(\frac{-1}{3}\right)^3$ **(c)** $\left(\frac{-6}{7}\right)^2$

Sol. (a) $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{2\times2\times2\times2}{5\times5\times5\times5} = \frac{16}{625}$

(b) $\left(\frac{-1}{3}\right)^3 = \frac{(-1)^3}{3^3} = \frac{-1\times-1\times-1}{3\times3\times3} = -\frac{1}{27}$

(c) $\left(\frac{-6}{7}\right)^2 = \frac{(-6)^2}{7^2} = \frac{-6\times-6}{7\times7} = \frac{36}{49}$

Example-5 : Simplify and express each of the following in the exponential form.

(a) $[(5^2)^3 \times 5^4] \div 5^7$ **(b)** $125^4 \div 5^3$ **(c)** $[(2^2)^3 \times 3^6] \times 5^6$

Sol. (a) $[(5^2)^3 \times 5^4] \div 5^7$

$= (5^{2\times3} \times 5^4) \div 5^7$

$= (5^6 \times 5^4) \div 5^7$

$= 5^{6+4} \div 5^7$

$= 5^{10} \div 5^7$

$= 5^{10-7} = 5^3$

(b) $125^4 \div 5^3$

$125^4 = (5 \times 5 \times 5)^4 = (5^3)^4 = 5^{12}$

$125^4 \div 5^3 = 5^{12} \div 5^3 = 5^{12-3} = 5^9$

(c) $[(2^2)^3 \times 3^6] \times 5^6 = (2^{2\times3} \times 3^6) \times 5^6$

$= (2 \times 3)^6 \times 5^6$

$= 6^6 \times 5^6$

$= (6 \times 5)^6$

$= 30^6$

Example-6 : Simplify and express each of the following in exponential form :

(i) $\frac{2^3\times3^4\times4}{3\times32}$ **(ii)** $(3^0 + 2^0) \times 5^0$ **(iii)** $\frac{25\times5^2\times a^8}{10^3\times a^4}$

Sol. (i) $4 = 2 \times 2 = 2^2$ and $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$\therefore \quad \frac{2^3\times3^4\times4}{3\times32} = \frac{2^3\times3^4\times2^2}{3\times2^5} = \frac{2^5\times3^4}{3^1\times2^5}$

$= 2^{5-5} \times 3^{4-1}$

$= 2^0 \times 3^3 = 1 \times 3^3 = 3^3$

(*ii*) $(3^0 + 2^0) \times 5^0 = (1 + 1) \times 1 = 2 \times 1 = 2 = 2^1$

(*iii*) $$\frac{25\times5^2\times a^8}{10^3\times a^4} = \frac{5^2\times5^2\times a^8}{(2\times5)^3\times a^4} = \frac{5^2\times5^2\times a^8}{2^3\times5^3\times a^4}$$

$$= \frac{5^{2+2-3}\times a^{8-4}}{2^3} = \frac{5a^4}{2^3}$$

Example-7 : Express each of the following rational numbers in exponential form :-

(*i*) $\frac{64}{343}$ **(*ii*)** $\frac{-27}{125}$ **(*iii*)** $\frac{-1}{243}$

Sol. (*i*) $\frac{64}{343} = \frac{4^3}{7^3} = \left(\frac{4}{7}\right)^3$

(*ii*) $\frac{-27}{125} = \frac{(-3)^3}{5^3} = \left(\frac{-3}{5}\right)^3$

(*iii*) $\frac{-1}{243} = \frac{-1}{243} = \frac{(-1)^5}{3^5} = \left(\frac{-1}{3}\right)^5$

Example-8: Simplify $(3^0 + 2^0 - 6^0) \div (100)^0$ and write the answer as a power of 5.

Sol. $(3^0 + 2^0 - 6^0) \div (100)^0$

$= (1 + 1 - 1) \div 1$ $(\because x^0 = 1)$

$= 1 \div 1 = 1 = 5^0$ $(\because 5^0 = 1)$

EXERCISE - 13.2

1. Using laws of exponents, simplify and write the following in the exponential form.
   (*i*) $2^7 \times 2^4$ (*ii*) $p^5 \times p^3$
   (*iii*) $(-7)^5 \times (-7)^{11}$ (*iv*) $20^{15} \div 20^{13}$
   (*v*) $(-6)^7 \div (-6)^3$ (*v*) $7^x \times 7^3$
2. Simplify and write the following in exponential form.
   (*i*) $5^3 \times 5^7 \times 5^{12}$ (*ii*) $a^5 \times a^3 \times a^7$
3. Simplify and write the following in the exponential form :-
   (*i*) $(2^2)^{100}$ (ii) $(5^3)^7$
4. Simplify and write in the exponential form :
   (*i*) $(2^3)^4 \div 2^5$ (*ii*) $2^3 \times 2^2 \times 5^5$
   (*iii*) $[(2^2)^3 \times 3^6] \times 5^6$
5. Simplify and write in the exponential form :
   (*i*) $5^4 \times 8^4$ (*ii*) $(-3)^6 \times (-5)^6$

6. Simplify and express each of the following in the exponential form :

(*i*) $\dfrac{\left(3^2\right)^3 \times (-2)^5}{(-2)^3}$ (*ii*) $\dfrac{3^7}{3^4 \times 3^3}$

(*iii*) $\dfrac{2^8 \times a^5}{4^3 \times a^3}$ (*iv*) $3^0 \times 4^0 \times 5^0$

7. Express each of the following rational number in the exponontial form :-

(*i*) $\dfrac{25}{64}$ (*ii*) $\dfrac{-64}{125}$

(*iii*) $\dfrac{-125}{216}$ (*iv*) $\dfrac{-343}{729}$

8. Simplify :-

(*i*) $\dfrac{(2^5)^2 \times 7^3}{8^3 \times 7}$ (*ii*) $\dfrac{2 \times 3^4 \times 2^5}{9 \times 4^2}$

9. Express each of the following as a product of prime factors in the exponential form

(*i*) 384×147 (*ii*) 729×64

(*iii*) 108×92

10. Simplify and write the following in the exponential form :

(*i*) $3^3 \times 2^2 + 2^2 \times 5^0$ (*ii*) $\left(\dfrac{3^7}{3^2}\right) \times 3^5$

(*iii*) $8^2 \div 2^3$

Multiple Choice Questions :-

11. $\left(\dfrac{-5}{8}\right)^0$ is equal to

(*i*) 0 (*ii*) 1

(*iii*) $\dfrac{-5}{8}$ (*iv*) $\dfrac{-8}{5}$

12. $(5^2)^3$ is equal to

(*i*) 5^6 (*ii*) 5^5

(*iii*) 5^9 (*iv*) 10^3

13. $a \times a \times a \times b \times b \times b$ is equal to

(*i*) $a^3 b^2$ (*ii*) $a^2 b^3$

(*iii*) $(ab)^3$ (*iv*) $a^6 b^6$

14. $(-5)^2 \times (-1)^1$ is equal to

(*i*) 25 (*ii*) –25

(*iii*) 10 (*iv*) –10

DECIMAL NUMBER SYSTEM

Consider the expansion of the number 753015, we know that

$$753015 = 7 \times 100000 + 5 \times 10000 + 3 \times 1000 + 0 \times 100 + 1 \times 10 + 5 \times 1$$

Using power of 10 in the exponent, we can write it as

$$753015 = 7 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 5 \times 10^0$$
$$= 7 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 1 \times 10^1 + 5 \times 10^0$$

In fact, the expansion of every number can be written using power of 10 in the exponent.

STANDARD FORM OF NUMBERS

The standard form of a number in of the form $k \times 10^n$ where k is a number between 1 and 10 and n is an integer.

Look at the following

$$76 = 7.6 \times 10 = 7.6 \times 10^1$$
$$763 = 7.63 \times 100 = 7.63 \times 10^2$$
$$7630 = 7.63 \times 1000 = 7.63 \times 10^3$$
$$76300 = 7.63 \times 10000 = 7.63 \times 10^4 \text{ and so on.}$$

SCIENTIFIC NOTATION

Scientific notation is a way of writing numbers that accommodates value too large to be conveniently written in decimal notation.

In scientific notation all numbers are written as $k \times 10^n$ where k is decimal number such that $1 < k < 10$ and n is a whole number. The decimal k is called significand. Scientific notation is also known as standard form.

Ordinary decimal notation	Scientific notation
500	5×10^2
47,000	4.7×10^4
9,830,000,000	9.83×10^9

Example-1 : Write the following numbers in the standard form.

(*i*) 763.4 (*ii*) 83,500 (*iii*) 573,000

Sol. (*i*) $763.4 = 7.634 \times 10^2$

(*ii*) $83,500 = 8.3500 \times 10^4 = 8.35 \times 10^4$

(*iii*) $573,000 = 5.73000 \times 10^5 = 5.73 \times 10^5$

Example-2 : Write the following as usual decimal notation.

(*i*) 5.37×10^4 (*ii*) 7.501×10^7 (*iii*) 2.3049×10^{11}

Sol. (*i*) $5.37 \times 10^4 = 53700$

(*ii*) $7.501 \times 10^7 = 75010000$

(*iii*) $2.3049 \times 10^{11} = 230490000000.$

Example-3 : Express the numbers appearing in the following statement in scientific notation (or standard form)

(*i*) **The radius of the earth is 6366000 metres**

(*ii*) **The distance between the sun and the earth is 149, 600, 000, 000 *m***

(*iii*) **The speed of light in vacum = 299, 800, 000 *m*/sec**

(*iv*) **The mass of the earth is 5, 976, 000, 000, 000, 000, 000, 000, 000 kg**

Sol. (*i*) The radius of the earth is = 6366000 = 6.366×10^6 m

(*ii*) The distance between the sun and the earth = 149, 600, 000, 000 *m* = 1.496×10^{11} m

(*iii*) The speed of light in vacuum = 299, 800, 000 *m*/sec

= 2.998×10^8 *m*/sec

(*iv*) The mass of earth = 5, 976, 000,000,000,000,000,000, 000 kg

= 5.976×10^{24} *kg*

Example-4 : Compare the following numbers

(*i*) **2.7×10^{12} ; 1.5×10^8** (*ii*) **3.547×10^9; 6.02×10^9**

Sol. (*i*) The given numbers are 2.7×10^{12} and 1.5×10^8,

Note that both the numbers are in standard form. Since the power of 10 in 2.7×10^{12} is greater than the power of 10 in 1.5×10^8,

$\therefore$ $2.7 \times 10^{12} > 1.5 \times 10^8$

(*ii*) The given numbers are 3.54×10^9 and 6.02×10^9 and 6.02×10^9. Note that both the numbers are in standard form. Also we note that both the numbers have equal power of 10. There fore, we compare their significands.

The significand in 3.547×10^9 is 3.547 and the significand in 6.02×10^9 is 6.02

As 6.02 > 3.547, so $6.02 \times 10^9 > 3.547 \times 10^9$

EXERCISE - 13.3

1. **Write the following numbers in the expanded exponential form :**

(*i*) 104278 (*ii*) 20068

(*iii*) 120719 (*iv*) 3006194

(*v*) 28061906

2. **Find the number from each of the following expanded form :**

(*i*) $4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(*ii*) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$

(*iii*) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$

(*iv*) $8 \times 10^7 + 3 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 8 \times 10^1$

3. **Express the following numbers in standard form :**

(*i*) 3, 43,000 (*ii*) 70,00,000

(*iii*) 3, 18,65,00,000 (*iv*) 530.7

(*vi*) 5985.3 (*v*) 3908.78

4. Express the number appearing in the following statements in standard form :

(*i*) The distance between the earth and the moon is 384, 000,000*m*

(*ii*) The diameter of the earth is 1,27, 56,000*m*.

(*iii*) The diameter of the sun is 1,400,000,000*m*.

(*vi*) The universe is estimated to be about 12,000,000,000 years old.

(*v*) Mass of uranis is 86,800,000, 000,000,000,000,000,000kg

5. Compare the following numbers :

(*i*) 4.3×10^{14}; 3.01×10^{17}.

(*ii*) 1.439×10^{12}; 1.4335×10^{12}

WHAT HAVE WE DISCUSSED ?

- **If a is any rational number and *n* is an integer then.**

  $a^n = a \times a \times a$ .... multiplied n times.

  Where a is called the base and n is called the exponent or index and a^n is the exponential form a^n is read as 'a' raised to the power n or a to the power.'n'

  In particular $a^1 = a$

  $(-1)^{\text{odd natural number}} = -1$

  and $(-1)^{\text{even natural number}} = 1$

- **Laws of exponents**

  Law 1. : If a is any rational number and m, n are integers then $a^m \times a^n = a^{m+n}$

  Law 2. : If a is any (non – zero) rational number and m, n are integers such that $m > n$, then $a^m \div a^n = a^{m-n}$.

  Law 3. : If a is any (non – zero) rational number, then $a^0 = 1$

  Law 4. : If a is any rational number and m, n are integers, then $(a^m)^n = a^{m \times n}$

  Law 5. : If a, b are any rational numbers and n is an integer then $a^n \times b^n = (ab)^n$

  Law 6. : If a, b $(b \neq 0)$ are any rational numbers and n is an integer, than $a^n \div b^n = \left(\frac{a}{b}\right)^n$ or $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$.

  Law 7. : If a is any (non – zero) rational number and n is an integer then $a^{-n} = \frac{1}{a^n}$

- **Standard form or scientific notation**

  A number is said to be in the standard form if it is expressed as $k \times 10^n$, where k is a decimal number such that $1 \leq k < 10$ and n is a whole number.

  The standard form of a number is also known as scientific notation

  The decimal number k is called significand.

- **Convert from decimal notation to standard form**

  (*i*) Move the decimal point to the left till you get just one digit to the left of decimal place.

  (*ii*) Write the given number as the product of the number obtained in step (*i*) and 10^n, where n is the number of places the decimal point has been moved to the left.

- **Conversion from standard form to usual form.**

  Take the significand and move the decimal point to the right by the number of places indicated by the exponent of 10^n adding trailing zeros as necessary.

- **Comparing numbers in standard form.**

  (*i*) The number with greater power of 10 is greater.

  (*ii*) If the power of 10 are equal in both numbers then compare their significands. The number with greater significand is greater.

LEARNING OUTCOMES

After completion of the chapter, students are now able to

1. Understand the base and the exponent.
2. Apply the laws of exponents including multiplication and division of power with same base.
3. Evaluate the zero exponents.
4. Understand that exponents with different base can't be multiplied or divided.
5. Write standard form of numbers.
6. Use exponential form of numbers to simplify problems involving multiplication and division of large numbers.

ANSWERS

EXERCISE 13.1

1.	(*i*) 3, 7	(*ii*) $\frac{2}{5}, 11$	
2.	(*i*) 64	(*ii*) 729	(*iii*) 3125
	(*iv*) 1296	(*v*) $\frac{-32}{243}$	
3.	(*i*) 6^4	(*ii*) b^4	(*iii*) $5^2 \times 7^3$
4.	(*i*) 2000	(*ii*) 225	(*iii*) 90000
5.	(*i*) 24	(*ii*) –1600	(*iii*) –1
	(*iv*) 225	(*v*) 1	
6.	(*i*) 3^4	(*ii*) 5^3	(*iii*) 8^2
	(*iv*) 4^5	(*v*) 2^{10}	
7.	(*i*) 2^3	(*ii*) 2^7	(*iii*) 2^{10}
8.	(*i*) 3^3	(*ii*) 2^7	(*iii*) 2^{10}
9.	(i) 3	(*ii*) 3	(*iii*) 3
10.	4		
11.	(*i*) $2^3 \times 3^2$	(*ii*) $2^3 \times 3^2 \times 5^1$	(*iii*) $5^1 \times 3^4$
	(*iv*) $2^3 \times 3^4$	(*v*) $2^4 \times 3^2 \times 5^2$	

EXERCISE 13.2

1. (*i*) 2^{11} (*ii*) p^8 (*iii*) $(-7)^{16}$
(*iv*) 20^2 (*v*) $(-6)^4$ (*vi*) 7^{x+3}

2. (*i*) 5^{22} (*ii*) a^{15}

3. (*i*) 2^{200} (*ii*) 5^{21}

4. (*i*) 2^7 (*ii*) 10^5 (*iii*) 30^6

5. (*i*) 40^4 (*ii*) 15^6

6. (*i*) $3^6 \times 2^2$ (*ii*) 1^1 (*iii*) $(2a)^2$
(*iv*) 1^1

7. (*i*) $\left(\frac{5}{8}\right)^2$ (*ii*) $\left(\frac{-4}{5}\right)^3$ (*iii*) $\left(\frac{-5}{6}\right)^3$ (*iv*) $\left(\frac{-7}{9}\right)^3$

8. (*i*) 98 (*ii*) 36

9. (*i*) $2^7 \times 3^2 \times 7^2$ (*ii*) $3^6 \times 2^6$ (*iii*) 28×34

10. (*i*) $2^4 \times 7^1$ (*ii*) 3^{10} (*iii*) 2^3

11. (*i*) **12.** (*i*)

13. (*iii*) **14.** (*ii*)

EXERCISE 13.3

1. (*i*) $104278 = 1 \times 10^5 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0$
(*ii*) $20068 = 2 \times 10^4 + 6 \times 10^1 + 8 \times 10^0$
(*iii*) $120719 = 1 \times 10^5 + 2 \times 10^4 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$
(*iv*) $3006194 = 3 \times 10^6 + 6 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$
(*v*) $28061906 = 2 \times 10^7 + 8 \times 10^6 + 6 \times 10^4 + 1 \times 10^3 + 9 \times 10^2 + 6 \times 10^0$

2. (*i*) 47561 (*ii*) 30705 (*iii*) 405302
(*iv*) 80037580

3. (*i*) 3.43×10^5 (*ii*) 7.0×10^6 (*iii*) 3.1865×10^9
(*iv*) 5.307×10^2
(*iv*) 5.9853×10^3 (*v*) 3.90878×10^3

4. (*i*) 3.84×10^8 (*ii*) $1.2756 \times 10^7 m$ (*iii*) $1.40 \times 10^9 m$.
(*iv*) 1.2×10^{10} years (*v*) 8.68×10^{28} kg.

5. (*i*) $3.01 \times 10^{17} > 4.3 \times 10^{14}$ (*ii*) $1.439 \times 10^{12} > 1.4335 \times 10^{12}$.

CHAPTER 14

C1S6F7

Symmetry

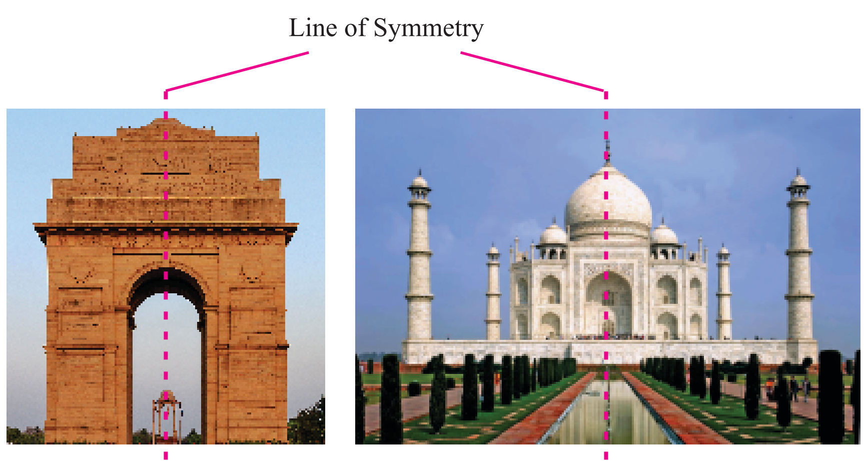
Learning Objectives :-

In this chapter, you will learn :-

1. To differentiate symmetrical and asymmetrical figures.
2. To draw lines of symmetry.
3. The concepts of rotational symmetry, centre of rotation, angle of rotation and order of rotational symmetry.
4. About the shapes that have both lines of symmetry and rotational symmetry.
5. To make use of symmetry in completing the missing half of symmetrical figures.
6. To relate the concept of symmetry with your daily life situations and develop aesthetic sense in you.

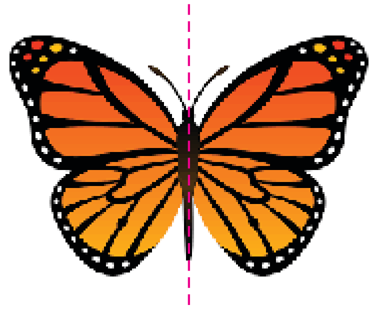
INTRODUCTION

Symmetry is an important geometrical concept that is commonly used in almost every activity of our daily life. Various professionals like architects, car manufacturers, engineers, and designers use the concept of symmetry. In class VIth we have learnt about the line of symmetry, which refers to the line that divides the shape in two identical parts. We have seen the presence of lines of symmetry in many man made things as well as in nature. Flowers, Leaves, Fish, birds, animals, human, architecture and in religious symbols everywhere we find symmetry.

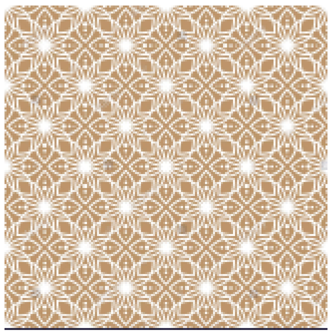


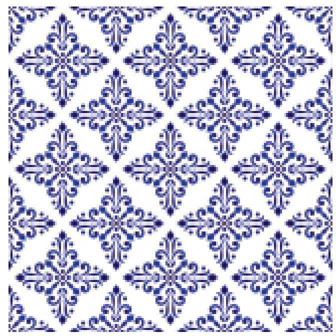
Architecture

The above architecture designs of India Gate and Taj Mahal look beautiful because of their symmetry.



Symmetry in nature

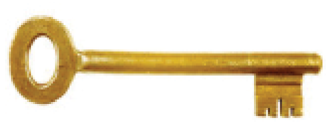




Symmetry in Cloth Designing

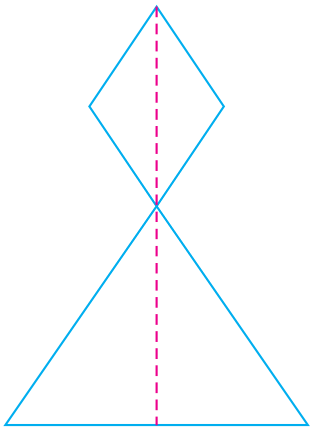
Symmetry in Enginnering

Asymmetrical figures : The objects or figures that do not have any line of symmetry are called asymmetrical figures.

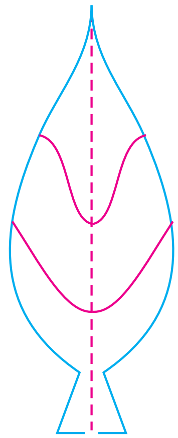


Line of symmetry : Look at the following plane figures and pictures of objects.

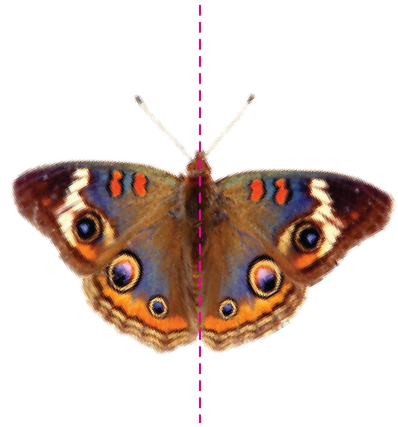
We observe that if these figures or pictures are folded along a dotted line shown in each figure on picture the left hand of dotted line fits exactly on the right side of dotted line i.e; each figure is divided into two coincident parts about the dotted line.



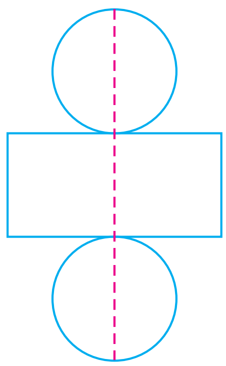
(*i*)



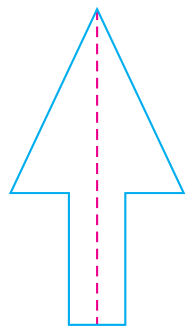
(*ii*)



(*iii*)



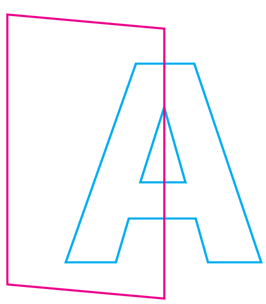
(*iv*)



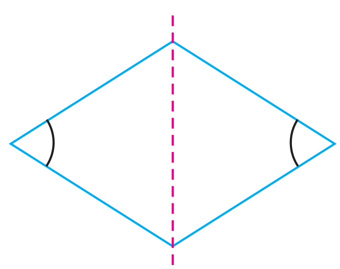
(*v*)

Thus if a figure can be divided into two coincident parts by a line then the figure is called symmetrical about the line and the line is called **line of symmetry or axis of symmetry.**

Mirror reflection : The concept of symmetry is closely related to mirror reflection. A shape has line symmetry when one half of it is the mirror image of the other half. as in (fig) (*i*) A mirror line thus helps to visualise a line of symmetry (fig *ii*)

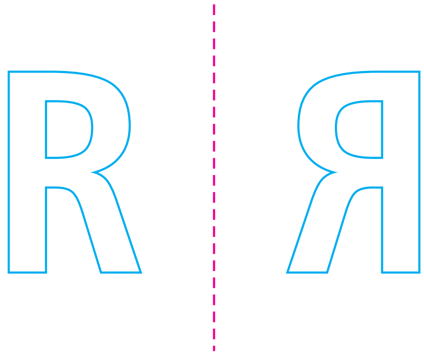


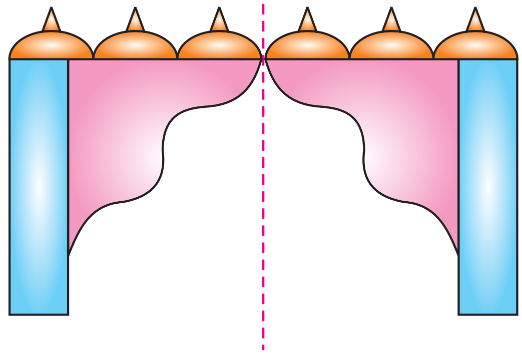
(*i*)



(*ii*)

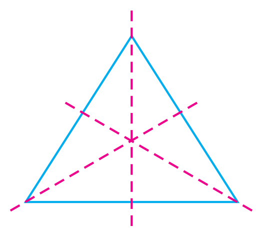
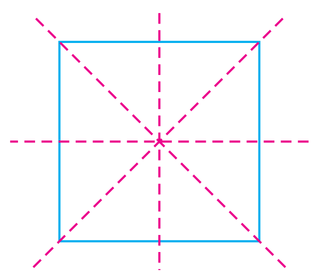
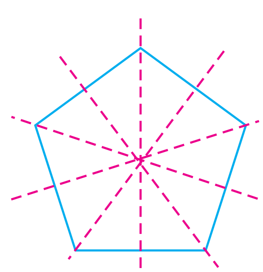
While dealing with mirror reflection, care is needed to note down the left right changes (directional changes) every thing of the image (Shape size are same but reversed). Some examples of mirror image are as shown below.



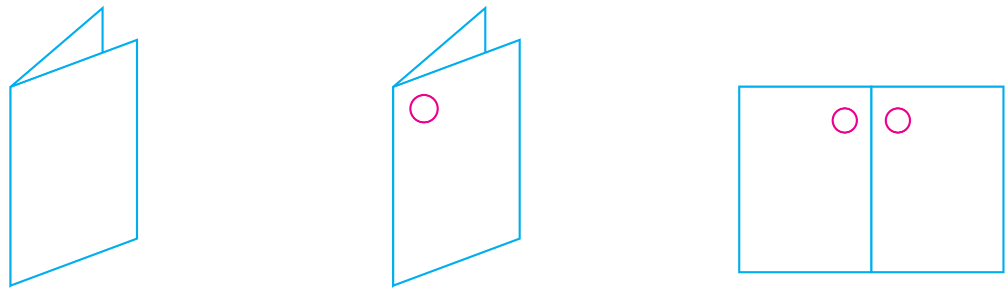


Lines of symmetry for regular polygons : A simple closed figure made up of several line segments is called a polygon. A polygon has minimum three line segments. If all the sides of a polygon are of equal length and its angles are also equal then it is said to be a regular polygon The regular polygons are symmetrical figures and have more than one line of symmetry. **In fact each regular polygon has as many line of symmetry as the number of sides.**

Some regular polygon with lines of symmetry

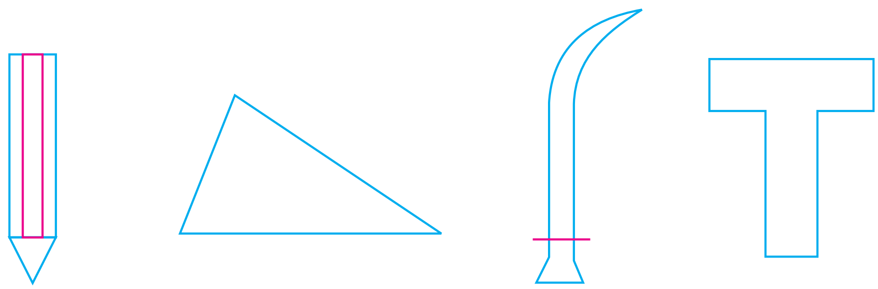
Sr. No.	Name of regular polygons and their features	Figure with lines of symmetry	No. of lines of symmetry
1.	**Equilateral triangle :** An equilateral triangle is regular because all of its sides have same length and measures of each interior angle is 60°.		3
2.	**Square :** All its four sides are of equal length and each of its interior angle is 90°. Its diagonals are prependicular bisector of each other.		4
3.	**Regular pentagon :** All its five sides are of equal length and measure of each interior angles is 108°.		5

Punching of Paper : In punching of a paper for symmetrical design, the line of fold is the line of symmetry as shown below.



(Fold a sheet into two halves) (Punch a hole) (Two holes about the symmetric fold)

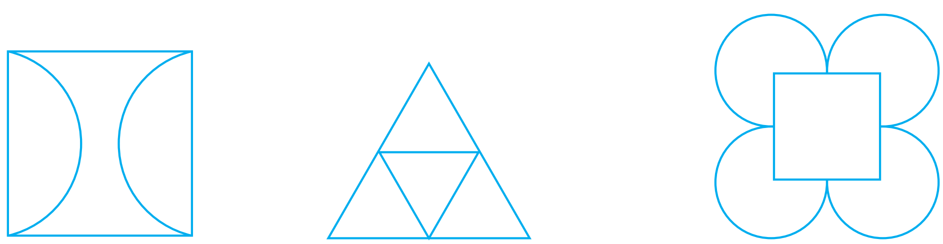
Example-1 : Which of the following figures are asymmetrical ?



(a) (b) (c) (d)

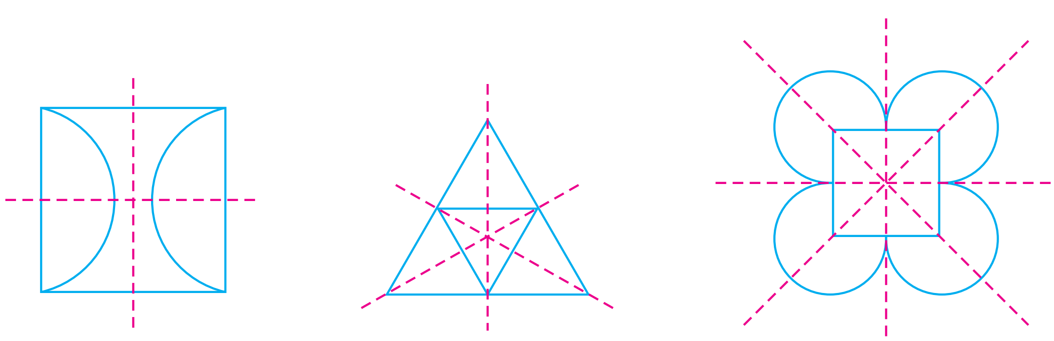
Sol. (*b*) and (*c*) are asymmetrical figures (*a*) and (*d*) are symmetrical figures

Example-2 : Draw lines of symmetry. If any in each of the following figures.

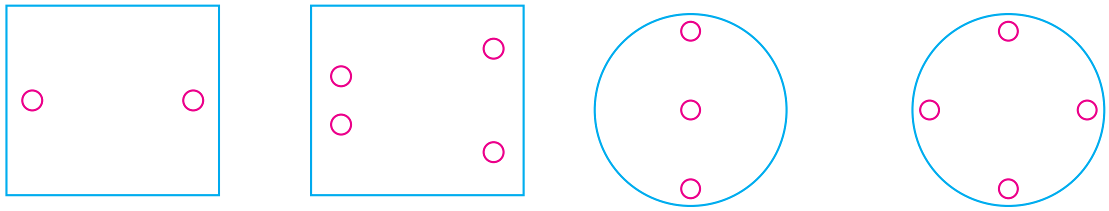


(a) (b) (c)

Sol.

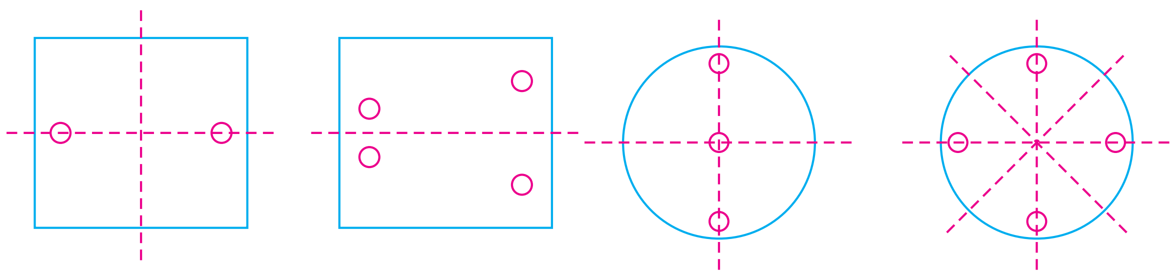


Example-3.: Copy the figures with punched holes and find the axes of symmetry for the following.

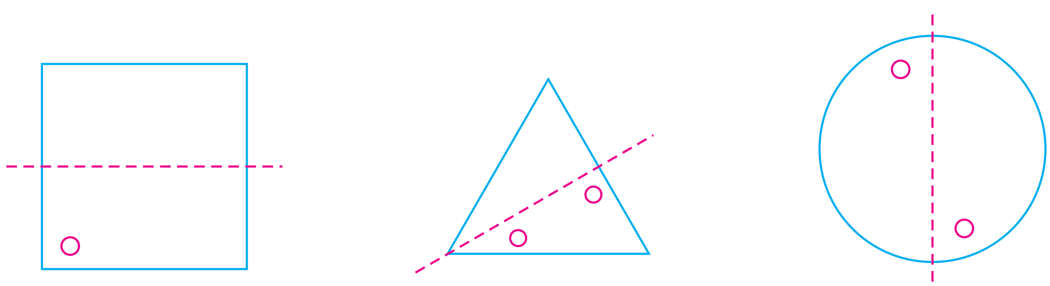


(a) (b) (c) (d)

Sol.

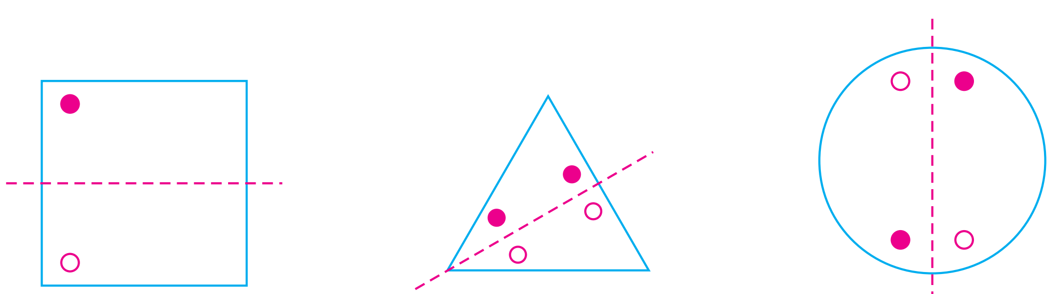


Example-4 : In the following figures mark the missing holes in order to make them symmetrical about the dotted line.

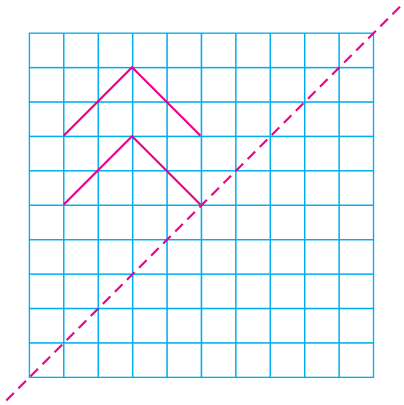


(a) (b) (c)

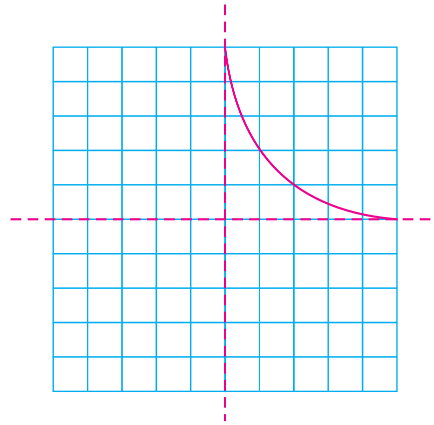
Sol. The missing holes are marked by dark punches (small circles) in each of the following figures.



Example-5 : Copy each diagram on a squared paper and complete each shape to be symmetrical about the mirror lines shown dotted.

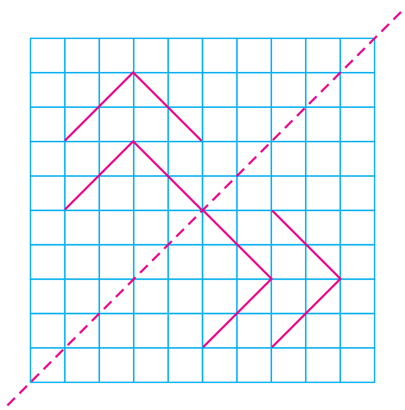


(a)

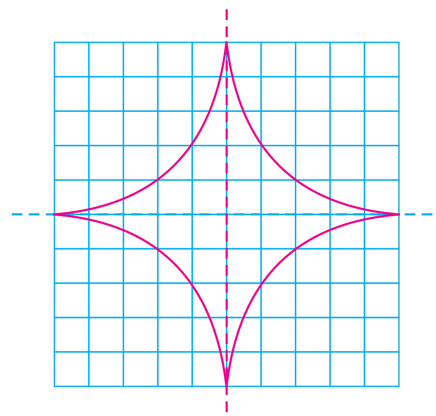


(b)

Sol. The complete shapes are given below



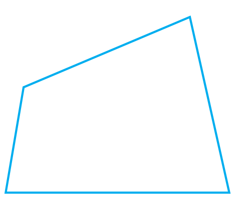
(a)



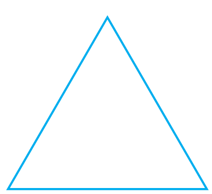
(b)

EXERCISE - 14.1

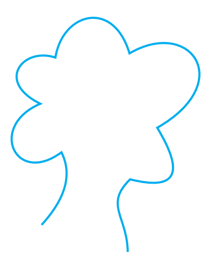
1. Which of the following figures are asymmetrical ?



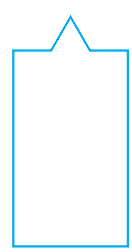
(a)



(b)

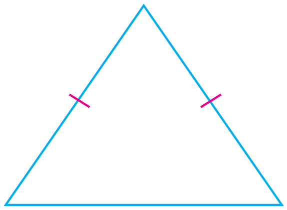


(c)

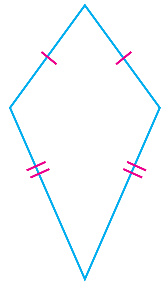


(d)

2. Draw the lines of symmetry in the following figures

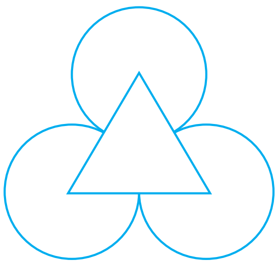

(a)

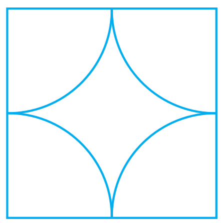

(b)

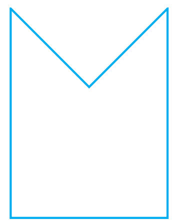

(c)

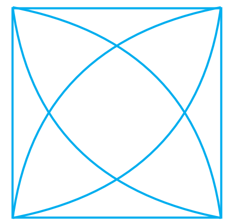

(d)

3. Draw all lines of symmetry if any in each of the following figures.

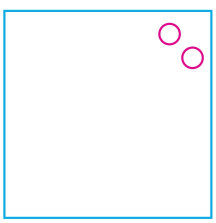

(a)

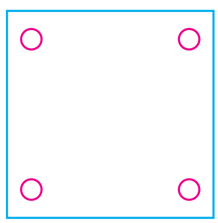

(b)

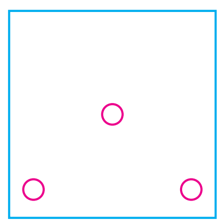

(c)

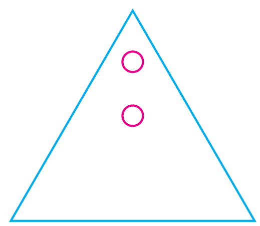

(d)

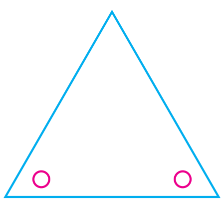
4. Copy the figures with punched holes and find the axes of symmetry for the following.

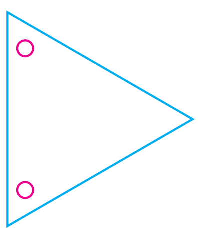

(a)

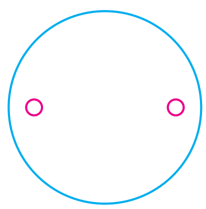

(b)

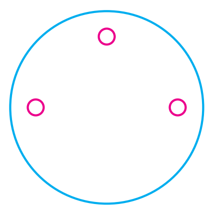

(c)


(d)

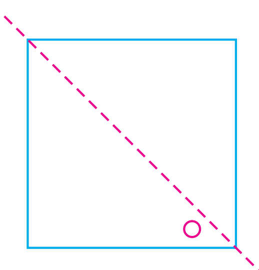

(e)

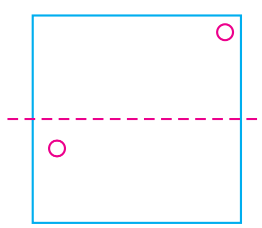

(f)

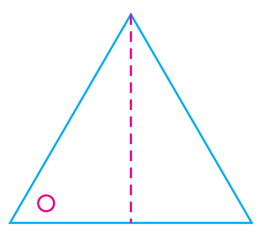

(g)

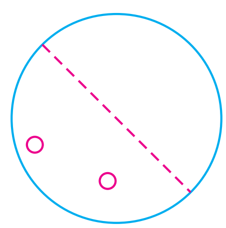

(h)

5. In the following figures mark the missing holes in order to make them symmetrical about the dotted line.

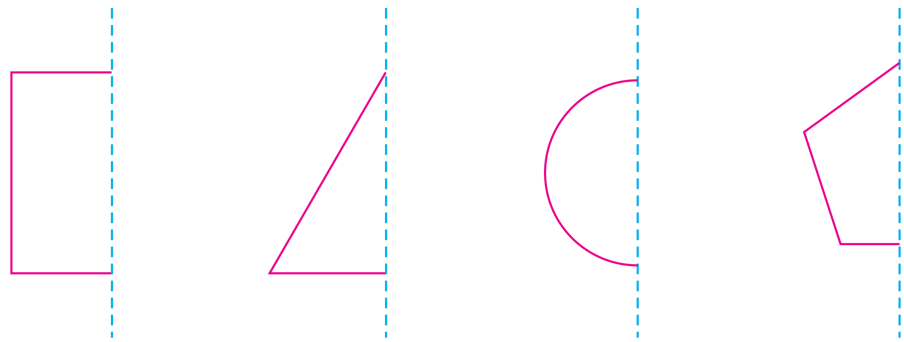

(a)


(b)


(c)

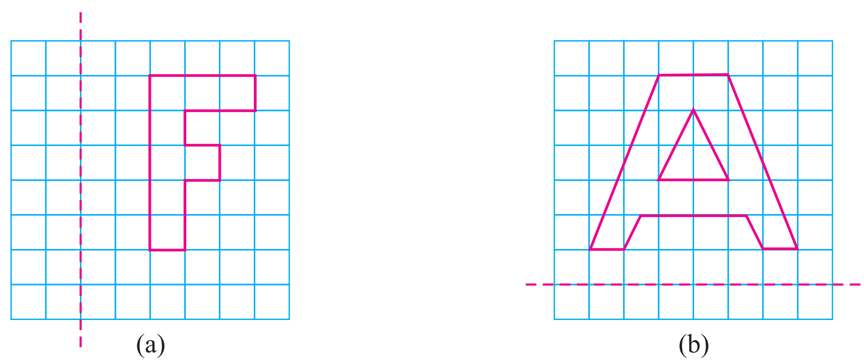

(d)

6. In each of the following figures, the mirror line (i.e; the line of symmetry) is given as dotted line complete each figure performing reflection in the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of figure you complete.



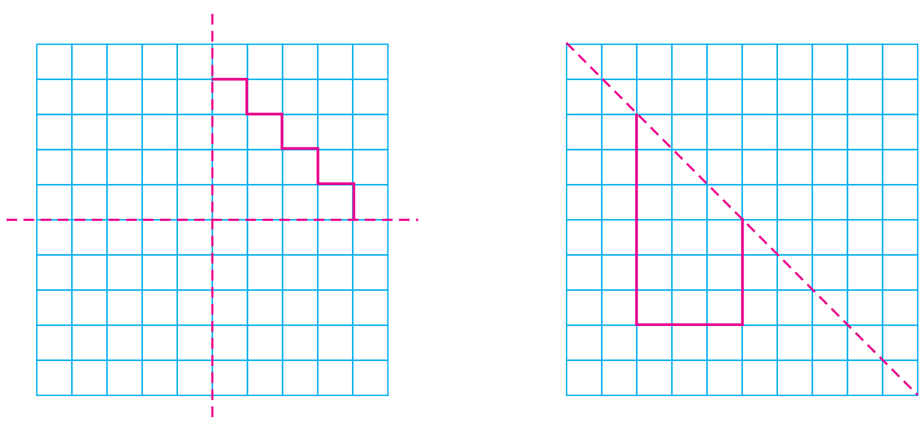
(a) (b) (c) (d)

7. Draw the reflection of the following letter in the given mirror line



(a) (b)

8. Copy each diagram on a squared paper and complete each shape to be symmetrical about the mirror lines shown dotted



(a) (b)

9. State the number of lines of symmetry for the following figures

(*a*) A scalene triangle (*b*) A rectangle

(*c*) A rhombus (*d*) A parallelogram

(*e*) A regular hexagon (*f*) A circle

10. Multiple Choice Questions.

(*i*) Which of the following triangles have no line of symmetry ?

(*a*) An equilateral triangle (*b*) An Isosceles triangle

(*c*) A scalene triangle (*d*) All of above

(*ii*) What is the other name for a line of symmetry of a circle ?

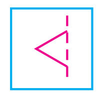
(*a*) An arc (*b*) A sector

(*c*) A diameter (*d*) A radius

(*iii*) How many lines of symmetry does a regular polygon have ?

(*a*) Infinitely many (*b*) As many as its sides

(*c*) One (*d*) Zero

(*iv*) In the given figure, the dotted line is the line of symmetry which figure is formed if the given figure is reflected in the dotted line 

(*a*) Square (*b*) Rhombus

(*c*) Triangle (*d*) Pentagon

(*v*) What is other name for a line of symmetry of an Isosceles triangle ?

(*a*) Side (*b*) Median

(*c*) Radius (*d*) Angle

(*vi*) Which of the following alphabets has a vertical line of symmetry ?

(*a*) M (*b*) Q

(*c*) E (*d*) B

(*vii*) Which of the following alphabets has a horizontal line of symmetry ?

(*a*) C (*b*) D

(*c*) K (*d*) All the above

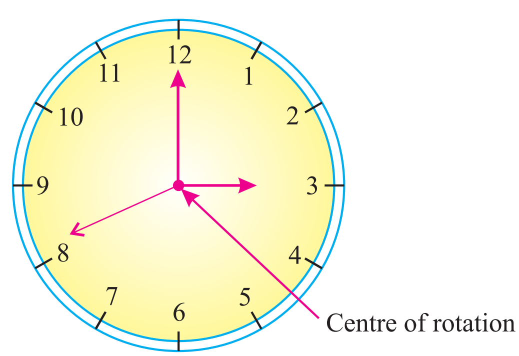
(*viii*) Which of the following alphabets has no line of symmetry ?

(*a*) A (*b*) B

(*c*) P (*d*) O

Rotational symmetry : In our daily life we see objects that rotate. Rotation is the circular movement of an object about a point. Rotation can be clockwise or anticlockwise for example, when we open the cap of a bottle the rotation is anticlockwise and when we close the cap of a bottle, the rotation is clockwise.

Centre of rotation : The fixed point about which an object rotated is called centre of rotation. For example, the centre of rotation for the hands of clock is the point where all the three hands are joined as shown in figure.



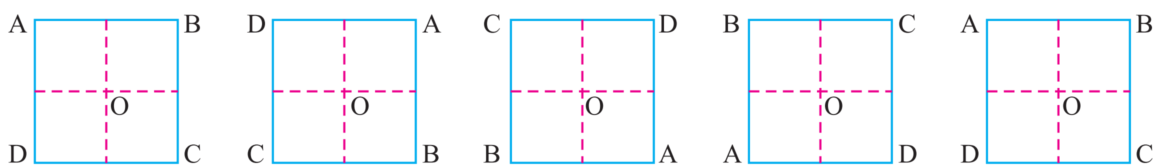
Angle of rotation : The smallest angle through which an object (or a figure) rotates about a fixed point (Centre of rotation) so that it looks the same is called angle of rotation. An object is said to take a full turn if it rotates by 360°. A half turn means a rotation by 180° and a quarter-turn means a rotation by 90°.

Order of rotational symmetry : If A° is the smallest angle through which a figure can be rotated and still looks the same, then it has a rotational symmetry of order = $\frac{360}{A°}$

For a figure having a rotational symmetry, A° must be less than or equal to 180°.

EXAMPLE OF ROTATION SYMMETRY

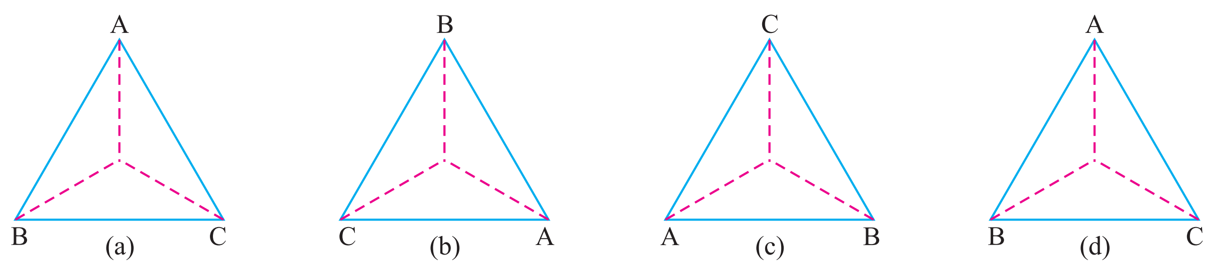
(*i*) **Rotational symmetry of a square :** Let us rotate a square ABCD in fig (*a*) to a full turn, i.e; through four positions *i.e*, 90° 180°, 270° and 360° to attain the positions shown in fig (*b*), fig (*c*), fig (*d*) and fig (*e*) respectively.



(a) Original form | (b) Rotated through 90° | (c) Rotated through 180° | (d) Rotated through 270° | (e) Rotated through 360°

Clearly, after four rotations square regains its original position so It has a rotational symmetry of order 4

(*ii*) **Rotational symmetry of Equilateral triangle :** Let us rotate an equilateral triangle ABC through an angle of 120° we observe that in a full turn, there are precisely three positions (on rotation through 120°, 240° and 360°)



(a) Original form | (b) Rotated through 120° | (c) Rotated through 240° | (d) Rotated through 360°

Thus an equilateral triangle has a rotational symmetry of order 3.

Note that in this case

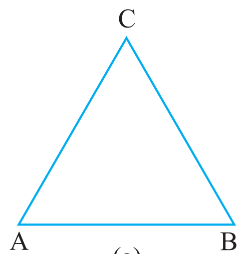
(*i*) The centre of rotation is the point of concurrence of the bisectors of the angles of triangle.

(*ii*) Angle of rotation is 120°.

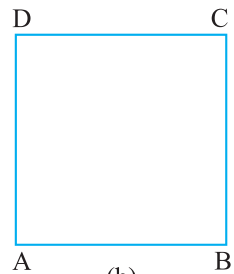
(*iii*) The direction of rotation is clockwise.

(*iv*) The order of rotational symmetry is 3.

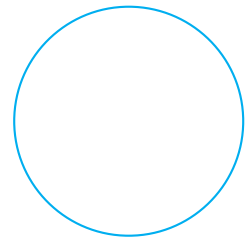
Example-1 : Write the order of rotation for the following figures



(a)



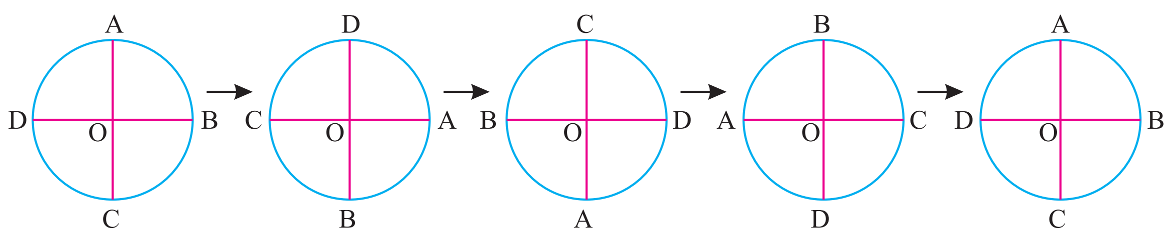
(b)


(c)

Sol.

Sr. No.	Name of figure	Order of rotation
1.	Equilateral triangle	3
2.	Square	4
3.	Circle	Infinite

Example-2 : Specify the centre of rotation, direction of rotation, angle of rotation and order of rotation for the following.



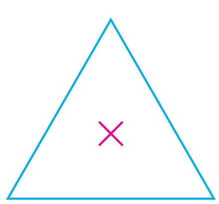
Sol. (*i*) The centre of rotation is O.

(*ii*) The direction of rotation is clockwise

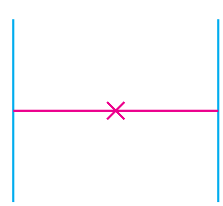
(*iii*) The angle of rotation is 90°.

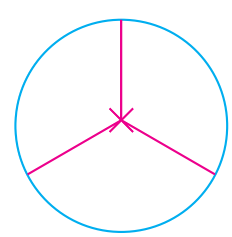
(*iv*) The order of rotation is 4.

Example-3 : Which of the following figures have rotational symmetry about the marked point, specify angle of rotation and order of rotation of the figures.


(a)


(b)


(c)


(d)

Sol. Figure (*a*) has rotational symmetry of order 3 about the marked point through an angle of 120°.

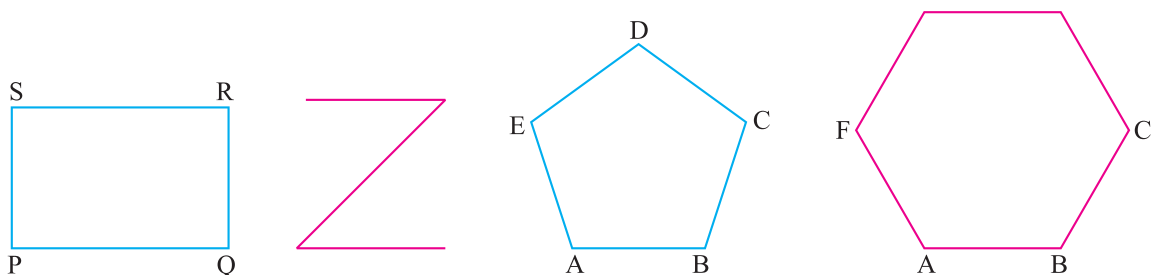
Figure (*b*) has no rotational symmetry about the marked point

Figure (*c*) has a rotational symmetry of order 2 about the marked point through an angle of 180°.

Figure (*d*) has a rotational symmetry of order 3 about the marked point through an angle of 120°.

EXERCISE - 14.2

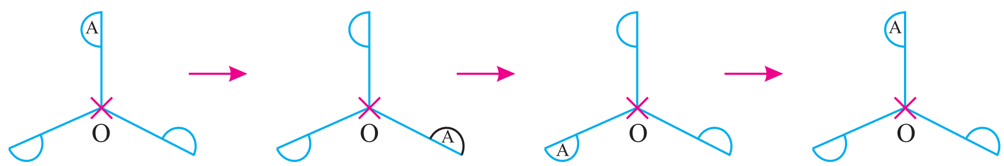
1. Write the order of rotation for the following figures.



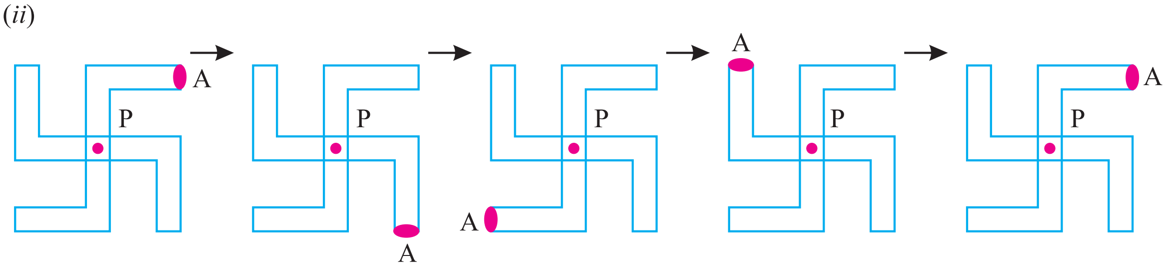
(a) (b) (c) (d)

2. Specify the centre of rotation, direction of rotation, angle of rotation and order of rotation for the following.

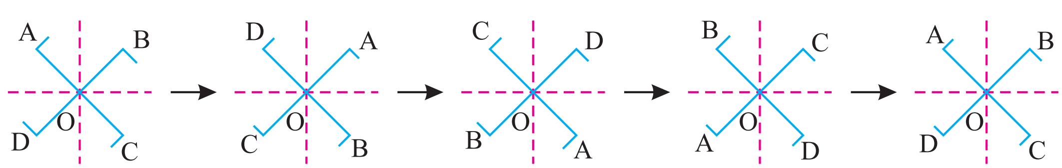
(*i*)



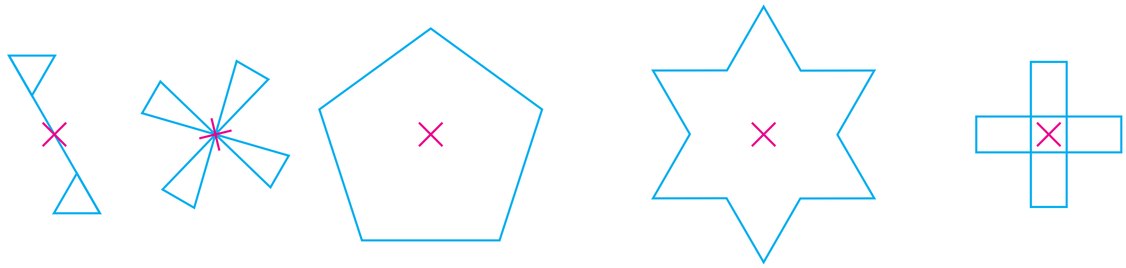
(*ii*)



(*iii*)



3. Which of the following figures have rotational symmetry about the marked point (×) give the angle of rotation and order of the rotation of the figures.



(a) (b) (c) (d) (e)

4. Multiple choice questions :-

(*i*) The angle of rotation in an equilateral triangle is

(*a*) 60° (*b*) 70°

(*c*) 90° (*d*) 120°

(*ii*) A square has a rotational symmetry of order 4 about its centre what is the angle of rotation ?

(*a*) 45° (*b*) 90°

(*c*) 180° (*d*) 270°

(*iii*) What is the order of rotational symmetry of the english alphabet Z ?

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

(*iv*) Which of these letters has only rotational symmetry ?

(*a*) S (*b*) E

(*c*) B (*d*) P

(*v*) If the smallest angle of rotation is 90° then order of symmetry is ?

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) 2

Line symmetry and rotational symmetry : We have learnt about various figure and their symmetries. Some figures have only line symmetry, some have rotational symmetry and some have both line symmetry and rotational symmetry for example :

(*i*) An isosceles triangle has a line symmetry but not rotational symmetry

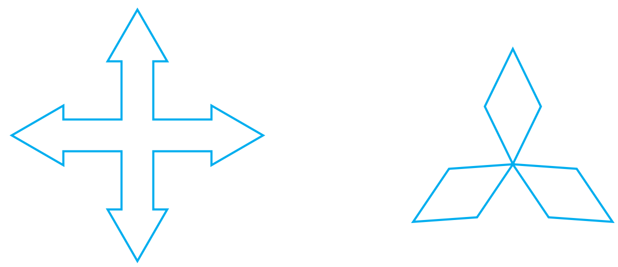
(*ii*) A parallelogram has rotational symmetry but no line symmetry.

(*iii*) A square has both line symmetry as well as rolational symmetry. In fact a square has four lines of symmetry and has rotational symmetry of order 4.

(iv) A circle is the most perfect symmetrical figure because its has an infinite number of lines of symmetry and it can be rotated about its centre at any angle and still works the same.

Note : If a figure has two or more lines of symmetry then it also has rotational symmetry.

Example-1 : In the following figures, find the number of symmetry and angle of rotation for rotational symmetry.



(a) (b)

Sol. (*a*) Number of lines of symmetry = 2

Angle of rotation = 90°

(*b*) Number of line of symmetry = 3

Angle of rotation = 120°

Example-2 : Following shapes have both line symmetry and rotational symmetry. Write number of lines of symmetry, also specify centre of rotation and write order of rotational symmetry.

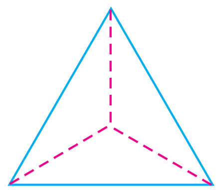
(a) (b) (c)

Sol.

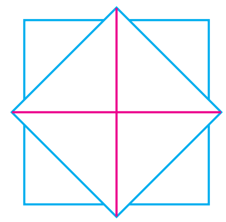
Sr. No	Name of figure	Number of lines of symmetry	Centre of rotation	Order of rotational symmetry
1.	Square	4	Intersection of diagonals	4
2.	Rectangle	2	Intersection of diagonals	2
3.	Circle	Infinite	Centre	Infinite

EXERCISE - 14.3

1. In the following figures, find the number of lines of symmetry and angle of rotation for rotational symmetry.

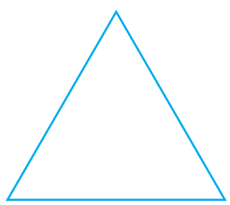


(a)

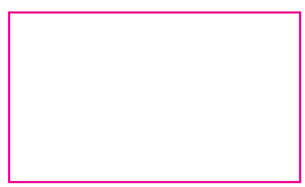


(b)

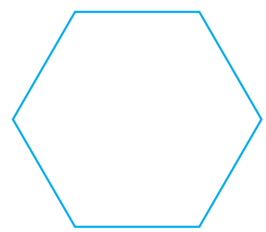
2. Name any two figures that have both line of symmetry and rotational symmetry.
3. If a figure has two or more lines of symmetry should it have a rotational symmetry of order more than 1 ?
4. Following shapes have both, line symmetry and rotational symmetry. Find the number of lines of symmetry, centre of rotation and order of rotational symmetry.



(a)



(b)



(c)

5. Some of the english alphabets have fascinating symmetrical structures. Which capital letters have just one line of symmetry (Like E) ? Which capital letters have a rotational symmetry of order 2 (Like I) ? By attempting to think on such lines, you will be able to fill in the following table.

English Alphabet	Line Symmetry	Number of lines Symmetry	Rotational Symmetry	Order of Rotational symmetry
Z	No		Yes	
S		0		2
H		2		
O	Yes			4
E	Yes	1		
N			Yes	
C	Yes			1

6. Multiple choice questions :-

(*i*) If 60° is the smallest angle of rotation for a given figure what will be the angle of rotation for same figure.

(*a*) 150° (*b*) 180°

(*c*) 90° (*d*) 330°

(*ii*) Which of these can not be a measure of an angle of rotation for any figure.

(*a*) 120° (*b*) 180°

(*c*) 17° (*d*) 90°

(*iii*) Which of the following have both line symmetry and rotational symmetry ?

(a) An isosceles triangle (b) A scalene triangle

(c) A square (d) A parallelogram

(*iv*) Which of the alphabet has both multiple line and rotational symmetries ?

(a) S (b) O

(c) H (d) L

(*v*) In the word 'MATHS' which of the following pairs of letters shows rotational symmetry?

(a) M and T (b) H and S

(c) A and S (d) T and S

WHAT HAVE WE DISCUSSED ?

1. A figure has line symmetry, if there is a line about which the figure may be folded so that two parts of figure will coincide.
2. Each regular polygon has as many lines of symmetry as the number of Its sides.
3. Mirror reflection leads to symmetry, under which the left right orientation have to be taken care of.
4. Rotation turns an object about a fixed point
   - This fixed point is the centre of rotation
   - The angle by which the object rotates is the angle of rotation.
   - A full turn means rotation by 360°, A half turn means rotation by 90° rotation may be clockwise or anticlockwise.
5. A plane figure has a rotational symmetry if on rotation through some angle ($\leq 180°$) about a point, it looks the same as it did in starting position.
6. If A° ($\leq 180°$) is the smallest angle through which a figure can be rotated and still looks the same, then it has a rotationals symmetry of order = $\dfrac{360°}{A°}$
7. If the order of rotational symmetry is 1, Then the figure is said to have no rotational symmetry
8. Some figure (or shapes) have only line symmetry, some have only rotational symmetry and some have both line symmetry as well as rotational symmetry.

LEARNING OUTCOMES

After completion of this chapter the students are now able to :

1. Differentiate between symmetrical and asymmetrical figures.
2. Draw lines of symmetry.

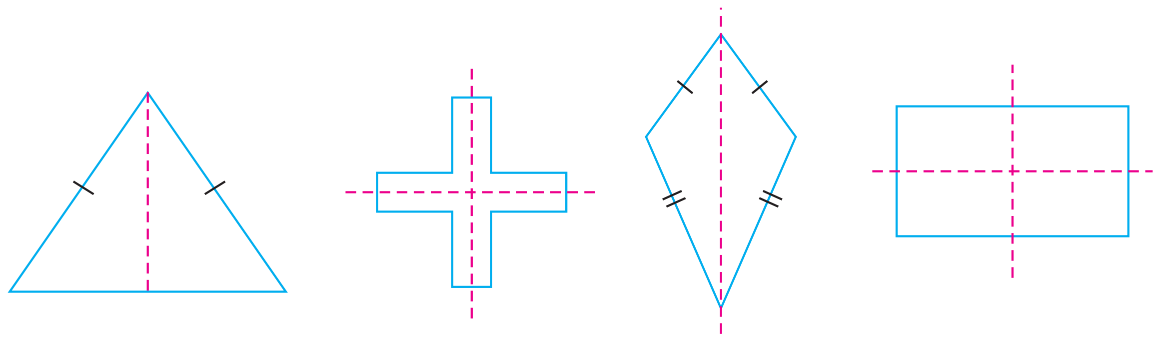
3. Differentiate between line symmetry and rotational symmetry.
4. Find the centre of rotation, the angle of rotation and the order of rotation.
5. Derive equivalent positions for mirrors and certain rotations.

ANSWERS

EXERCISE 14.1

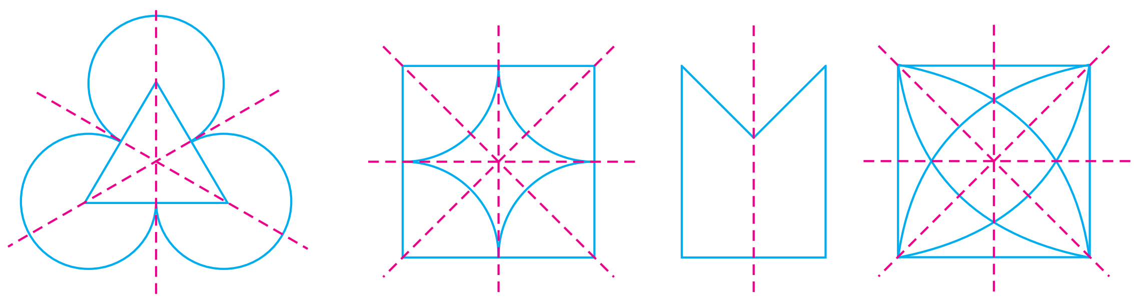
1. Figure (*a*) and (*c*) are asymmetrical

2.



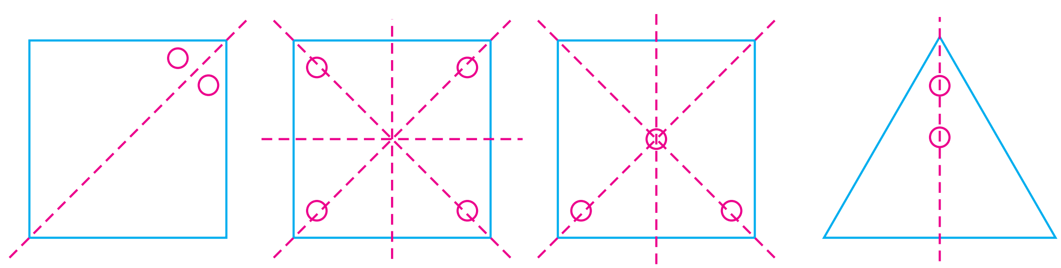
(a) (b) (c) (d)

3.

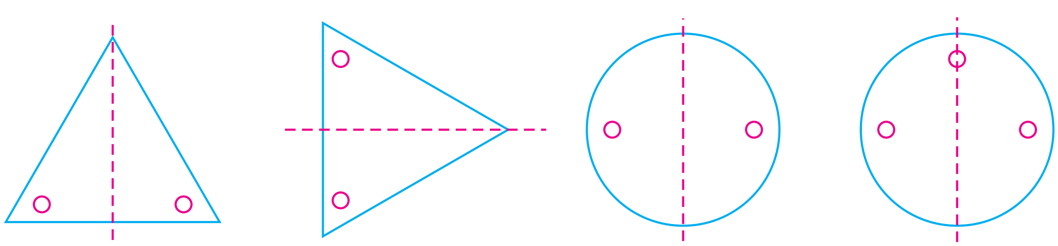


(a) (b) (c) (d)

4.

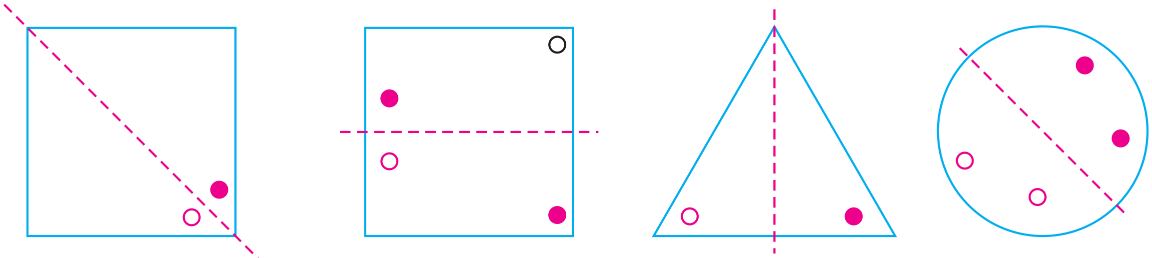


(a) (b) (c) (d)



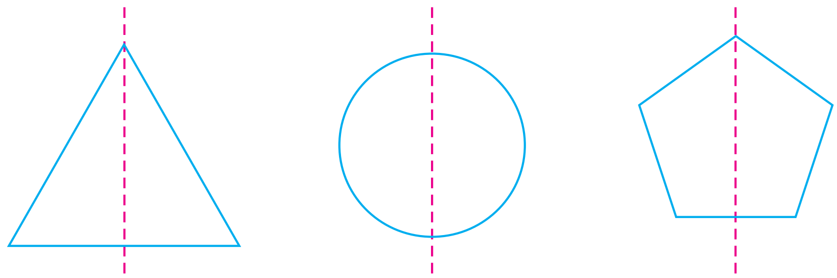
(e) (f) (g) (h)

5. The missing holes are marked by dark punches (small circles) in each of following figures



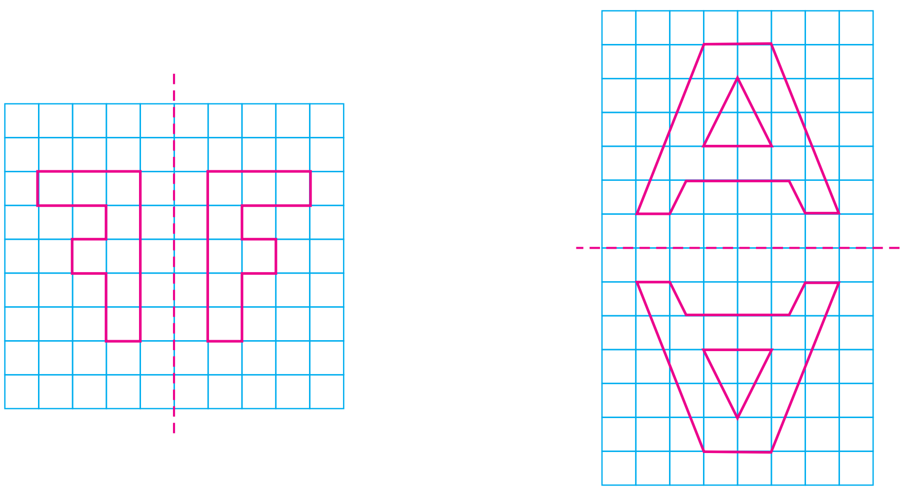
(a) (b) (c) (d)

6.



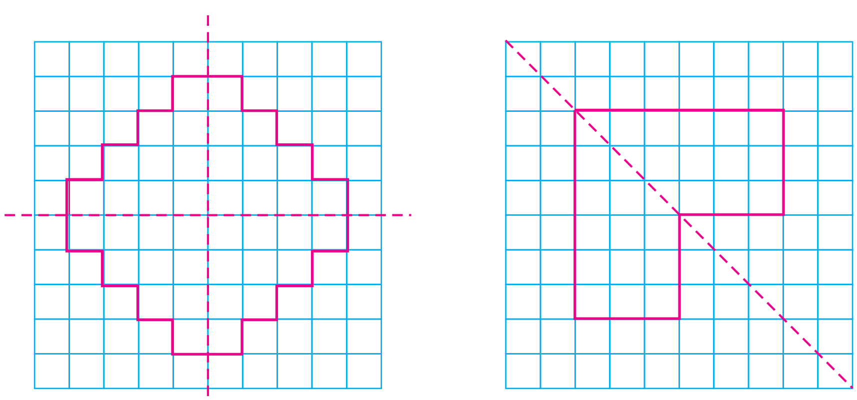
(a) Square (b) Triangle (c) Circle (d) Pentagon

7.



(a) (b)

8.



(a) (b)

9. (*a*) 0 (*b*) 2 (*c*) 2
(*d*) 0 (*e*) 6 (*f*) Infinite

10. (*i*) *a* (*ii*) *c* (*iii*) *b*
(*iv*) *b* (*v*) *b* (*vi*) *a*
(*vii*) *d* (*viii*) *c*

EXERCISE 14.2

1. (*a*) 2 (*b*) 2
(*c*) 5 (*d*) 6

2. (*i*) Centre of rotation is O, direction of rotation is clockwise, Angle of rotation is 120° and order of rotation is 3.
(*ii*) Centre of rotation is P, direction of rotation is clockwise, Angle of rotation is 90° and order of rotation is 4.
(*iii*) Centre of rotation is O, direction of rotation is clockwise, Angle of rotation is 90° and order of rotation is 4.

3. (*a*) It has rotational symmetry, angle of rotation 180° and order of rotation 2
(*b*) It has rotational symmetry angle of rotation is 90° and order of rotation 4
(*c*) It has rotational symmetry angle of rotation is 72° and order of rotation 5
(*d*) It has rotational symmetry angle of rotation is 60° and order of rotation 6
(*e*) It has rotational symmetry angle of rotation is 90° and order of rotation 4

4. (*i*) *d* (*ii*) *b* (*iii*) c
(*iv*) *a* (*v*) *c*

EXERCISE 14.3

1. (*a*) Line of symmetry 3, angle of rotation 120°.
(*b*) Line of symmetry 4, angle of rotation 90°.

2. Equilateral triangle and circle

3. Yes, Square has four lines of symmetry and rotational symmetry of order 4.

4. (*a*) 3, centroid , 3 (*b*) 2, Intersection of diagonals, 2
(c) 6, centre of hexagon, 6

5.

Alphabet Letters	Line Symmetry	No of lines Symmetry	Rotational Symmetry	Order of Rotational symmetry
Z	No	0	Yes	2
S	No	0	Yes	2
H	Yes	2	Yes	2
O	Yes	2	Yes	4
E	Yes	1	Yes	1
N	No	0	Yes	2
C	Yes	1	Yes	1

6. (*i*) *b* (*ii*) *c* (*iii*) *c*
(*iv*) *b* (*v*) *b*

CHAPTER 15

Q6E9Y6

Visualising Solid Shapes

Learning Objectives :-

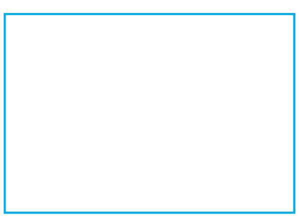
In this chapter, you will learn :-

1. To associate 2–D shapes with 3-D shapes.
2. To understand and identify faces, edges and vertices of solid figures.
3. To identify nets of different 3–D shapes and use them to form those 3–D shapes.
4. About oblique sketches and isometric sketches and also their differences.
5. To visualize solid shapes in different ways and also to see the hidden parts of a solid.
6. To apply your knowledge of solids in your day to day life.

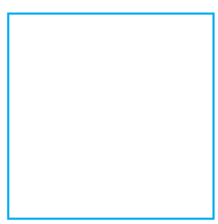
INTRODUCTION

In this chapter we will discuss about plane figures and solid shapes.

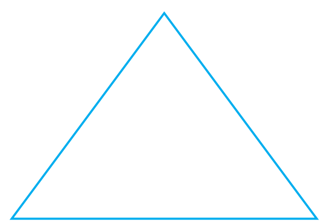
Plane figures : In previous classes, we have learnt to draw some figures like square, rectangle, triangle, quadrilateral, circle etc. These figure have two dimensions namely length and breadth and can be drawn on a paper, these figure are called two dimensional (2 – D) figure or plane figures. Some types of plane figure are as follows.



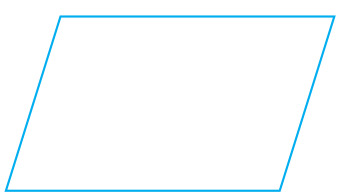
(i) Rectangle



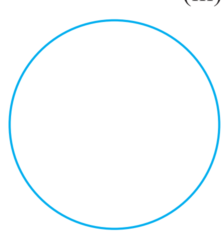
(ii) Square



(iii) Triangle



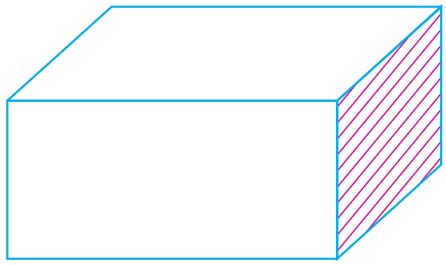
(iv) Quadrilateral



(v) Circle

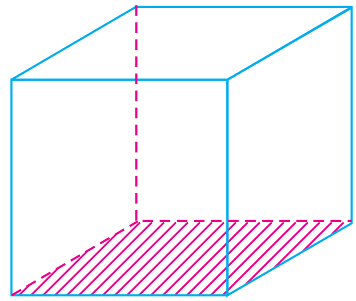
Solid Shapes : In Our daily life we come across various objects like books, packing boxes, road rollers, balls and ice cream cones etc. These type of objects having length, breadth and height are known as three dimensional (3 – D) figures or solids, because they have a definite shape and occupy space. Some types of solid shapes are as follows.

(i)



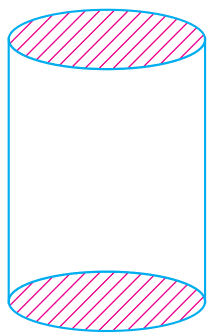
Cuboid

(ii)



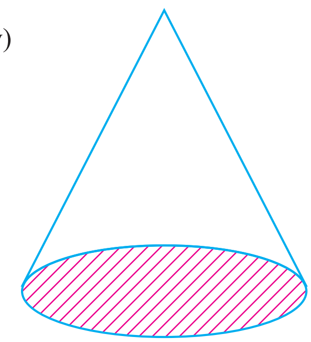
Cube

(iii)



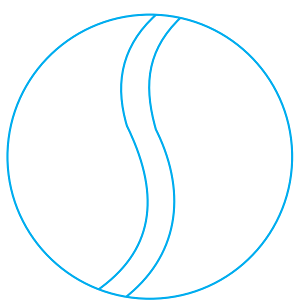
Cylinder

(iv)



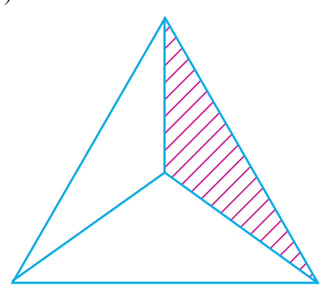
Cone

(v)



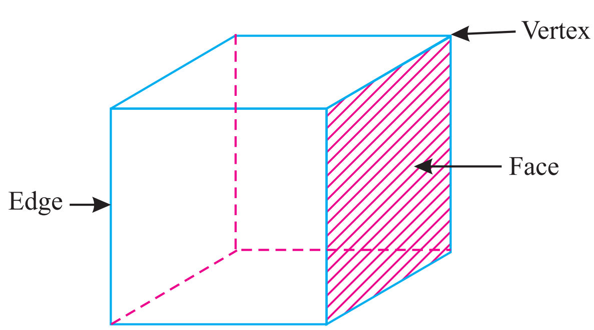
Sphere

(vi)



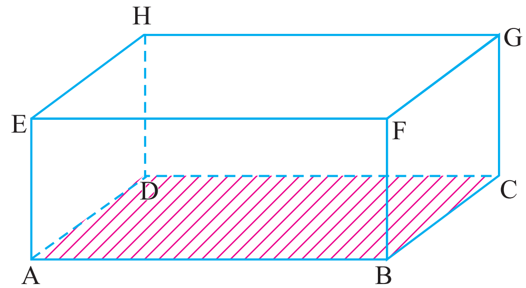
Triangular pyramid

Faces, edges and vertices : You have studied faces, edges and vertics of solid shapes. Let us revise the terms related to solid shapes.



DIFFERENT SOLID SHAPES AND THEIR FEATURES

Cuboid : A solid bounded by six rectangular faces at right angle to each other is called a cuboid. The figure shows a cuboid ABCDEFGH having :

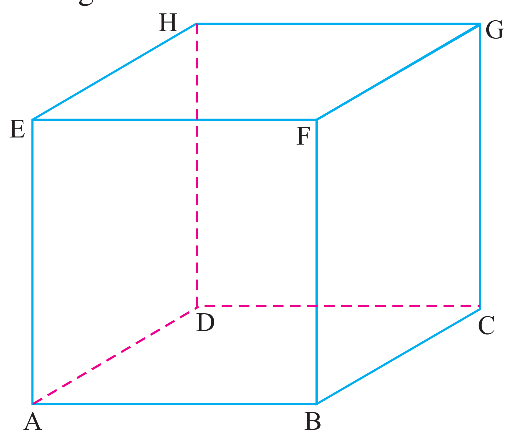


(*i*) **Faces :** It has 6 rectangular faces ABCD, EFGH, ADHE, BCGH, ABEF and DCGH. Out of these six faces ABFE, DCGH, BCGE and ADHE are called lateral faces of cuboid

(*ii*) **Edges :** It has 12 edges AB, BC, CD, DA, EF, FG, GH, HE, BF, CG, AE and DH

(*iii*) **Vertices :** It has 8 vertices A, B, C, D, E, F, G and H

Cube : A cuboid whose length, breadth and height are equal is called a cube the figure shows a cube ABCDEFGH having :



(*i*) **Faces :**It has 6 square faces ABCD, EFGH, ADHE, BCGH, ABEF and DCGH and lateral faces are ABFE, DCGH, BCGF and ADHE.

(*ii*) **Edges :** It has 12 edges AB, BC, CD, DA, EF, FG, GH, HE, BF, CG, AE and DH

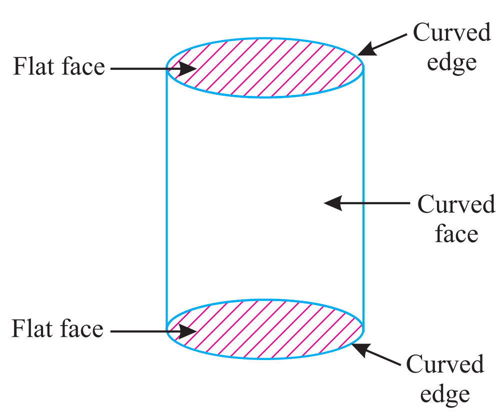
(*iii*) **Vertices :** It has 8 vertices A, B, C, D, E, F and H.

Cylinder : A cylinder is a three dimensional solid that contains two parallel bases connected by a curved surface. The bases are usally circular in shape for example :- pipes, cold drink cans, road roller. The figure shows a cylinder having

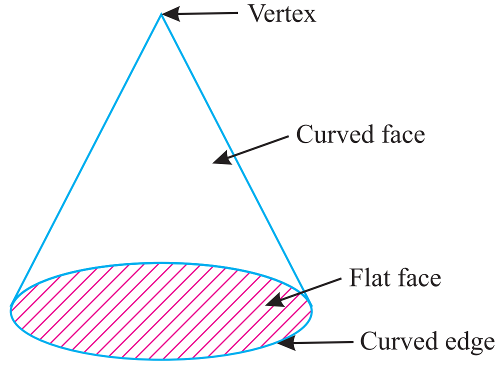
(*i*) **Faces :** It has two flat faces and one curved face.

(*ii*) **Edges :** It has two curved edges.

(*iii*) **Vertices :** It has no vertex.



Cone : A cone is a three dimensional shape that tapers smoothly from a flat base to a point called vertex for example : Ice cream cone, funnel, a conical tent. The figure shows a cone having

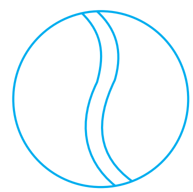


(*i*) **Faces :** It has one flat and one curved face.

(*ii*) **Edge :** It has one cured edge

(*iii*) **Vertex :** It has one vertex

Sphere : A three dimensional figure which is absolutely round like a ball is called a sphere.



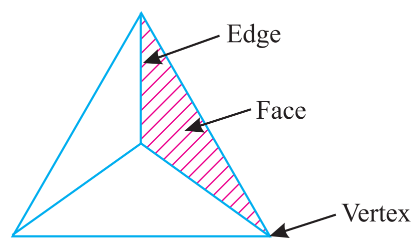
The figure shows a sphere

(*i*) It has a curved surface

(*ii*) It has no edge

(*iii*) It has no vertex

Triangular Pyramid : A triangular pyramid is a Pyramid which has a triangular base. The figure shows a triangular pyramid having :-



Faces : It has 4 faces

Edges : It has 6 edges

Vertices : It has 4 vertices

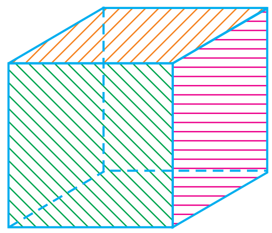
SUMMARY

Sr No.	Name of the solid	Number of faces	Number of edges	Number of vertices
1.	Cuboid	6	12	8
2.	Cube	6	12	8
3.	Cylinder	3	12	NIL
4.	Cone	2	1	1
5.	Sphere	1	NIL	NIL
6.	triangular pyramid	4	6	4

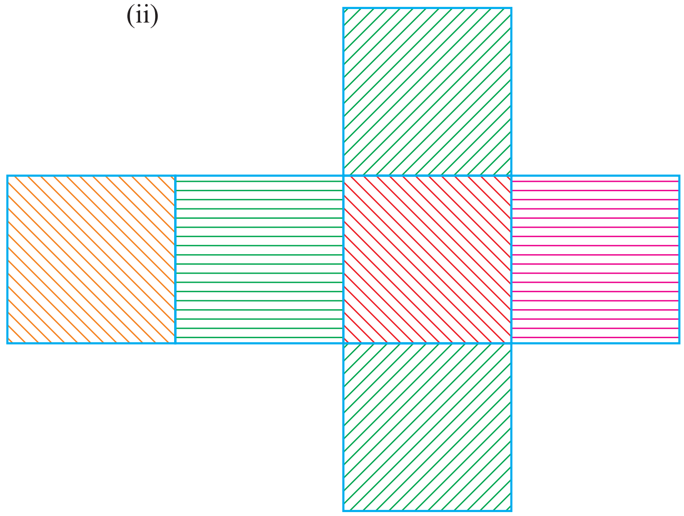
Nets for Building 3 – D shapes : Net is a two dimensional shape that can be folded to form a three dimensional shape or a solid. A solid may have different nets.

The figure (*i*) Shows a cube and figure (*ii*) is net of cube.

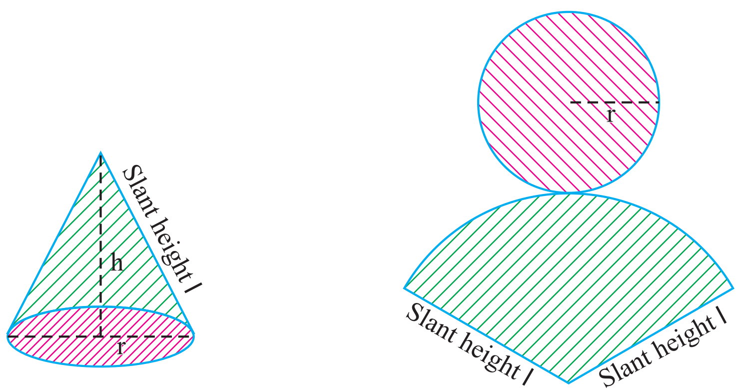
(i)



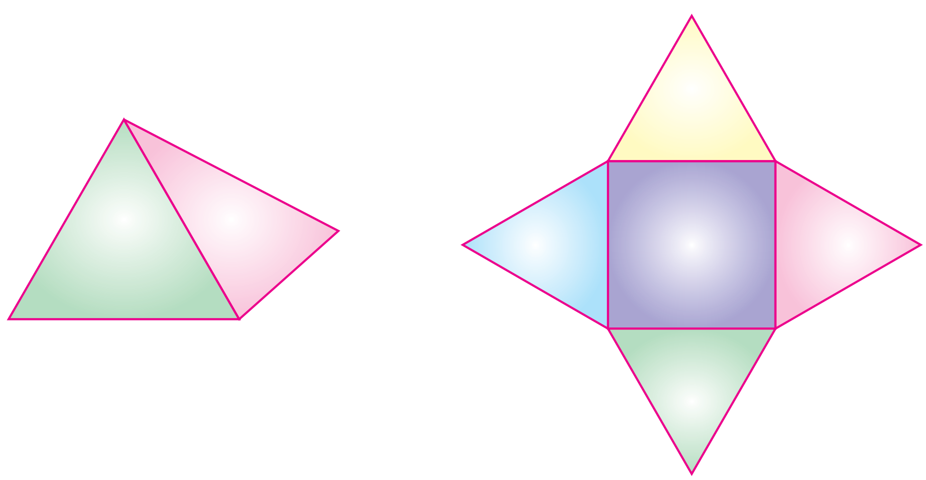
(ii)



Similarly, you can get a net for a cone by cutting a slit along its slant surface.

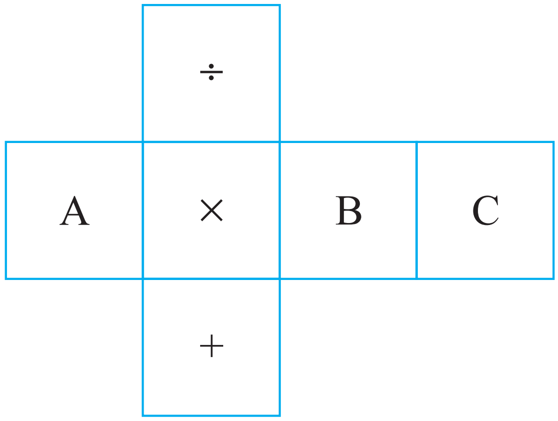


Net of great pyramid of Egypt which has a square base and triangles on the four side is as follows.

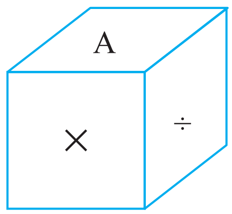


We can also make 3 – D shapes from different nets.

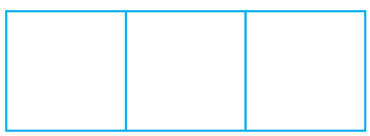
Example-1 : Draw the correct solid shape with the help of the given net.



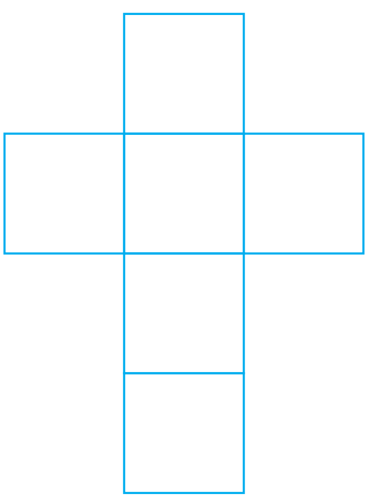
Sol.

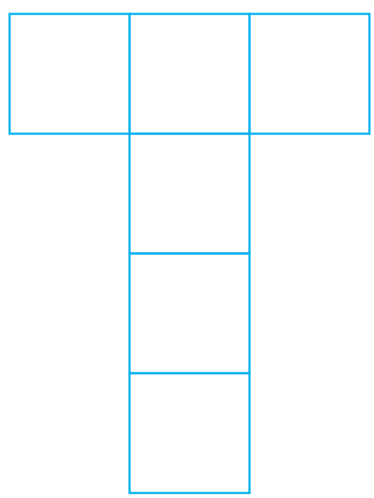


Example-2 : The following net is an in complete net for making a cube complete it in atleast two ways (seprate diagrams) Remember that a cube has six faces.

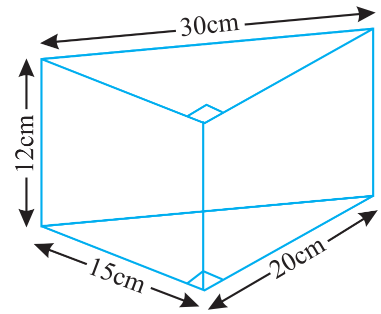


Sol. **A cube has six faces so net for making a cube in atleast two different ways are as follows.**

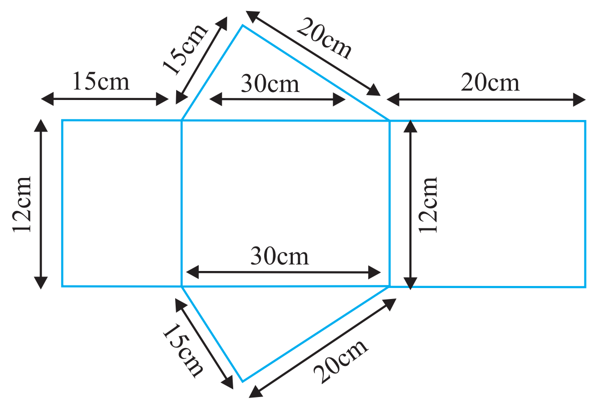




Example-3 : Draw the net of the solid given in the figure

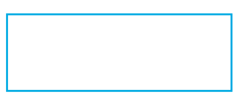


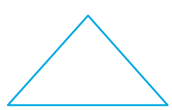
Sol. The net of given solid figure is

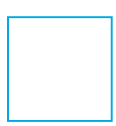


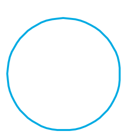
EXERCISE - 15.1

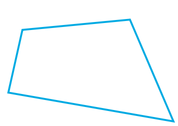
1. Match the two dimensional figure with the names

(*i*)  (*a*) Square

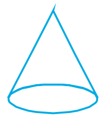
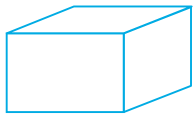
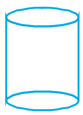
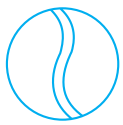
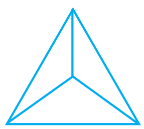
(*ii*)  (*b*) Circle

(*iii*)  (*c*) Quadrilateral

(*iv*)  (*d*) Triangle

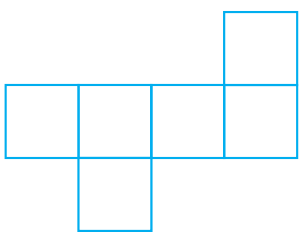
(*v*)  (*e*) Rectangle

2. Match the three dimension shapes with the names.

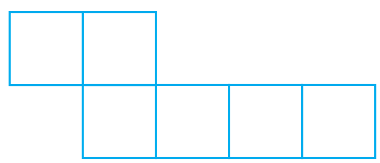
(*i*)		(*a*)	Cylinder
(*ii*)		(*b*)	Triangular pyramid
(*iii*)		(*c*)	Sphere
(*iv*)		(*d*)	Cone
(*v*)		(*e*)	Cuboid

3. Identify the nets which can be used to make cubes (cut out copies of the nets and try it)

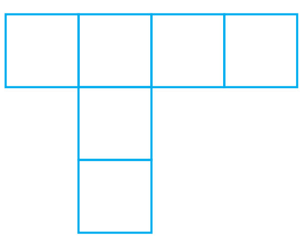
(i)



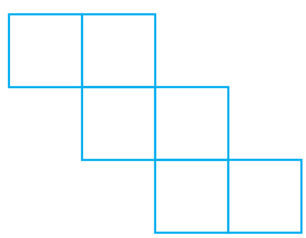
(ii)



(iii)

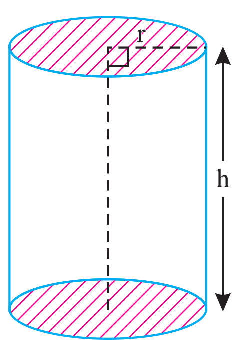


(iv)

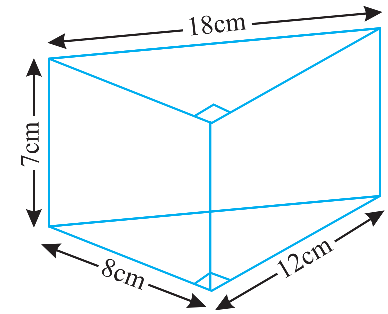


4. Draw the net for a square pyramid with base as square of sides 5*cm* and slant edges 7*cm*.

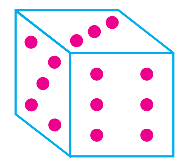
5. Draw a net for the following cylinder.



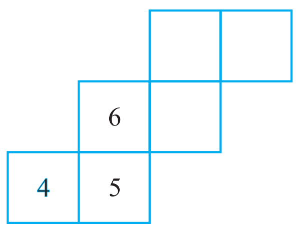
6. Draw the net of the solid given in figure.



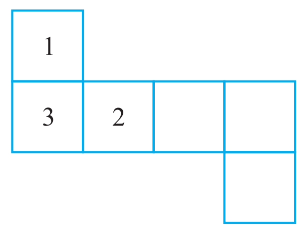
7. Dice are cubes with dots on each face oppsite faces of a die always have a total of seven dots on them following are two nets to make dice (cuber) the number inserted in each square indicate the number of dots in that box insert suitable number in the blank squares.



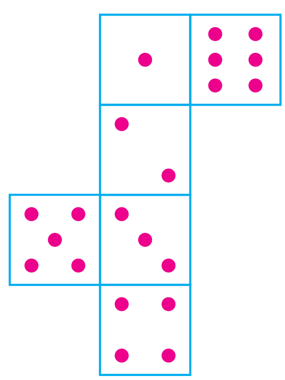
(i)



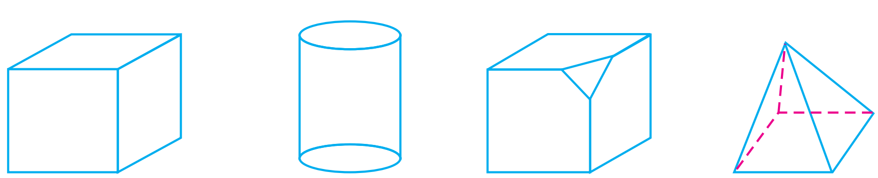
(ii)



8. Which solid will be obtained by folding the following net.



9. Complete the following table



(i) (ii) (iii) (iv)

Face		3		
Edges	12			8
Vertices	8		10	

10. Multiple choice questions

(*i*) Out of following which is 3–D figure ?

(*a*) Square (*b*) Triangle

(*c*) Sphere (*d*) Circle

(*ii*) Total number of faces a cylinder has

(*a*) 0 (*b*) 2

(*c*) 1 (*d*) 3

(*iii*) How many edges are there in a square pyamid ?

(*a*) 5 (*b*) 8

(*c*) 7 (*d*) 4

(*iv*) Sum of number on the opposite faces of a die is

(*a*) 8 (*b*) 7

(*c*) 9 (*d*) 6

(*v*) Which is not a solid figure ?

(*a*) Cuboid (*b*) Sphere

(*c*) Quadrilateral (*d*) Pyramid

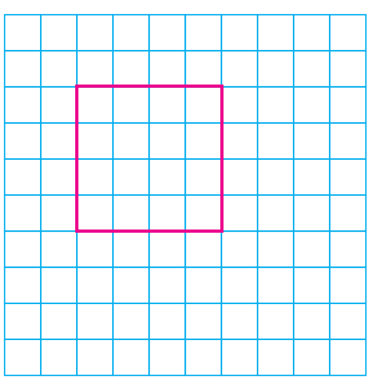
DRAWING SOLIDS ON A FLAT SURFACE

When we draw a solid shape, the images are somewhat distorted because our drawing surface is paper, which is flat. To make them appear three dimensional, there are two ways to draw 3–D figures.

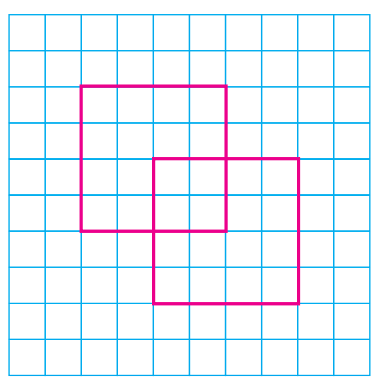
1. Oblique sketches

2. Isometric sketches

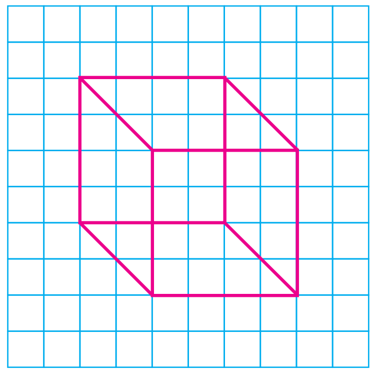
Oblique sketches : The oblique sketching is a pictorial representation of an object, in which the diagram is intended to depict the perspective of object in three dimensions. Following are the steps to draw oblique sketch of a cube.



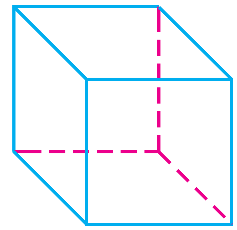
Step 1
Draw a square



Step 2
Draw the second square where the midpoints of two sides of both square coincide



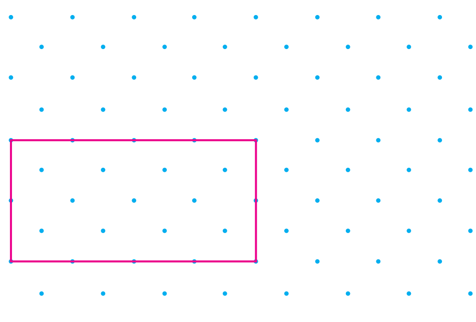
Step 3
Join the corresponding vertices
of both the squares



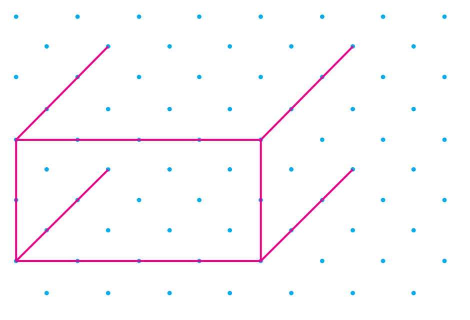
Step 4
Redraw using dotted lines
for hidden edges

Similarly we could make an oblique sketch of cuboid (Remember faces of cuboid are rectangles)

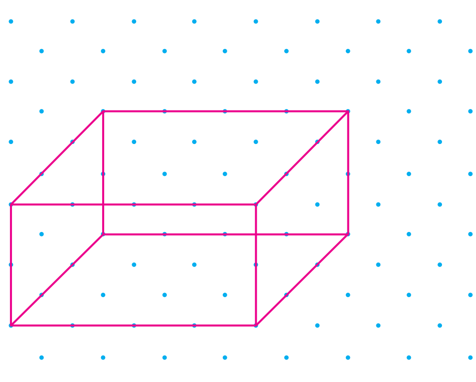
***(ii)* Isometric sketches :** An Isometric sheet is a special sheet on which dots are formed a pattern of equilateral triangles. In an isometric sketch measurement are kept proportional. Following are steps to draw isometric sketch of a cuboid having dimensions $4 \times 3 \times 2$



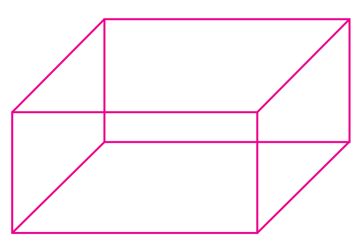
Step 1
Draw a rectangle to depict the front face



Step 2
Draw four parallel line segments
of length starting from each of
the four corners rectangle



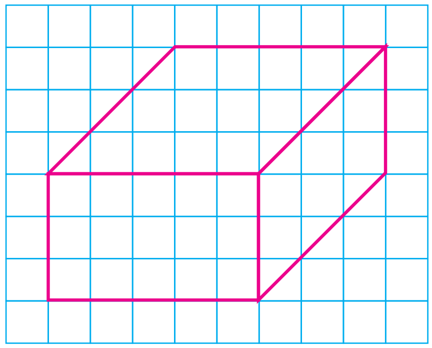
Step 3
Join the matching corners with an
appropriate line segments



Step 4
This is an isometric sketch
of a cuboid

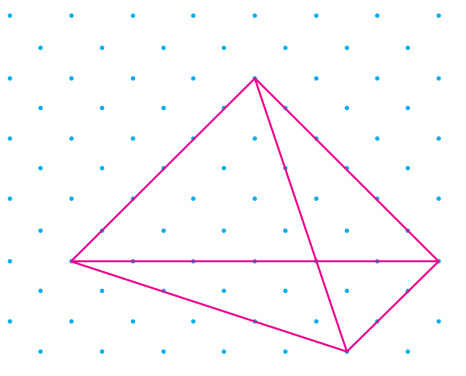
Example-1 : Length of a cuboid is 5*cm*, Breadth is 4*cm* and height is 3*cm* draw oblique sketch of this cuboid.

Sol. Oblique sketch of a cuboid with length 5*cm*, breadth 4*cm* and height 3*cm* is as follows.



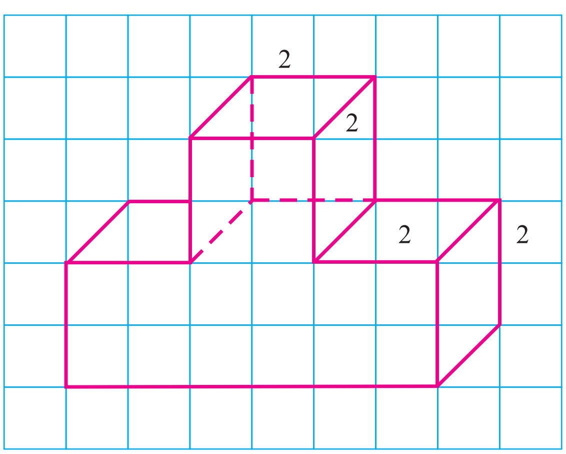
Example-2 : Draw an Isometric sketch of a trianglar pyramid

Sol. Isometric sketch of triangular pyramid is as follows.

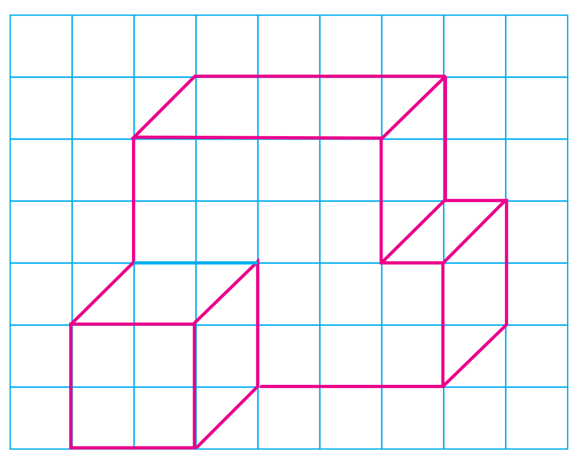


EXERCISE - 15.2

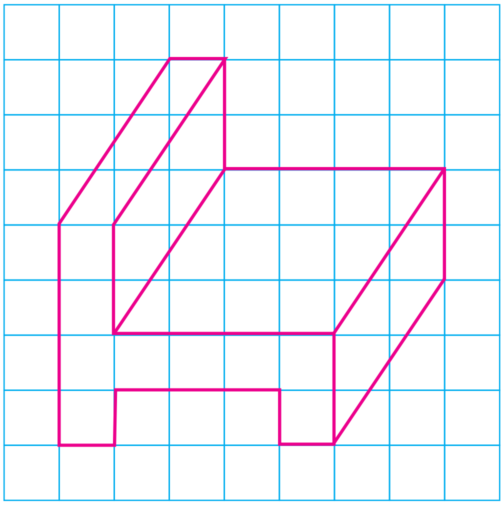
1. Use Isometric dot paper to make an Isometric sketch of the following figures



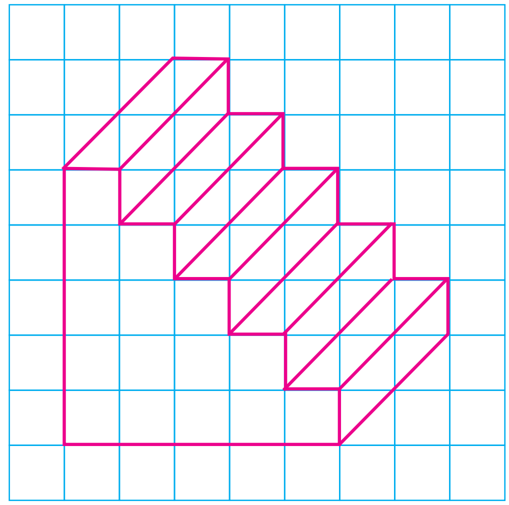
(i)



(ii)



(iii)



(iv)

2. Draw (*i*) an oblique sketch (*ii*) Isometric sketch for

(*a*) A cube with a edge of 4*cm* long

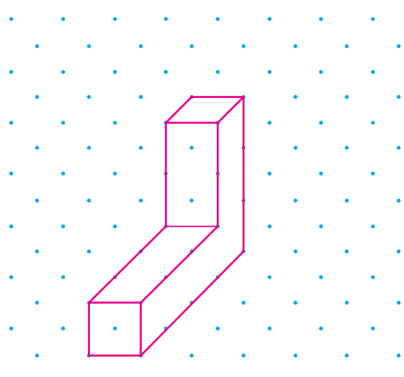
(*b*) A cuboid of length 6*cm*, breadth 4*cm* and height 3*cm*

3. Two cubes each with edge 3*cm* are placed side by side to form a cuboid, sketch oblique and isometric sketch of this cuboid.

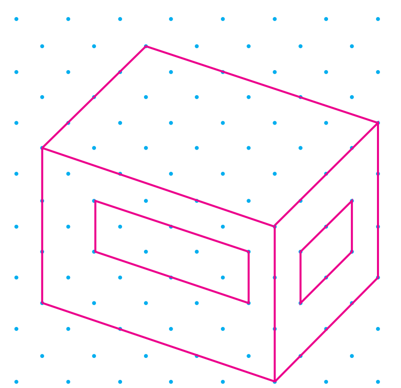
4. Draw an Isometric sketch of triangular pyramid with base as equilateral triangle of 6*cm* and height 4*cm*.

5. Draw an Isometric sketch of square pyramid

6. Make an oblique sketch for each of the given Isometric shapes

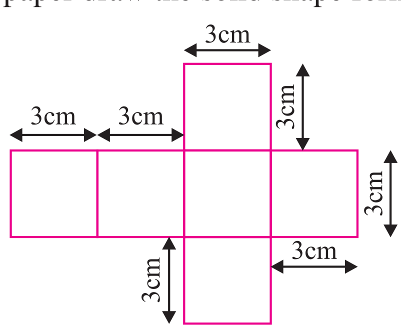


(i)



(ii)

7. Using an isometric dot paper draw the solid shape formed by the given net.



8. Multiple choice questions :-

(*i*) An oblique sheet is made up of

(*a*) Rectangles (*b*) Squares

(*c*) Right angled triangles (*d*) Equilateral triangles

(*ii*) An isometric sheet is made up of dots forming

(*a*) Squares (*b*) Rectangles

(*c*) Equilateral triangles (*d*) Right angled triangle

(*iii*) An oblique sketch has

(*a*) Proportional lengths (*b*) Parallel lengths

(*c*) Non proportional lengths (*d*) Prependicular lengths

(*iv*) An Isometric sketch has.

(*a*) Non propertional lengths (*b*) Parallel lengths

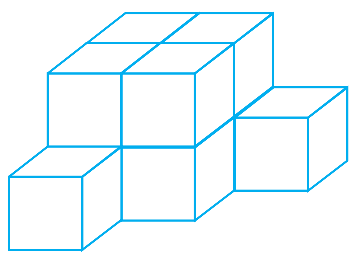
(*c*) Prependicular lengths (*d*) Proporional lengths

(*v*) Isometric sketches shows objects of

(*a*) Two dimensions (*b*) Shadows

(*c*) Three dimensions (*d*) One dimension

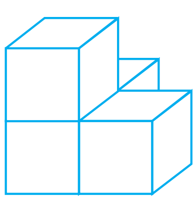
Visualising solid objects : When we look at a solid object, It is not necessary that the whole of it can be seen from one place. The view of the solid, also depend upon the direction from where it is seen. when some combined shapes are viewed, some of the shapes remains hidden from the viewer. There fore visualising solid shapes is a very useful skill through which we can see the hidden parts of a solid shape.



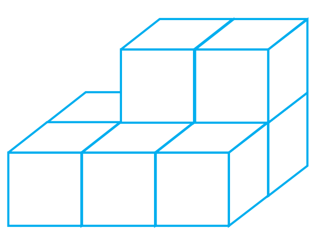
Look at above combined shape and how many cubes do you think have been used to make this structure. A little thinking will help you to find the right answer. This structure contains 10 cubes.

Example-1 : Count the number of cubes in the following structures

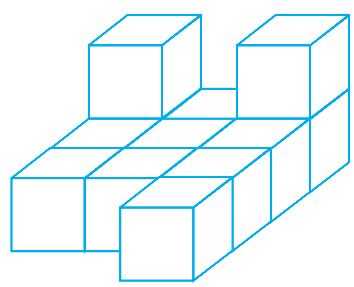
(i)



(ii)



(iii)

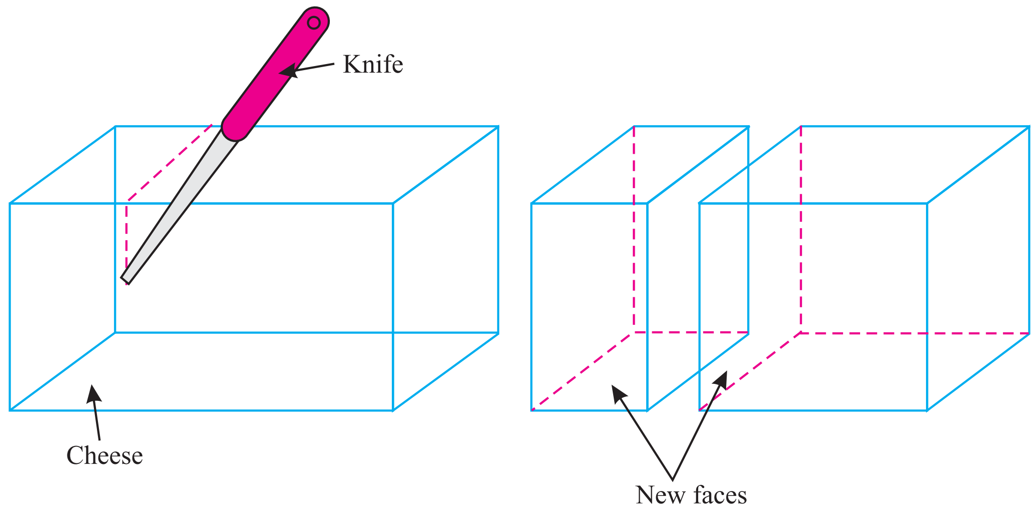


Sol. (*i*) 4 (*ii*) 8 (*iii*) 12

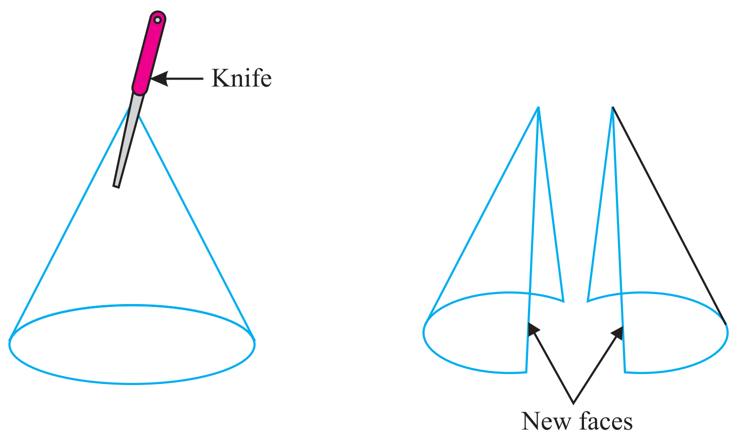
VIEWING DIFFERENT SECTIONS OF A SOLID

Different ways of viewing *a* 3-D object are

1. By cutting or slicing : A solid or *a* 3-D object can be cut into a number of parts. When we cut a 3-D object into two parts by using a knife, we get two new faces of solid. These new faces are called cross-sections of a solid. For example, If we cut a piece of cheese vertically we get two new faces as shown in figure.



Similarly, If we cut a solid cone vertically we get two new faces as shown in figure.



From above it is clear that when we give a cut we get a plane face. This plane face is called a 'Cross Section' and its boundary is a plane curve.

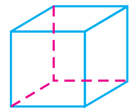
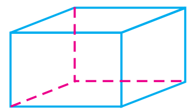
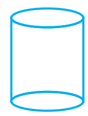
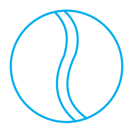
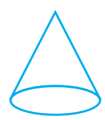
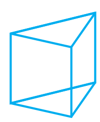
Example-2 : What cross section is made by (*i*) Vertical cut (*ii*) Horizontal cut in the following solid :

(*a*) Cube (*b*) Cuboid

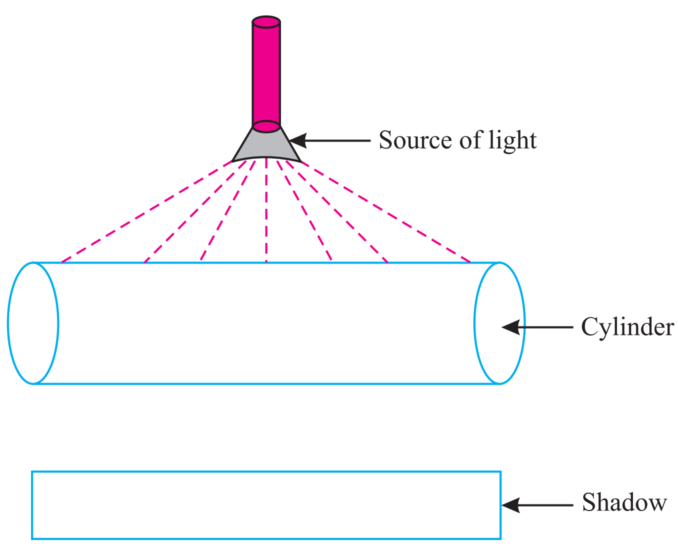
(*c*) Cylinder (*d*) Sphere

(*e*) Cone (*f*) Triangular prism

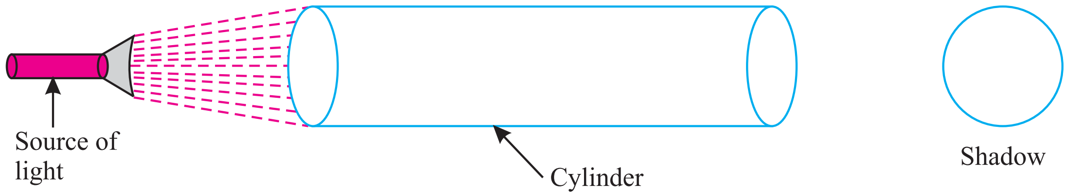
also draw rough sketch of the solids

	Name of solid	Sketch	Vetical Cut	Horizontal Cut
a	Cube		Square	Square
b	Cuboid		Rectangle	Rectangle
c	Cylinder		Rectangle	Circle
d	Sphere		Circle	Circle
e	Cone		Triangle	Circle
f	Triangular Prism		Rectangle	Triangle

2. By shadow of a 3-D object : The shadow of a 3-D object is a 2-D Image. The shadow of an object is not a fixed image with a change in the position of the source of light, the shadow of the object changes. If we throw light on the cylinder from the top we get a shadow in the shape of a rectangle.



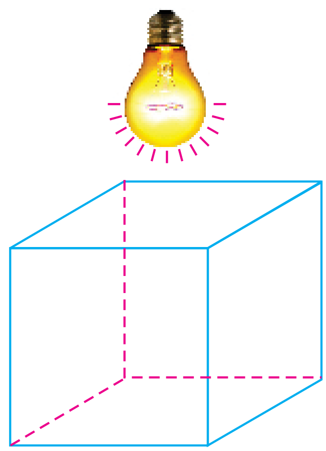
But, if we throw the light on a cylinder from left, we get the shadow in the shape of a circle.



We observe that the shadow of an object depends not only on different position of the solid but also on the position of the source of light.

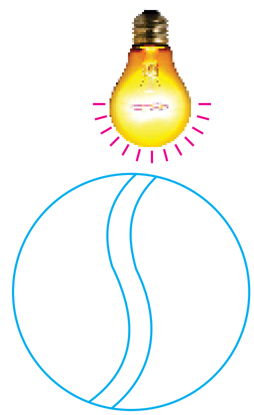
Example-3 : If we throw light on the following solids from the top. Name the shape of shadow obtained in each case and also give a rough sketh of the shadow.

(i)



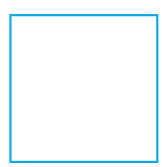
Cube

(ii)



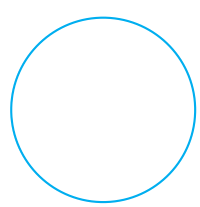
Ball

Sol. (*i*) Shadow of cube looks like a square.



Square

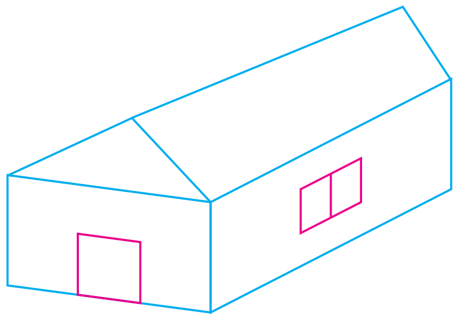
(*ii*) The shape of shadow of a ball looks like a circle.

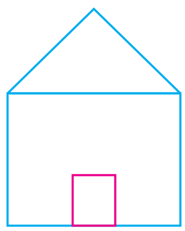


Circle

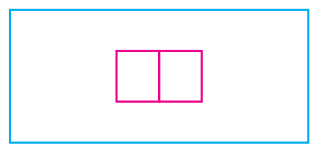
3. By looking at a solid from certain angles : Another way is to look at the shape from different angles i.e; the front view, the side view and the top view, which can provide a lot of information about the shape observed.

We can see three views i.e; front, side and top views of a house as follows.

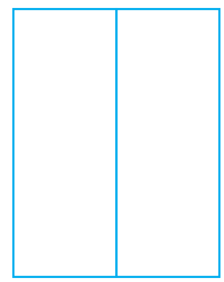




Front view



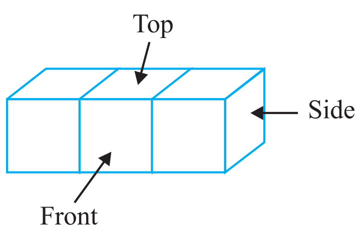
Side view



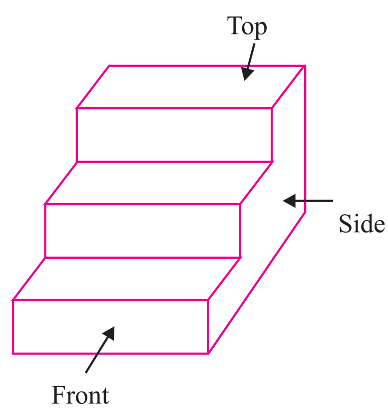
Top view

Example-4 : For the given solids sketch the front side and top view.

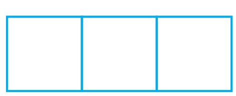
(i)



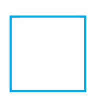
(ii)



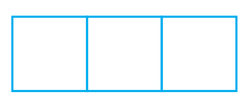
Sol. (i)



Front view



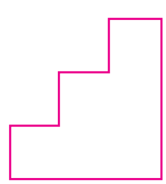
Side view



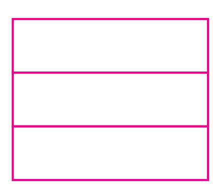
Top view

(ii)

Front view



Side view

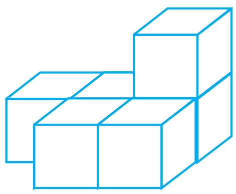


Top view

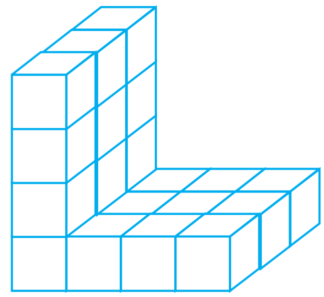
EXERCISE - 15.3

1. Count the number of cubes in each of the following figures.

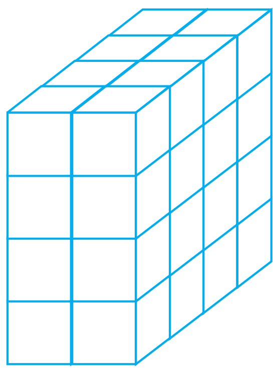
(i)



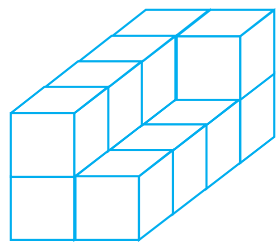
(ii)



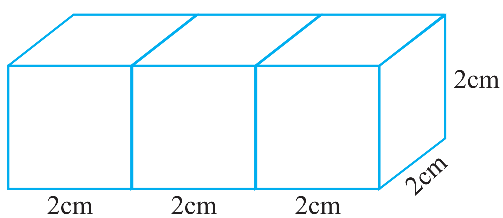
(iii)



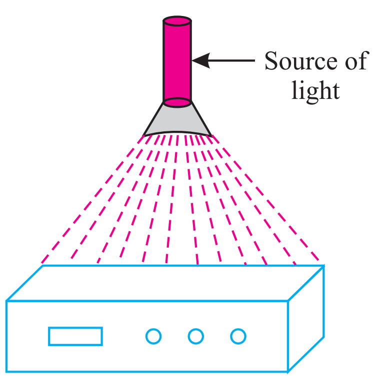
(iv)



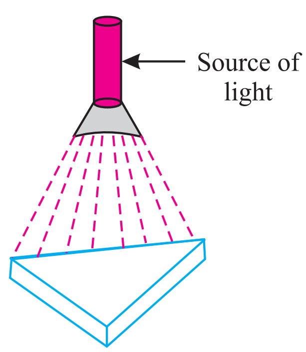
2. If three cubes of dimensions $2cm \times 2cm \times 2cm$ are placed side by side, what would be the dimensions of resulting cuboid ?



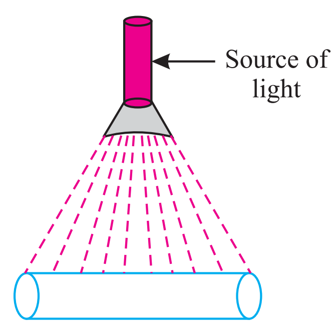
3. If we throw light on the following solids from the top name the shape of shadow obtained in each case and also give a rough sketch of the shadow.



(i) DVD player



(ii) Sandwich



(iii) Straw

3. What cross-sections do you get when you given a.

(*i*) Vertical cut (*ii*) Horizontal cut

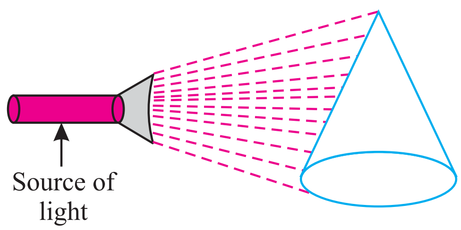
to the following solids ?

(*a*) A die (*b*) A square pyramid

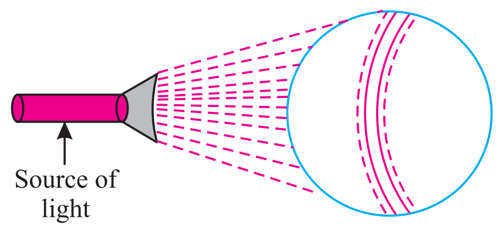
(*c*) A round melon (*d*) A circular pipe

(*e*) A brick (*f*) An ice cream cone

5. If we throw light on following solids, from left name the shape of shadow in each and also give a rough sketch of the shadow.

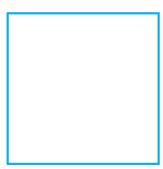


A cone

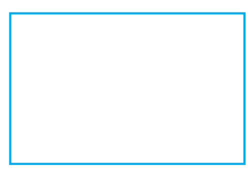


A ball

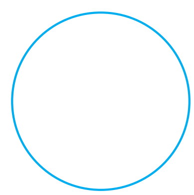
6. Here are the shadows of some 3 – D objects, when seen under the lamp of an overhead projector. Identify the solids that match each shadow (There may be multiple answers for these)



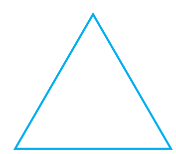
A Square



A Rectangle



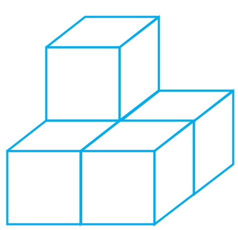
A Circle



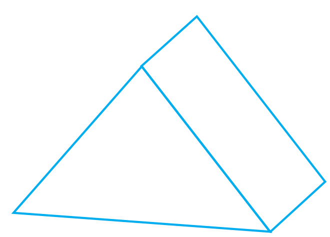
A Triangle

7. Sketch the front, side and top view of the following figures.

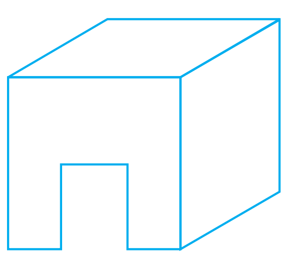
(i)



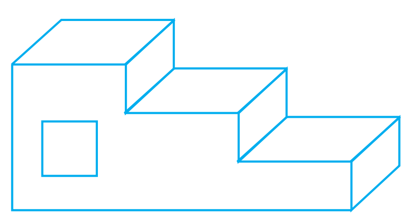
(ii)



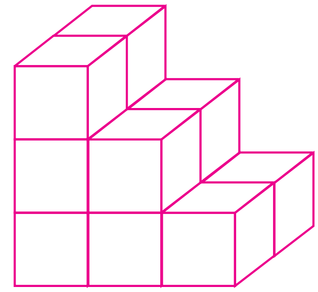
(iii)



(iv)



8. Multiple choice questions :-



(*i*) The number of cubes in the given structure is ?

(*a*) 12

(*b*) 10

(*c*) 9

(*d*) 8

(*ii*) The number of unit cubes to be added in above students to make a cuboid of dimensions 4 unit × 2 unit × 3 unit is ?

(*a*) 11 (*b*) 12

(*c*) 13 (*d*) 14

(*iii*) What cross-section is made by vertical cut in a cuboid

(*a*) Square (*b*) Rectangle

(*c*) Circle (*d*) Triangle

(*iv*) What cross section is made by horizontal cut in a cone

(*a*) Triangle (*b*) Circle

(*c*) Square (*d*) Rectangle

(*v*) Which solid cost a shadow of triangle under the effect of light

(*a*) Sphere (*b*) Cylinder

(*c*) Cone (*d*) Cube

WHAT HAVE WE DISCUSSED ?

1. The circle, the square, the rectangle, the quadrilateral and the triangle are examples of plane figures; the cube, the cuboid, the sphere, the cylinder, the cone and the pyramid are examples of solid shapes.

2. Plane figures are two dimensional (2–D) and solid shapes are three dimensional (3–D)

3. A face is a flat surface, An edge is where two faces meet and a vertex is a corner where edges meet and the plural of vertex is vetices.

4. Net is a two dimensional shape that can be folded to form a three dimensional shape or a solid. A solid can have several type of nets

5. Solid shapes can be drawn on a flat surface i.e, on paper, as 2–D representation of a 3–D shape. The two methods to draw sketches of 3–D shapes are :-

(*i*) **Oblique sketch :** An oblique sketch does not have proportional lengths, still it conveys all important aspects of the appearance of a solid.

(*ii*) An isometric sheet is a special sheet on which dots are formed on a pattern of equilateral triangles. In an isometric sketch measurement are kept proportional.

6. Visualising solid shapes is a very usefull skill through which we can seen the hidden parts of a solid shape. Different ways of viewing a 3–D object are :-

(*i*) **By cutting or slicing :** A solid or a 3–D object can be cut into a number of parts. When we cut it into two parts, we get two new faces called crossections of a solid.

(*ii*) **By shadow of a 3–D object :** In this way we observe a 2–D shadow of a 3–D shape.

(*iii*) **By looking at a solid from certain angles :** In this way we look at the shape from different angles, i.e. the front view, the side view and the top view. Which can provide a lot of information about the shape observed.

LEARNING OUTCOMES

After completion of the chapter, the students are now able to :

1. Relate 2–D shapes with 3–D shapes.
2. Identify and draw nets of various 3–D shapes and are able to form solids using these nets.
3. Count the faces, edges and vertices of solids (3–D Shapes)
4. Draw 3–D shapes on a flat surface by oblique and isometric sketch method.
5. See the hidden parts of a solid by cutting or slicing and will also be able to visualize solids from different angles.
6. Apply their knowledge about shapes in their day to day life.

ANSWERS

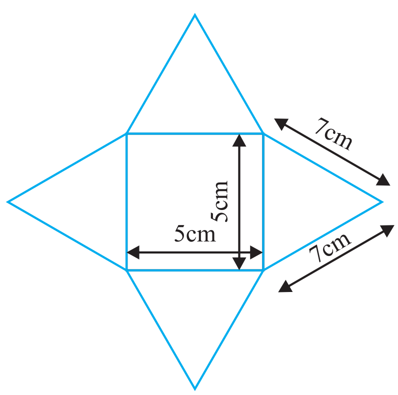
EXERCISE 15.1

1. (*i*) *e* (*ii*) *d*
(*iii*) *a* (*iv*) *b*
(*v*) *c*

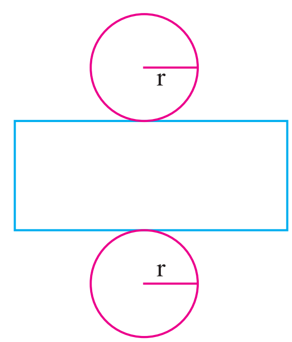
2. (*i*) *d* (*ii*) *e*
(*iii*) *a* (*iv*) *c*
(*v*) *b*

3. (*i*), (*iv*)

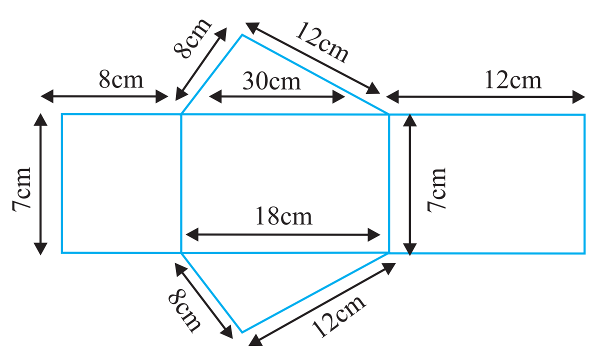
4.



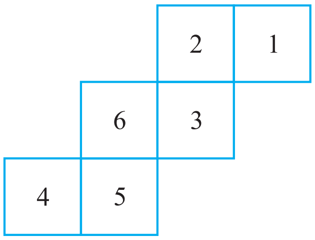
5.



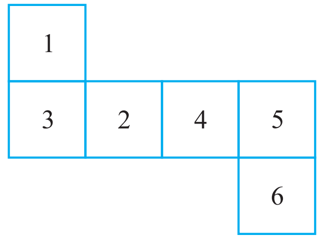
6.

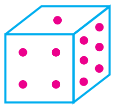


7. (i)



(ii)



8. Dice 

9. (*i*) Faces : 6 (*ii*) Edges : 2, vertices : NIL (*iii*) Faces : 7, Edges : 15 (*iv*) faces : 5, vertices : 5

10. (*i*) *c* (*ii*) *d* (*iii*) *b* (*iv*) *b* (*v*) *c*

EXERCISE 15.2

8. (*i*) *b* (*ii*) *c* (*iii*) *c*
(*iv*) *d* (*v*) *c*

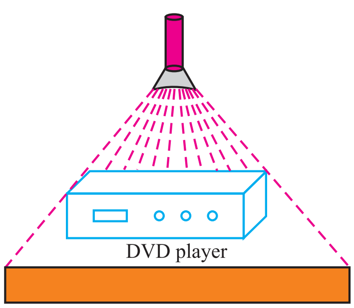
EXERCISE 15.3

1. (*i*) 6 (*ii*) 21
(*iii*) 32 (*iv*) 13

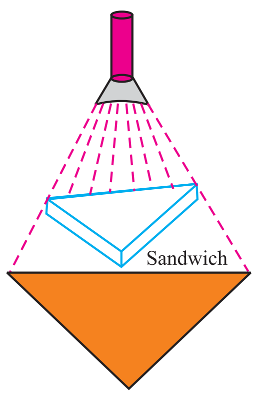
2. Length 6*cm*, breadth 2*cm* and height 2*cm* (3) (*a*) Svuare, Svuare (*b*)Triangle, Svuare.

3. Circle, circle (*d*) Circle, Rectangle (*e*) Rectangle, Rectangle (*f*) Triangle, Circle

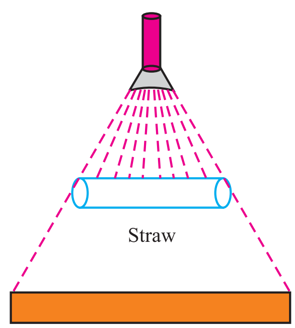
4.



(i) Rectangle

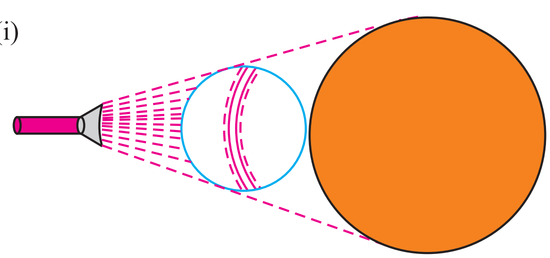


(ii) Triangle

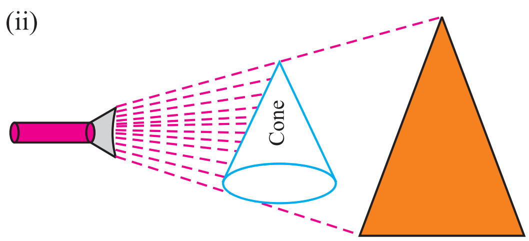


(iii) Rectangle

5. (i)



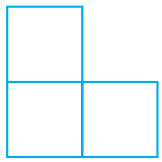
(ii)

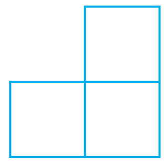


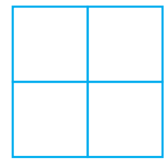
6. (*i*) Dice, chalk box etc (*ii*) Book, Mobile, DVD player etc

(*iii*) Cricket ball, Disc etc (*iv*) Birthday cap, Icecream cone etc.

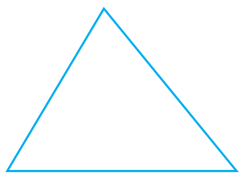
7. (i)

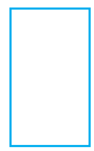

Front view

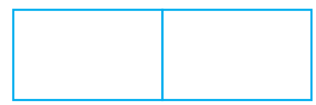

Side view


Top view

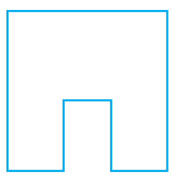
(ii)

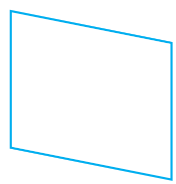

Front view

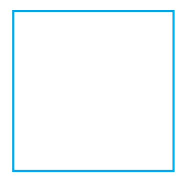

Side view


Top view

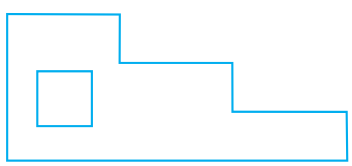
(iii)

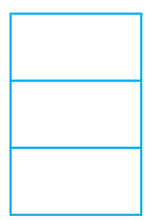

Front view

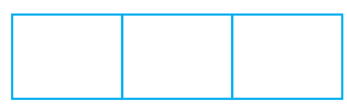

Side view


Top view

(iv)


Front view


Side view


Top view

8. (*i*) *a* (*ii*) *b* (*iii*) *b*

(*iv*) *b* (*v*) *c*

ISOMETRIC SHEET

ISOMETRIC SHEET